Notes on the Food of Plants. By Cuthbert C. Grundy, F.C.S. (London: Simpkin, Marshall, and Co., 1871.)

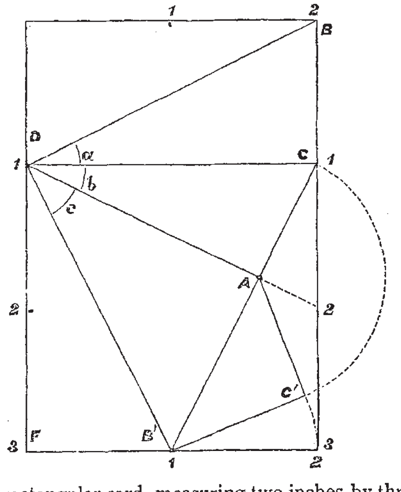
THIS is a useful elementary sketch of the form and manner in which food is obtained by plants. Faults in it there are. Thus, notwithstanding the conclusive experiments of Prillieux and Duchartre, proving that plants have no power of absorbing moisture through their leaves, and the author's own reference to this now established fact in the preface, we still find the assertion (p. 14) that "the leaves withdraw from the atmosphere aqueous vapour." The statement (p. 25) that the sap descends in dicotyledonous plants *through* the bark is not strictly correct; and a Fellow of the Chemical Society ought not to have described (p. 23) carbonic acid as "carbon dioxide combined with water." These blemishes apart, this little book may be recommended to those who desire an explanation of the mode in which vegetable organisms are built up from inorganic materials, and who are unable to devote the time to the more elaborate works of Mr. Johnson, "How Crops Grow" and "How Crops Feed." The portion relating to the effect on crops of different soils strikes us as the best.

LETTERS TO THE EDITOR

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. No notice is taken of anonymous communications.*]

Proof of Napier's Rules

As the following graphical construction is easily executed, representing to the eye the figure usually employed for the proof of Napier's rules of the parts of right-angled triangles in spherical geometry, it will perhaps remove difficulties from their proof for beginners, like those which Mr. W. D. Cooley's work on "Elementary Geometry" must, from his description of some interesting parts of its contents in NATURE of the 19th of October, have proposed to itself to meet, and to render at least as easily accessible as possible to the inquiring student in mathematics.



BF is a rectangular card, measuring two inches by three inches in the sides, and divided by the lines DB, DC, DA, DB′, and B′C in the directions shown in the figure, and in such a manner that the three corners of the rectangle are completely cut away by the last two, and by the first of these lines; while DC and DA are only cut or scored lightly in the card, so as to allow the remaining three triangles, DBC, DCA, DAB′, to be folded towards each other, until, DB and DB′ coinciding, they form a solid angle of three faces at the point D. The property possessed by this solid angle, that the inclination of the two faces, DCB, DCA, to each other is a right angle (the angle shown at C′ in the base, AB′C′ of the solid angle), and that the base AB′C′ of the resulting tetrahedron cuts the two faces ADC, ADB′, perpendicularly (or at right angles to their common intersection DA) in the line AC, AB′, so that the plane angle A of the plane right-angled triangle B′AC′ is also the inclination between those faces, or the angle of the right-angled spherical triangle formed by the intersection of a sphere, about the centre D, with the three planes meeting each other at that point, affords a ready proof of all Napier's rules, excepting that connecting the two angles of a right-angled spherical triangle, from the simple definitions of the trigonometrical "ratios" of plane angles.*

Calling the angles of the faces which meet together at the point D, as shown in the figure a, b, c, opposite to the spherical angles A, B, C, formed by the inclination of the other two faces to each other, these angles, and those of inclination of the faces are, respectively, the sides and angles of a right-angled spherical triangle, whose right angle is C, its hypothenuse is c, and the angle A, between b and c is equal to the plane angle A, of the right-angled triangle AB′C′.

Taking, firstly, as the radius, DA, equal to unity, AC (or AC′), and AB′ are the tangents of b and c; and the right-angled triangle AC′B′ gives the rule,

$$\frac{\tan b}{\tan c} = \cos A\,;\ \text{or}\ \cos A = \tan b\,.\,\cot c \qquad (1)$$

Taking, in the next place, DB, (or BB′), as the radius, equal to unity; BC (or B′C′), and B′A are the sines; and DC, DA are the cosines of the angles a and c. In the first case the right-angled triangle AB′C′ affords the ratio

$$\frac{\sin a}{\sin c} = \sin A\,;\ \text{or}\ \sin a = \sin c\,.\,\sin A\,; \qquad (2)$$

And in the second case we obtain from the right-angled triangle ADC the rule

$$\cos c = \cos a\,.\,\cos b \qquad (3)$$

The rules for the angle B, corresponding to (1) and (2) for the angle A, are simply obtained from them by transposing in them the sides and angles aA for bB; thus—

$$\cos B = \tan a\,.\,\cot c \qquad (4)$$
$$\sin b = \sin c\,.\,\sin B \qquad (5)$$

Finally, dividing (1) by (5), a rule for connecting together the two angles of the right-angled spherical triangle is found as follows:—

$$\cos A \div \sin B = \frac{\tan b}{\tan c} \div \frac{\sin b}{\sin c} = \frac{\cos c}{\cos b} = \cos a, \text{ by (3)};$$
$$\text{or } \cos A = \cos a \sin B \qquad (6)$$

If, as in Napier's rules, the two sides and the differences from 90° of the two angles and of the hypothenuse arranged in their natural order round the triangle are regarded as constituting its five parts, it will be seen that all the above consequences may be included in the two rules known as Napier's rules, that the sine of the middle (that is, of any chosen) part is equal to the product of the tangents of the two adjacents, as well as to the product of the cosines of the two opposite parts.

As a rule to assist the memory, the laconic brevity and completeness of Napier's formula possess a most uniquely felicitous, and, happily for mathematicians, a not unfrequently enduring charm. But should the student desire to divest himself of their artificiality, and to retrace for himself the steps of the demonstration upon which any one example of these rules is based, he must first draw a solid tetrahedron ABCD, in which the facial angles at A, C, are as represented in the figure, but as they cannot all be correctly shown on account of the embarrassing effects of the perspective in the drawing, right angles. By having recourse to a model, on the other hand, which may very readily be cut from a card like that illustrated in the above description, and folded so as to form the solid figure required for their demonstration, all the cases of Napier's rules may be exhibited, and proved, almost as speedily, and satisfactorily to a learner's apprehension in solid geometry, as the definitions of the simple trigonometrical ratios of plane angles, and the least complicated relations connecting together the parts of plane triangles may be made intelligible to him; and that by a plain series of immediate deductions from the figure, which his familiarity with the processes of plane trigonometry will already have taught him very easily to supply.

Newcastle-on-Tyne, Oct. 30 A. S. HERSCHEL

* Another similar property, with a somewhat less important application of the same tetrahedron, is described in the *Quarterly Journal of Mathematics* for October 1862, p. 306.

Remarkable Paraselene seen at Highfield House on October 25th, 1871

THE phenomenon first became visible at $7^h\ 12^m$ P.M., and finally vanished at $7^h\ 33$ P.M. The upper portion of a halo of