MUSICAL INTERVALS

A BRIEF summary of the remarkable papers on musical intervals, by MM. Cornu and Mercadier, published in the *Comptes Rendus* of February in last year, may perhaps interest those of the readers of NATURE who have not met with the original.

The authors remark, in the first place, that two schemes of musical intervals have been proposed, in which the ratios of the number of vibrations in a given time are as follows:—

	Octave.	Fifth.	Fourth.	Major Third.	Minor Third.	Sixth.	Seventh.
(1)	2	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{3^4}{2^5}$	$\frac{2^5}{3^3}$	$\frac{3^3}{2^4}$	$\frac{3^5}{2^7}$
(2)	2	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{6}{5}$	$\frac{5}{3}$	$\frac{15}{8}$

and the object of the paper is to examine the claims of each for adoption.

On comparing these two systems, it may be observed that the Octave, Fifth, and Fourth are the same in both, and that the other intervals are connected by the following relations:—

$$\frac{3^4}{2^6} = \frac{5}{4} \times \frac{81}{80} = \frac{2^5}{3^3} = \frac{6}{5} : \frac{81}{80} = \frac{3^3}{2^4} = \frac{5}{3} \times \frac{81}{80} = \frac{3^5}{2^7} = \frac{15}{8} \times \frac{81}{80}$$

A third view of the subject, according to which either of the two systems may be adopted indifferently, because they differ only by a "comma" (an interval represented by 81 : 80), may be at once dismissed, since, as a matter of fact, the ear is capable of appreciating intervals much smaller than the comma.

As regards the scheme (2), it seems impossible not to admit that a major third is most harmonious when the resultant tone is exactly the double octave below the fundamental note, *i.e.* when the interval is represented by the ration 5 : 4. Any deviation from this proportion produces unpleasant beats. This argument seems decisive in favour of (2).

On the other hand, an ear which hears successively the notes emitted by an entire string, and by $\frac{1}{3}$ ths of its length, will pronounce the third so formed to be too low. And, in fact, stringed instruments for concerted music are tuned by perfect fifths. Thus the intervals given by the violoncello, alto, and violins, &c., in a concerto would be *c, g, d', a', e''*.

But this involves a third (*c, e*) defined by the ratio $\frac{3^4}{2^6}$; for *e* must be the double octave below *e''*, and the interval (*c, e''*) is by hypothesis $\left(\frac{3}{2}\right)^4$; hence

$$(c, e) = \frac{1}{2^2} \left(\frac{3}{2}\right)^4 = \frac{3^4}{2^6}$$

which is the ratio for a major third according to scheme (1).

Thus, experiment apparently gives contradictory results.

The authors of the paper then proceed to describe a series of experiments made with the voice, violoncello, violin, organ-pipes, and monochord, all of which lead to conclusions reconciling this apparent contradiction, *viz.* :—

That musical intervals do not belong to any single scheme, but that the ear is capable of distinguishing between the intervals of the two schemes in question, and requires,

(a) When notes are heard in succession, forming what is called melody, that the intervals should belong to a series of fifths, in accordance with scheme (1);

(b) When notes are heard simultaneously, forming chords, or harmony, that the intervals should be adjusted according to scheme (1).

The details of the experiments, which are well worth study, would be too long to give in *extenso*; but the sub-

joined table will enable the reader to form a judgment of the results.

Notes produced by	Major Third.		Fifth.	
	Harmony.	Melody.	Harmony.	Melody.
Voice	—	1.260	—	1.497
Violoncello	1.251	1.266	1.499	1.508
Violin	1.249	1.264	1.504	1.504
Organ pipes	1.252	1.267	1.493	1.497
Monochord	—	1.271	—	1.500
Mean of observation	1.251	1.266	1.499	1.501
Calculation	$\frac{1}{2} = 1.250$	$\frac{3}{2} = 1.2656$	$\frac{4}{3} = 1.500$	1.500

The direct experiments were made with these two intervals only; but the same conclusions are shown to apply to the other intervals.

