mum wave velocity for sea-water may be expected to be not very different from this. (It would of course be the same if the cohesive tension of sea water were greater than that of pure water in precisely the same ratio as the density.)

About three weeks later, being becalmed in the Sound of Mull, I had an excellent opportunity, with the assistance of Prof. Helmholtz, and my brother from Belfast, of determining by observation the minimum wave velocity with some approach to accuracy. The fishing-line was hung at a distance of two or three feet from the vessel's side, so as to cut the water at a point not sensibly disturbed by the motion of the vessel. The speed was determined by throwing into the sea pieces of paper previously wetted, and observing their times of transit across parallel planes, at a distance of 912 centimetres asunder, fixed relatively to the vessel by marks on the deck and gunwale. By watching carefully the pattern of ripples and waves, which connected the ripples in front with the waves in rear, I had seen that it included a set of parallel waves slanting off obliquely on each side, and presenting appearances which proved them to be waves of the critical length and corresponding minimum speed of propagation. Hence the component velocity of the fishing-line perpendicular to the fronts of these waves was the true minimum velocity. To measure it, therefore, all that was necessary was to measure the angle between the two sets of parallel lines of ridges and hollows, sloping away on the two sides of the wake, and at the same time to measure the velocity with which the fishing-line was dragged through the water. The angle was measured by holding a jointed two-foot rule, with its two branches, as nearly as could be judged, by the eye, parallel to the sets of lines of wave-ridges. The angle to which the ruler had to be opened in this adjustment was the angle sought. By laying it down on paper, drawing two straight lines by its two edges, and completing a simple geometrical construction with a length properly introduced to represent the measured velocity of the moving solid, the required minimum wave-velocity was readily obtained. Six observations of this kind were made, of which two were rejected as not satisfactory. The following are the results of the other four:—

Velocity of Moving Solid.	Deduced Minimum Wave-Velocity.
51 centimetres per second.	23·0 centimetres per second.
38 " "	23·8 " "
26 " "	23·2 " "
24 " "	22·9 " "
	Mean 23·22

The extreme closeness of this result to the theoretical estimate (23 centimetres per second) was, of course, merely a coincidence, but it proved that the cohesive force of sea-water at the temperature (not noted) of the observation cannot be very different from that which I had estimated from Gay Lussac's observations for pure water.

I need not trouble you with the theoretical formulæ just now, as they are given in a paper which I have communicated to the Royal Society of Edinburgh, and which will probably appear soon in the *Philosophical Magazine*. If 23 centimetres per second be taken as the minimum speed they give 1·7 centimetres for the corresponding wave-length.

I propose, if you approve, to call ripples, waves of lengths less than this critical value, and generally to restrict the name waves to waves of lengths exceeding it. If this distinction is adopted, ripples will be undulations such that the shorter the length from crest to crest the greater the velocity of propagation; while for waves the greater the length the greater the velocity of propagation. The motive force of ripples is chiefly cohesion; that of waves chiefly gravity. In ripples of lengths less than half a centimetre the influence of gravity is scarcely sensible; cohesion is nearly paramount. Thus the motive of ripples is the same as that of the trembling of a dew drop and of the spherical tendency of a drop of rain or spherule of mist. In all waves of lengths exceeding five or six centimetres, the effect of cohesion is practically insensible, and the moving force may be regarded as wholly gravity. This seems amply to confirm the choice you have made of dimensions in your models, so far as concerns escaping disturbances due to cohesion.

The introduction of cohesion into the theory of waves explains a difficulty which has often been felt in considering the patterns of standing ripples seen on the surface of water in a finger-glass made to sound by rubbing a moist finger on its lip. If no other levelling force than gravity were concerned, the length from crest to crest corresponding to 256 vibrations per second would be a fortieth of a millimetre. The ripples would be quite undistinguishable without the aid of a microscope, and the disturbance of the surface could only be perceived as a dimming of the reflections seen from it. But taking cohesion into account, I find the length from crest to crest corresponding to the period of $\frac{1}{256}$ of a second to be 1·9 millimetres, a length which quite corresponds to ordinary experience on the subject.

When gravity is neglected the formula for the period (P) in terms of the wave-length (l), the cohesive tension of the surface (T), and the density of the fluid (ρ), is

$$P = \sqrt{\frac{l^3 \rho}{2\pi T}},$$

where T must be measured in kinetic units. For water we have $\rho = 1$, and (according to the estimate I have taken from Poisson and Gay Lussac) $T = 982^* \times \cdot 074 = 73$. Hence for water

$$P = \frac{l^{\frac{3}{2}}}{\sqrt{2\pi \times 73}} = \frac{l^{\frac{3}{2}}}{21 \cdot 4}$$

When l is anything less than half a centimetre the error from thus neglecting gravity is less than 5 per cent. of P. When l exceeds $5\frac{1}{2}$ centimetres the error from neglecting cohesion is less than five per cent. of the period. It is to be remarked that, while for waves of sufficient length to be insensible to cohesion, the period is proportional to the square-root of the length, for ripples short enough to be insensible to gravity, the period varies in the sesquiplicate ratio of the length.

WILLIAM THOMSON

Mr. Froude having called my attention to Mr. Scott Russell's Report on Waves (British Association, York, 1844) as containing observations on some of the phenomena which formed the subject of the preceding letter to him, I find in it, under the heading "Waves of the Third Order," or, "Capillary Waves," a most interesting account of the

* 982 being the weight of one gramme in kinetic units of force-centimetres per second.

