

been already proved in the seventh proposition. Having proved that *from a point which is not the centre three equal straight lines cannot be drawn to the circumference of a circle* (Prop. 7), it was wholly unnecessary to prove that *the point from which three equal straight lines can be drawn to the circumference must be the centre of the circle* (Prop. 9).

The two theorems are, in fact, contra-positive forms, one of the other; the truth of each is implied, when that of the other is asserted, and to demonstrate both geometrically is more than superfluous; it is a mistake, since the true relation between the two is thereby masked. There can be no better proof of this than the fact that the above defect in exposition remained undetected for centuries. Another, though less striking, example of the same kind is presented by the 16th and 27th propositions of the first book. Few intelligent boys fail on first reading the 27th to note the oddity of giving to two parallel lines a dagger-like shape in order to prove indirectly that "if a straight line falling on two other straight lines make the alternate angles equal to each other, these two straight lines shall be parallel." It is certain, however, that few of them ever discover that the proposition has virtually been proved before, that it is in fact the contra-positive form of the 16th, since the latter is obviously susceptible of being thus enunciated: "If two straight lines meet one another, a straight line falling on them will not make the alternate angles equal."

The late Prof. de Morgan, to whose keen penetration we owe the detection, not merely of the above defects in Euclid, but of many others, strongly and justly insisted upon the necessity of a more logical study of the elements of geometry.

I do not advocate the introduction of more *formal* logic into elementary geometry, but simply the cultivation of a logically severer habit of thought, and the more frequent application of those simple rules of reasoning by means of which tedious reiteration may be so often obviated, and, as a consequence, clearness of insight promoted. As an instance of such a rule I may mention that very useful one according to which "the converse of each of a series of demonstrated theorems is necessarily true if of their several hypotheses, as well as of their predicates, it can be said that one must be true, and that no two of them can be so at the same time." A conviction of the general validity of this rule is readily imparted, even to your pupils, by first selecting familiar instances and then generalising; and, once imparted, they are put in possession of the instrument whereby converse propositions in geometry are most frequently and satisfactorily established.

In conclusion, I may observe that it is chiefly by the aid of general rules, such as those just alluded to, that the mechanical details of demonstration become sufficiently subordinated to allow a complete grasp of the whole subject to be acquired; they serve, in fact, as the thread on which the isolated propositions of geometry, like beads, have to be strung before they can be properly viewed.

THE YELLOWSTONE PARK

THE following, reprinted from the "Reports to Congress" of the United States, will serve to show the zeal displayed by the American Government for the improvement of the people. We regret that we are unable to reproduce the accompanying maps:—

"The Bill now before Congress has for its object the withdrawal from settlement, occupancy, or sale, under the laws of the United States, a tract of land fifty-five by sixty-five miles, about the sources of the Yellowstone and Missouri Rivers; and dedicates and sets it apart as a great national park or pleasure-ground for the benefit

and enjoyment of the people. The entire area comprised within the limits of the reservation contemplated in this Bill is not susceptible of cultivation with any degree of certainty, and the winters would be too severe for stock-raising. Whenever the altitude of the mountain districts exceed 6,000ft. above tide-water, their settlement becomes problematical unless there are valuable mines to attract people. The entire area within the limits of the proposed reservation is over 6,000ft. in altitude, and the Yellowstone Lake, which occupies an area 15 miles by 22 miles, or 330 square miles, is 7,427ft. The ranges of mountains that hem the valleys in on every side rise to the height of 10,000ft. and 12,000ft., and are covered with snow all the year. These mountains are all of volcanic origin, and it is not probable that any mines or minerals of value will ever be found there. During the months of June, July, and August, the climate is pure and most invigorating, with scarcely any rain or storms of any kind; but the thermometer frequently sinks as low as 26°. There is frost every month of the year. This whole region was in comparatively modern geological times the scene of the most wonderful volcanic activity of any portion of our country. The hot springs and the geysers represent the last stages—the vents or escape-pipes—of these remarkable volcanic manifestations of the internal forces. All these springs are adorned with decorations more beautiful than human art ever conceived, and which have required thousands of years for the cunning hand of nature to form. Persons are now waiting for the spring to open to enter in and take possession of these remarkable curiosities, to make merchandise of these beautiful specimens, to fence in those rare wonders so as to charge visitors a fee, as is now done at Niagara Falls, for the sight of that which ought to be as free as the air or water.

"In a few years this region will be a place of resort for all classes of people from all portions of the world. The geysers of Iceland, which have been objects of interest for the scientific men and travellers of the entire world, sink into insignificance in comparison with the hot springs of the Yellowstone and Fire-Hole Basins. As a place of resort for invalids it will not be excelled by any portion of the world. If this Bill fails to become a law this session, the Vandals who are now waiting to enter into this wonderland will, in a single season, despoil beyond recovery these remarkable curiosities which have required all the cunning skill of nature thousands of years to prepare.

"We have already shown that no portion of this tract can ever be made available for agricultural or mining purposes. Even if the altitude and the climate would permit the country to be made available, not over fifty square miles of the entire area could be settled. The valleys are all narrow, hemmed in by high volcanic mountains like gigantic walls.

"The withdrawal of this tract, therefore, from sale or settlement takes nothing from the value of the public domain, and is no pecuniary loss to the Government, but will be regarded by the entire civilised world as a step of progress and an honour to Congress and the nation.

Department of the Interior,
Washington, D. C., January 29, 1872

Sir,—I have the honour to acknowledge the receipt of your communication of the 27th instant relative to the Bill now pending in the House of Representatives dedicating that tract of country known as the Yellowstone Valley as a national park.

I hand you herewith the report of Dr. F. V. Hayden, United States geologist, relative to said proposed reservation, and have only to add that I fully concur in his recommendations, and trust that the Bill referred to may speedily become a law.

Very respectfully, your obedient servant,
C. DELANO, Secretary.

Hon. M. H. Dunnell, House of Representatives.

"The committee therefore recommend the passage of the bill without amendment."