The authors then proceed to inquire whether there is any reason for limiting the prime numbers which enter into the ratios of the harmonic intervals to those (1, 2, 3, 5) actually occurring in scheme (1). An answer to this question is found in the chord of the dominant seventh, usually defined as the common chord with the addition of a minor third (*e.g.* Do, Mi, Sol, Sib). The ratios of these intervals, according to scheme (2) are

$$1 : \frac{5}{4} : \frac{3}{2} : \frac{3}{2} \times \frac{6}{5} = 20 : 25 : 30 : 36$$

and the simplest whole numbers near to these are 4 : 5 : 6 : 7; and these, it is argued, are in fact an improvement on the former. For the ear, which alone can decide the question, will choose those notes which will form a chord devoid of beats, and whose difference tones do not introduce any notes foreign to the chord itself. Now

$$\begin{aligned} 7-6 &= 1, & 6-5 &= 1, & 5-4 &= 1 \\ 7-5 &= 2 & 6-4 &= 2 \\ 7-4 &= 3 \end{aligned}$$

So that from the chord 4 : 5 : 6 : 7, we obtain the group of difference tones 1, 2, 3, all of which belong to the natural series terminating with the chord itself. While from the chord 20 : 25 : 30 : 36 = 4 : 5 : 6 : 7¹⁴², we at once derive an inharmonious difference tone. This *a priori* conclusion may be verified on the violin, by first tuning the two upper strings in unison; then by shortening one of them so as to form a minor third (6 : 5) with a difference tone 1; and finally, shortening the other until the difference tone 1 is again heard. This will, of course, give an interval (7 : 6) perfectly agreeable when sounded simultaneously, but not so when sounded in succession. In the same way the ear might be called upon to decide whether the numbers 11, 13, &c., are or are not admissible in harmony.

I trust that this very brief abstract may induce some of your readers to examine the paper itself.

W. SPOTTISWOODE

ON THE GREAT MOVEMENTS OF THE ATMOSPHERE *

MY original paper was based on the mean monthly pressures calculated for 516 places, and on the mean monthly direction of the wind calculated for 203 places over the whole surface of the earth. From these mean pressures the monthly isobars were drawn for every tenth of an inch, a pressure of thirty inches and upwards being represented on the charts by red-coloured isobars, and pressures of 29.9 inches and less by isobars coloured blue. Thus the distribution of the mass of the earth's atmosphere from month to month was graphically represented, the red lines showing

* This paper, presented to the recent meeting of the "British Association" at Liverpool, is a brief *résumé* of a paper, and the discussion which followed, "On the Mean Pressure of the Atmosphere, and the Prevailing Winds over the Globe for the Months and for the Year," originally read before the Royal Society of Edinburgh, and published in the Transactions of that Society in the beginning of December last.

where there was an excess, and the blue lines where there was a deficiency. The general results are that, in each hemisphere, pressures are highest in winter, and lowest in summer; that in winter the highest pressures are grouped over the continents, and in summer the lowest, and that in winter very low pressures prevail in the northern parts of the Atlantic and Pacific oceans respectively. The position of the isobars is wholly determined by the relative distribution of land and water.

As regards wind, those places were selected which are favourably situated for observing its direction. In calculating the average direction, the element of force was wholly excluded. The years were, so far as possible, the same as those for which the mean pressure had been calculated; but where this was not possible, care was taken to see that a good average was in every case obtained. The mean direction of the wind for each month is shown in the charts by arrows.

Thus two quite distinct facts were exhibited on the charts, viz. lines showing the mean pressure of the atmosphere, and arrows showing the prevailing winds at the earth's surface, each being independently arrived at by the summing and averaging of observed facts. What relation is there between these two classes of facts?