"ripples" (as I have called them), seen in advance of a body moving uniformly through water; also a passage quoted by Russell from a paper of date, Nov. 16, 1829, by Poncelet and Lesbros,* where it seems this class of waves was first described.

Poncelet and Lesbros, after premising that the phenomenon is seen when the extremity of a fine rod or bar is lightly dipped in a flowing stream, give a description of the curved series of ripples (which first attracted my attention in the manner described in the preceding letter). Russell's quotation concludes with a statement from which I extract the following:— . . . . "on trouve que les rides sont imperceptibles quand la vîtesse est moyennement au dessous de 25c. per seconde."

Russell gives a diagram to illustrate this law. So far as I can see, the comparatively long waves following in rear of the moving body have not been described either by Poncelet and Lesbros or by Russell, nor are they shown in the plan contained in Russell's diagram. But the curve shown above the plan (obviously intended to represent the section of the water-surface by a vertical plane) gives these waves in the rear as well as the ripples in front, and proves that they had not escaped the attention of that very acute and careful observer. In respect to the curves of the ripple-ridges, Russell describes them as having the appearance of a group of confocal hyperbolas, which seems a more correct description than that of Poncelet and Lesbros, according to which they present the aspect of a series of parabolic curves. It is clear, however, from my dynamical theory that they cannot be accurate hyperbolas; and, as far as I am yet able to judge, Russell's diagram exhibiting them is a very good representation of their forms. Anticipating me in the geometrical determination of a limiting velocity, by observing the angle between the oblique terminal straight ridge-lines streaming out on the two sides, Russell estimates it at 8½ inches (21½ centimetres) per second.

Poncelet and Lesbros's estimate of 25 centimetres per second for the smallest velocity of solid relatively to fluid which gives ripples in front, and Russell's terminal velocity of 21½ centimetres per second, are in remarkable harmony with my theory and observation which give 23 centimetres per second as the minimum velocity of propagation of wave or ripple in water.

Russell calls the ripples in front "forced," and the oblique straight waves streaming off at the sides "free"—appellations which might seem at first sight to be in thorough accordance with the facts of observation, as, for instance, the following very important observation of his own:—

"It is perhaps of importance to state that when, while these forced waves were being generated, I have suddenly withdrawn the disturbing point, the first wave immediately sprang back from the others (showing that it had been in a state of compression), and the ridges became parallel; and, moving on at the rate of 8½ inches per second, disappeared in about 12 seconds."

Nevertheless I maintain that the ripples of the various degrees of fineness seen in the different† parts of the fringe are all properly "free" waves, because it follows from dynamical theory that the motion of every portion of fluid in a wave, and, therefore, of course, the velocity of propagation, is approximately the same as if it were part of an infinite series of straight-ridged parallel waves, provided that in the actual wave the radius of curvature of the ridge is a large multiple of the wave-length, and that there are several approximately equal waves preceding it and following it.

No indication of the dynamical theory contained in my communication to the *Philosophical Magazine*, and described in the preceding letter to Mr. Froude, appears either in the quotation from Poncelet and Lesbros, or in any other part of Mr. Scott Russell's report; but I find with pleasure my observation of a minimum velocity below which a body moving through water gives no ripples, anticipated and confirmed by Poncelet and Lesbros, and my experimental determination of the velocity of the oblique straight-ridged undulations limiting the series of ripples, anticipated and confirmed by Russell. W. T.

* Memoirs of the French Institute, 1829.

† The dynamical theory shows that the length from crest to crest depends on the corresponding component of the solid's velocity. For very fine ripples it is approximately proportional to the reciprocal of the square of this component velocity, and therefore to the square of the secant of the angle between the line of the solid's motion and the horizontal line perpendicular to the ridge of the ripple.

ALLBUTT ON THE OPHTHALMOSCOPE

On the Use of the Ophthalmoscope in Diseases of the Nervous System and of the Kidneys; also in certain other General Disorders. By Thomas Clifford Allbutt, M.A., M.D., Cantab. &c. (London and New York: Macmillan and Co., 1871.)

THE advances that have been made in the knowledge of the diseases of the eye since the introduction of the ophthalmoscope are now very widely known, not alone in the medical profession but to the general public. This little instrument, essentially consisting of a mirror with a hole in the centre by which a ray of light can be thrown into the interior of the eye, lighting up its recesses, and enabling, with the aid of a common hand lens, almost every portion of it to be explored, may be said to have revolutionised the surgery of the eye. Many separate and distinct types of disease have been distinguished in conditions that were formerly grouped together under the general term of amaurosis, and the ophthalmic surgeon, no longer administering, as was too often formerly the case, his remedies in rash ignorance, is now able either to infuse well-grounded hope of recovery, or to spare his patient the annoyance of protracted treatment when treatment would be hopeless. For nearly twenty years the use of the ophthalmoscope has been, as was natural, almost entirely restricted to those who devoted themselves to the study of ophthalmic diseases. Like other mechanical aids to diagnosis, as the stethoscope and laryngoscope, its employment requires practice, the opportunities for acquiring a mastery over it were till recently rare, and its value in the practice of medicine was by no means generally recognised. Within the last few years, however, several excellent surgeons and physicians, amongst whom Mr. Hutchinson, Dr. Hughlings Jackson, Dr. John Ogle, and the author of the treatise before us may be especially mentioned, have gradually begun to recognise that the ophthalmoscope may be made available not only to determine the nature of any defect of vision of which the patient may complain, but as a means of reading within certain limits changes in the conditions of the system at large, and of the nervous system in particular.