

as all astronomers know, by the *direction* of the line of nodes, not by its *actual place*.

By-the-bye, Sir John Herschel is sometimes very careful to use the words "actual place" where my critics contend that the word "position" would be sufficiently definitive.

It seems overlooked that I pointed out in the beginning that "position" was often but erroneously used as synonymous with "place." It is not my fault if this error appears in the technical use of the word "position" in some mathematical treatises. I say again with Colonel Manning, *Abusus non tollit usum*—"The abuse of anything doth not abrogate the lawful use thereof." It was a *lapsus calami* of mine to say that "position" could not be misunderstood. It could be, for it has been misused.

Prof. Hirst is quite right in saying I should be unable to describe the aspect of a horizontal plane. I should not think of trying to. He says, however, that Mr. Wilson would unhesitatingly pronounce its aspect *vertical*. (Does it look vertically *up* or vertically *down*?) What would Mr. Wilson assign—unhesitatingly or otherwise—as the aspect of the "prime vertical"?

Has a true plane (as distinguished from a plane face of a solid) one aspect or two? It has one *position* or *situation*, and one *place* or *location*, but I conceive that it has two aspects.

Mr. Laughton seems quite unaware of Sir J. Herschel's repeated use of the word "tilt."

His comment on my remark about the books which I have written is unworthy. He must surely perceive that I only sought to indicate how much occasion I had had to consider the subject of plane-position; more occasion, I think, than any of my critics, save Prof. Hirst, the weight of whose opinion I recognise fully, though I cannot agree with him. But I have *not* felt free to use the word "position" so systematically as I should wish, precisely because of its misuse to indicate *place*. I have only been able to use it where there could be no fear of that wrong meaning being assigned to it.

As I claim no credit for the invention of any word for indicating plane-position, and as I could not take from Mr. Laughton that which is not his—the credit for Hamilton's word "aspect"—perhaps I may be permitted to say that if I am "pertinacious" (as Mr. Laughton asserts) there is nothing personal in my pertinacity. It is not my custom to admit that I am wrong when I consider that I am right.

[My objections to the word "aspect" are confirmed by Mr. Wilson's letter. I wrote that the word could not be used in the sense indicated, "unless a new and artificial meaning were assigned to it." Mr. Wilson obligingly proves this by assigning to it just such a meaning. "The aspect of a plane is the direction of its normal," it would seem. Now no special objection need be urged against this definition, if it is to be confined rigidly within the limits of mathematical text-books. The definition is strange and artificial no doubt; but it is nothing new to see the familiar and natural banished from such works. As a writer on astronomy, however, I must decline to accept the proposed usage, which seems to me altogether objectionable. If I write respecting the celestial equator-plane that "its position is at right angles to the polar axis of the heavens," I find that I am understood; but I am sure my readers would be very much perplexed if I wrote that "the aspect of the equator-plane is the direction of the polar axis." Again, I should be understood, I think, if I said that "the positions of two hour-planes determine the direction of the polar axis," or that "the directions of the polar axis and the vertical determine the position of the meridian-plane." But if I wrote "aspect" where I have here written "position," I scarcely know what my readers would think.

By the way, what would be the "aspect" of the meridian-plane according to the proposed usage? Would it be "east" or "west"? The normal to that plane would lie east and west; but we could not hear of an "east-and-west" aspect without thinking of certain "clear stories towards the south-north, lustrous as ebony."

I am bound to point out, however, though I may seem to weaken my position by doing so, that a very eminent authority long since used the word "aspect" in the sense suggested by Mr. Laughton. In one of his well-known "Letters to a Lady," on quaternions, Sir W. R. Hamilton uses the words "position," "slope," "ledge," and "aspect," to express the relations which I have called respectively "place," "slope," "aspect," and "position." (See Nichol's "Cyclopædia of the Physical Sciences," 2nd edition, p. 708.) I app eh-nd, however, that he lays no special stress on this verbiage. He had used the word "position" for "place," and this left him without any word to indicate position. Besides, his illustrative plane is the surface of

a desk, and a surface may be conceived to have an aspect definable by the direction of its normal, but a geometrical plane is two-faced.]*

This is my last letter on the present subject—unless one of your correspondents should employ arguments showing me to be in error, in which case I shall crave two lines of your space to admit as much.

RICHD. A. PROCTOR

Brighton, Nov. 3

P.S.—Let it be noticed that the question is not how the word "position" has been used by some, but how it ought to be used by all.

I CANNOT agree with Mr. Wilson that "aspect" is *exactly* the word wanted. The same wall has *two* aspects; if a southern, then also a northern aspect on the other side. In fact the word seems adapted, according to its common usage, to express the "sense" (*sens*), as well as the direction of the plane's normal, whereas I take it that the word sought for should express the direction only without connoting the "sense."

I think a word sometimes used by geologists would be, if we dare use it, exactly the word. As they speak of the *lie* of strata, defined (with respect to the horizon) by its two elements, *strike* and *dip*, so geometers might well speak of the *lie* of a plane; but would our English language permit us to say that "two lies determine one direction," and "two directions determine one lie"? I fear the moral connotation of the word, although an etymological accident, is too ugly.

If we are reduced to coin a new word, I would suggest that the Latin root "pand" (spread), would afford for a plane the fitting analogue of the root "reg" (rule, make straight), for a line, and so the word "dispansion" would be the analogue of "direction." "Parallel planes have the same dispersion." "Two dispansions determine one direction, and two directions determine one dispersion." Will not the neatness of this mode of expressing Mr. Wilson's test propositions atone for the strangeness of the word?

The word "aspect," however, is too good to be rejected from geometrical science, though I believe its chief use will be found beyond the domain of pure geometry. Should it not be appropriated to cases where the plane presents different aspects to the portions of space on either side of it? For instance, if two bodies revolve in the same or parallel planes, their orbits might be said to have the same or contrary aspects, according as the bodies revolve in the same or contrary directions, and so the positive aspect of a planet's orbit would determine, not only the "lie" or "dispansion" of the plane of the orbit, but also the direction of revolution in that orbit. So, too, the statement that all the planetary orbits have nearly the same aspect, would imply not only that their planes nearly coincide, but also that they all revolve in the same direction. I cannot help thinking that Mr. Proctor would find his account in adopting this sense of the word "aspect" in his astronomical writings, especially since he might, as Dr. Hirst suggests, retain the word where he has hitherto employed it, by simply qualifying it with an appropriate adjective. (Would the adjective "azimuthal" satisfy him?)

May I conclude with a question which I have often wished to propound? What is the proper English equivalent for the French "*sens*"? English mathematicians generally seem shy of using the word "sense," while, to use the word "direction" as well for the "*sens*" as the "direction" of a line, is very awkward and inconvenient. The difficulty, I imagine, is the same as appears to me almost fatal to the word "lie" proposed above, namely, that the proposed technical use diverges too widely from the familiar use of the word. Is not the superior flexibility of the German language in the formation of new terms in part due to a less degree of fastidiousness in this respect?

Harrow, Nov. 6

ROBT. B. HAYWARD

AFTER all, I fear the word "aspect" is not quite the right thing. What is wanted is a word to express "plane-direction;" something in the plane, and not looking out from it. And I am not sure that the compound word "plane-direction," which is not ambiguous nor colloquial, will not be better even than "aspect."

We should then have axioms on planes analogous to those on straight lines: that planes may have the same or different plane-directions: that intersecting planes have different plane-directions; and conversely.

Parallel planes will be defined as those which have the same plane-direction.

* The matter between brackets was written on October 27.—ED.