For the last two years he has been engaged in carrying through the press, for the Indian Government, a photo-lithographic edition of the "Mahâbhâshya," of which 300 pages still remain to be done. By his decease, what may be called the "traditional" school of Vedic criticism, which gives to the interpretations of native tradition the preference over those derived from comparative philology, ceases to have a European representative. His manuscript of a Sanskrit grammar has long been finished, and it is hoped that this work, which is likely to revolutionise the teaching of Sanskrit in many respects, may be allowed to see the light. The great psychological value as an educational instrument which he attached to the Sanskrit language, if properly taught, was well known to his friends; and it was through his advocacy that a committee of the professors of University College, London, was appointed to report on the desirableness of making Sanskrit an integral part of all the degree examinations in the University of London.

Of the philosophical literature of India, the "Mîmânsa," from its close connection with grammatical researches, engaged his chief attention; some fruit of his labours in this field is a nearly finished edition, prepared for the Sanskrit Society, of Mâdhava's "Jaiminîya-nyâya-mâlâ-vistara" (1865).

It was however Goldstücker's thorough familiarity with the legal and ceremonial literature of the Hindus which rendered his advice of so much value to the Indian Government. A paper recently published by him "On the Deficiencies in the Present Administration of Hindu Law" (Trübner, 1871), contains an exposure of the frequent failures of justice arising from the misunderstandings of native codes, which disgrace our Indian administration.

Besides some papers in the *Reader* and the *Athenæum*, Goldstücker contributed an excellent essay on the "Mahâbhârata" to the *Westminster Review* in April 1868; and among his papers will be found a copy of the great Eastern epic collated with the best European MSS. His library is, we are glad to hear, to be kept together.

Dr. Goldstücker was Professor of Sanskrit in University College, London, President of the Philological Society, a member of the Council of the Asiatic Society and of the Association of the Friends of India.

REPORT OF THE ASSOCIATION FOR THE IMPROVEMENT OF GEOMETRICAL TEACHING

AT the Second Annual Meeting of this Association, held at University College, London, on January 12, Dr. Hirst, the president of the association, delivered the following address:—

In opening the proceedings of this, the Second Annual Meeting of the Association for the Improvement of Geometrical Teaching, I am glad to be able to congratulate you on the decided progress which has been made during the past year towards the realisation of your views. The discussions recorded in English journals, and the reception given to recently published text-books on geometry, unquestionably indicate that public opinion is far more inclined now than it was a few years ago to entertain the notion of an improved exposition of the elements of geometry. We are no longer warned that to touch that edition of Euclid to which, for more than a century, we have paid such literal homage, would be to ruin the teaching of geometry. On the contrary, it is now generally admitted that, without departing from the admirable exactitude and geometrical purity of Euclid's elements, we ought to be able, by judicious revision and extension, to bring them more into harmony with the scientific methods and the habits of thought of our own day. I alluded last year to the retrograde step that had been taken in Italy on this question of the teaching of geometry. The announcement excited much interest in England, though the true purport of the Italian movement was, I fear, slightly misunderstood. I have, therefore, thought it my duty to procure original documents, to make inquiries into the success of the Italian movement of 1867, and also to ascertain the present aspect of geometrical instruction in that country. I hold in my hand the historically interesting document which was issued by the Italian Government in 1871. It contains instructions and programmes relative to the teaching of mathematics in their *Ginnasi* and *Licei*.* Before quoting it I may observe that the *Ginnasio* is essentially a classical school, mathematics being studied only in its fifth or highest class, and then only for five hours a week; and that in the *Liceo* the instruction is still to a great extent classical, though less exclusively so. Here, as the pupil advances through its three classes, mathematics, physics, natural history, and philosophy become more and more prominent as subjects of study. The instructions, as already observed, relate solely to the teaching of mathematics in these classical schools; nevertheless, the following introductory remarks on the objects of mathematical study are, I venture to think, applicable to all schools in which the foundation of a truly liberal education is to be secured: "Mathematics should not be looked upon as a mere collection of intrinsically useful propositions or theorems of which boys ought to acquire a knowledge in order to be able to apply them subsequently to the practical purposes of life. The study should be regarded principally as a means of intellectual culture, directed towards the development of the faculty of reasoning, and to the strengthening of that just and healthy judgment which serves as the light whereby we distinguish truth from that which has but the semblance thereof."

After describing the course of instruction in arithmetic and algebra best suited to the end in view, the document before me proceeds thus:—"In order to give to the instruction in geometry its maximum intellectual efficacy, and at the same time to bring the subject-matter within reasonable limits, it will suffice to follow, in our schools, the example of English ones by returning to the elements of Euclid, universally admitted to be the most perfect model of geometrical rigour." It would be a grave error to suppose that it was the good results on geometrical teaching of our adherence to the elements of Euclid that induced the Italians to return to them. Although England is made, in some measure, responsible for the step taken, we know from sources alluded to in my address last year that the main object in taking it was to purge from Italian schools the many worthless books which private enterprise had succeeded in introducing, and by no other means than the one adopted could the Italian Government, in the opinion of their advisers, have achieved this end with sufficient promptitude and impartiality.

The real motive of the order issued in 1867 is a little more apparent in the following passage from the Instructions, wherein allusion is made to the practice, then prevalent, of striving after a deceptive facility of treatment by the introduction of algebraical processes in place of geometrical reasoning: "The instruction in geometry is to extend to the first six, and to the eleventh and twelfth, books of Euclid, and to be followed by lessons on the most essential propositions of Archimedes relating to the measure of the circle, of the cylinder, of the cone, and of the sphere. Taught by the method of the ancients, geometry is easier and more attractive than the abstract science of number; hence, instead of postponing geometry to algebra, one part of the subject (the first book) is assigned to the fifth class in the *Ginnasio*, and another (the second and third book) to the first class of the *Liceo*. The teacher is recommended to adhere to the method of

* Instruzione e Programmi, per l'Insegnamento della Matematica nei Ginnasi e nei Licei, approvati con R. Decreto, 10 Ottobre, 1867.

Euclid, as the one best fitted to establish in the youthful mind the habit of thoroughly rigorous reasoning; above all, he is not to impair the purity of the ancient geometry by transforming geometrical theorems into algebraic formulæ, that is to say, by substituting in place of concrete magnitude—such as lines, angles, superfices, volumes—their respective measures; on the contrary he is to accustom his pupils to reason always on the magnitudes themselves even when their ratios are under contemplation. It is only after the propositions of Euclid and of Archimedes, mentioned in the programme, have been mastered that formulæ are to be deduced for practically determining the areas of rectilineal figures, the area of the circle, the length of its circumference, and the magnitudes of the surfaces and volumes of prisms, pyramids, cylinders, cones, and spheres."

The measures taken by the Italian Government in 1867 have, I am informed, fully answered the expectations of the mathematicians who recommended them. A taste for rigorous and purely geometrical methods has been revived, and the ground has been cleared for further advances. That such advances were contemplated from the first is obvious from the following passages, with which the Professors Betti and Brioschi—two of the most distinguished mathematicians of Italy—concluded their preface to the new edition (based on that of Viviani) of the Elements of Euclid, with which classical schools were supplied in 1867 · "Profoundly convinced that it is only through the eminent qualities of precision and clearness which distinguished Euclid's Geometry that we can hope, in seeking to promote the intellectual development of our youth, to secure those results at which all civilised nations aim when they give to geometrical instruction so important a place in public instruction, we have undertaken the publication of an edition of the elements with the fixed intention of improving it whenever new foreign publications and the experience gained in our own schools shall have shown that improvements are desirable. We trust that professors in *Licei* will help us in this work. We shall gratefully accept their observations and suggestions."

Experience, however, has gone further than was here anticipated; already there appears to be a demand for something beyond a revision of Viviani's edition of Euclid's Elements. In the *Gazzetta Ufficiale* of the kingdom of Italy, published at Rome, I find that on the 2nd of December last an announcement was made by the authority of the Minister of Public Instruction, to the effect that in 1873 a prize of 2,500 lire (about 100*l*) would be given to the author of the best "Treatise on Elementary Geometry which shall adhere rigorously to the method of Euclid, and contain, besides the subject-matter in the programme of 1867, those portions of the science, developed since Euclid's time, which are now to be found in all elements of geometry adopted as text-books in the classical schools of the most cultured nations." I forbear to attempt to determine what would be the rank of England amongst cultured nations if she were judged by this standard of the introduction of post-Euclidean matter into school text-books. I prefer to see in the announcement merely an encouragement to proceed with our self-imposed task of endeavouring to bring up the teaching of geometry and the text-book we employ to the level of the science of our day. In Italy this can be done more promptly than in England. Our Government cannot, with a stroke of the pen, alter the entire character of the instruction given in English schools. With us improvements are of slower growth, and it is by operations less surgical in their character that obstructions to their growth have first to be removed. It is, in fact, the function of associations like our own to endeavour to remove unreasonable prejudices against changes in the English habit of teaching geometry by bringing prominently forward the defects which we find to exist, and the improvements which we desire to see introduced. Let me now turn, therefore, to the work done by this association during the past year. You will recollect that members were invited to prepare programmes and syllabuses of text-books on geometry in accordance with their own views. The primary object in making this request was to ascertain what amount of unanimity at present prevails amongst teachers. The invitation was accepted by many, and the syllabuses received were referred to two committees, one meeting at London and the other at Rugby. Although the occupations of many of us, and our distances asunder, rendered it very difficult to secure concerted action, a report has at length been prepared, and will be this day submitted to you. With respect to the resolutions and recommendations embodied in this report, I will for the present confine myself to the statement that the main object they are intended to further is a practically useful degree of conformity amongst teachers during the present transitional state of matters. No attempt has been made to prepare any detailed scheme or programme of elementary geometrical study. This last difficult task, however, although postponed, is not, as you will hereafter see, abandoned.

Although the assertion may partake of the character of a truism, it cannot be too often insisted upon, that however necessary it may be to have good text-books, it is far more necessary to have good teachers; that, in fact, good text-books are useful principally by the aid they render in forming good teachers and in furnishing students with an accurate record of what they have been taught. In teaching, one might say, there is *vis viva*—actual energy; whereas in a text-book, however good it may be, the disciplinal energy is at most potential. The text-book, indeed, to be properly used, should always be subordinated to the teaching; but to do this it is absolutely essential that the teacher should, by his own study, have risen not merely up to, but above, the level of the text-book he employs. Until he has so mastered the subject that it has become plastic in his hands, his teaching must necessarily remain defective; for geometrical truth, it must be remembered, has, like all other truth, many sides, and no text-book can present all, or necessarily the one which, to individual pupils, is the most accessible. Alternative methods of demonstration, inquiries into the interdependence of propositions, judicious variation of data, and just discrimination between the contingent and necessary properties of figures; these and numerous other matters, all essential to geometrical culture, can only be properly supplied by the teacher; no text-book could be weighted with them. Above all, it is to him that we must mainly look for the cultivation of that scientific method of inquiry under whose guidance solely problem-solving can be raised in character above what has been termed "exalted conundrum guessing," and acquire its full educational value.

The interdependence of geometrical propositions above alluded to, as one of the subjects to which teachers should habitually direct the attention of their pupils, is mainly logical in character, but nevertheless most essential to geometrical culture. Every one will admit the primary importance of habituating the student to extract its full logical significance from every proposition he establishes, to recognise each proposition readily under different, although logically equivalent forms of enunciation, and thus to discriminate accurately between the cases where mere *logical* deduction from antecedent propositions is requisite, from those which require the introduction of further *geometrical* considerations. Obvious as this may be, it is rarely sufficiently attended to by teachers, and even in approved text-books, ancient as well as modern, we not unfrequently find remarkable instances of the absence of the discrimination to which I refer. The ninth proposition of the third book of Euclid is now a well-known case of the kind. Geometrical apparatus is there employed to demonstrate, indirectly, what had virtually

been already proved in the seventh proposition. Having proved that *from a point which is not the centre three equal straight lines cannot be drawn to the circumference of a circle* (Prop. 7), it was wholly unnecessary to prove that *the point from which three equal straight lines can be drawn to the circumference must be the centre of the circle* (Prop. 9).

The two theorems are, in fact, contra-positive forms, one of the other; the truth of each is implied, when that of the other is asserted, and to demonstrate both geometrically is more than superfluous; it is a mistake, since the true relation between the two is thereby masked. There can be no better proof of this than the fact that the above defect in exposition remained undetected for centuries. Another, though less striking, example of the same kind is presented by the 16th and 27th propositions of the first book. Few intelligent boys fail on first reading the 27th to note the oddity of giving to two parallel lines a dagger-like shape in order to prove indirectly that "if a straight line falling on two other straight lines make the alternate angles equal to each other, these two straight lines shall be parallel." It is certain, however, that few of them ever discover that the proposition has virtually been proved before, that it is in fact the contra-positive form of the 16th, since the latter is obviously susceptible of being thus enunciated: "If two straight lines meet one another, a straight line falling on them will not make the alternate angles equal."

The late Prof. de Morgan, to whose keen penetration we owe the detection, not merely of the above defects in Euclid, but of many others, strongly and justly insisted upon the necessity of a more logical study of the elements of geometry.

I do not advocate the introduction of more *formal* logic into elementary geometry, but simply the cultivation of a logically severer habit of thought, and the more frequent application of those simple rules of reasoning by means of which tedious reiteration may be so often obviated, and, as a consequence, clearness of insight promoted. As an instance of such a rule I may mention that very useful one according to which "the converse of each of a series of demonstrated theorems is necessarily true if of their several hypotheses, as well as of their predicates, it can be said that one must be true, and that no two of them can be so at the same time." A conviction of the general validity of this rule is readily imparted, even to your pupils, by first selecting familiar instances and then generalising; and, once imparted, they are put in possession of the instrument whereby converse propositions in geometry are most frequently and satisfactorily established.

In conclusion, I may observe that it is chiefly by the aid of general rules, such as those just alluded to, that the mechanical details of demonstration become sufficiently subordinated to allow a complete grasp of the whole subject to be acquired; they serve, in fact, as the thread on which the isolated propositions of geometry, like beads, have to be strung before they can be properly viewed.

THE YELLOWSTONE PARK

THE following, reprinted from the "Reports to Congress" of the United States, will serve to show the zeal displayed by the American Government for the improvement of the people. We regret that we are unable to reproduce the accompanying maps:—

"The Bill now before Congress has for its object the withdrawal from settlement, occupancy, or sale, under the laws of the United States, a tract of land fifty-five by sixty-five miles, about the sources of the Yellowstone and Missouri Rivers; and dedicates and sets it apart as a great national park or pleasure-ground for the benefit and enjoyment of the people. The entire area comprised within the limits of the reservation contemplated in this Bill is not susceptible of cultivation with any degree of certainty, and the winters would be too severe for stock-raising. Whenever the altitude of the mountain districts exceed 6,000ft. above tide-water, their settlement becomes problematical unless there are valuable mines to attract people. The entire area within the limits of the proposed reservation is over 6,000ft. in altitude, and the Yellowstone Lake, which occupies an area 15 miles by 22 miles, or 330 square miles, is 7,427ft. The ranges of mountains that hem the valleys in on every side rise to the height of 10,000ft. and 12,000ft, and are covered with snow all the year. These mountains are all of volcanic origin, and it is not probable that any mines or minerals of value will ever be found there. During the months of June, July, and August, the climate is pure and most invigorating, with scarcely any rain or storms of any kind; but the thermometer frequently sinks as low as 26°. There is frost every month of the year. This whole region was in comparatively modern geological times the scene of the most wonderful volcanic activity of any portion of our country. The hot springs and the geysers represent the last stages—the vents or escape-pipes—of these remarkable volcanic manifestations of the internal forces. All these springs are adorned with decorations more beautiful than human art ever conceived, and which have required thousands of years for the cunning hand of nature to form. Persons are now waiting for the spring to open to enter in and take possession of these remarkable curiosities, to make merchandise of these beautiful specimens, to fence in those rare wonders so as to charge visitors a fee, as is now done at Niagara Falls, for the sight of that which ought to be as free as the air or water.

"In a few years this region will be a place of resort for all classes of people from all portions of the world. The geysers of Iceland, which have been objects of interest for the scientific men and travellers of the entire world, sink into insignificance in comparison with the hot springs of the Yellowstone and Fire-Hole Basins. As a place of resort for invalids it will not be excelled by any portion of the world. If this Bill fails to become a law this session, the Vandals who are now waiting to enter into this wonderland will, in a single season, despoil beyond recovery these remarkable curiosities which have required all the cunning skill of nature thousands of years to prepare.

"We have already shown that no portion of this tract can ever be made available for agricultural or mining purposes. Even if the altitude and the climate would permit the country to be made available, not over fifty square miles of the entire area could be settled. The valleys are all narrow, hemmed in by high volcanic mountains like gigantic walls.

"The withdrawal of this tract, therefore, from sale or settlement takes nothing from the value of the public domain, and is no pecuniary loss to the Government, but will be regarded by the entire civilised world as a step of progress and an honour to Congress and the nation.

Department of the Interior,
Washington, D. C., January 29, 1872

Sir,—I have the honour to acknowledge the receipt of your communication of the 27th instant relative to the Bill now pending in the House of Representatives dedicating that tract of country known as the Yellowstone Valley as a national park.

I hand you herewith the report of Dr. F. V. Hayden, United States geologist, relative to said proposed reservation, and have only to add that I fully concur in his recommendations, and trust that the Bill referred to may speedily become a law.

Very respectfully, your obedient servant,
C. DELANO, Secretary.

Hon. M. H. Dunnell, House of Representatives.

"The committee therefore recommend the passage of the bill without amendment."