1. As regards regions of Low Pressures:

In every case where such occur at any season, it is universally seen that the relations of the winds to the isobaric lines is exactly the same that is illustrated in every storm of wind when the winds and pressures are set down in synoptic charts.—The relation is this; the wind neither blows round the centre of least pressure in circles, nor as tangents to the concentric isobaric curves, nor does it blow directly towards that centre, but it takes an intermediate direction, approaching however more nearly to the direction and course of the circular curves than of the radii to the centre. The angle is not a right angle but from about 60° to 80° . Thus the whole system of winds seem to blow vortically upon the spaces of low pressure. This is the relation known as *Buy's Ballot Law of the Winds*.

2. As regards regions of High Pressures:

In all cases where such occur, the winds are seen to flow out in every direction. In these cases also, the behaviour of the winds differs in no respect from what occurs on particular days on which the isobaric lines present the same conditions of pressure. The winds flow out of these spaces of high pressure in courses exactly the opposite to what takes place when they flow in upon spaces of low pressure, and hence such meteorological phenomena have been called by Mr. Francis Galton, "Anticyclones." This is also, it will be seen, in strict accordance with *Buy's Ballot Law*. Hence then this broad result is arrived at, viz., that the prevailing winds over the globe at all seasons obey *Buy's Ballot Law*, with reference to the distribution of atmospheric pressure.

The outflow of the air from a region of high pressure, and the inflow upon a region of high pressure is reducible to the single principle of gravitation, and so marked is this, that if there be any other force or forces which put the winds in motion, they must be altogether insignificant as compared with gravitation.

The annual averages of the 115 places distributed over the north temperate zone were minutely examined with the view of ascertaining how far the commonly-alleged prevalence of equatorial and polar currents is borne out by observation. The result of the analysis is this:—There are two maximum directions of prevailing winds at the stations of which the S.W. and N.E. at Greenwich may be taken as an example. The chief prevailing winds of the north temperate zone blow from some point from S.S.W. to W. (the true equatorial direction) at 41 per cent. of the stations, leaving 59 per cent. at which they are from other points of the compass. And the secondary prevailing winds come from some point from N.N.E. to E. (the true polar direction) at 34 per cent. of the stations or only a third of the whole. Hence whilst the largest percentage of prevailing winds are in the direction in which truly equatorial and polar winds should blow, the percentages from the direction are so large as to preclude the supposition of a general flow of the surface winds of the temperate zone towards and from the polar regions. The truth is, it is not the poles of the earth, but the regions of high and low pressures which must be regarded as the true poles of the winds towards which and from which the great movements of the atmosphere proceed; and owing to the unequal distribution of land and water, the poles of pressure and movements of the atmosphere are, as in the case of the poles of temperature, very far from being coincident with the North Pole.

The causes which bring about an unequal distribution of the mass of the earth's atmosphere are chiefly two—viz., the temperature principally, and secondarily the moisture of the atmosphere, their relations to the geographical distribution of land and water. The charts show that where there is an excess of temperature, as in the interior of Asia in summer, and where there is a relative excess of moisture, as in the belt of calms and in the north of the Atlantic in winter, and where there is an excess of cold, as in the interior of Asia in winter, and where there is a deficiency of moisture—there atmospheric pressure is high. Hence where pressures are low there the air is specifically light, and where pressures are high there the air is heavy relatively to that of surrounding regions.

Further, since vast columns of air are poured by the prevailing winds into those regions where pressures are low without increasing that pressure, we must suppose that there is an outflow from these regions through upper currents, and this inference is all the more inevitable, seeing that the specifically light air resting over these regions supplies the conditions of an ascending current. Again, since vast volumes of air are driven off from the regions of high pressure without diminishing the pressure, it must be inferred that the high pressure is maintained by accessions poured in upon it by the upper currents, and this inference is the more certain because in such regions the air is relatively heavier than in surrounding regions, thus supplying the conditions of a descending current.

It is evident that the currents from the regions of low pressure will continue to ascend as long as their pressure is less than that in regions surrounding them at the same height, but that as soon as they reach a height where the pressure is less, towards and over that region will they flow as upper currents of the atmosphere. The courses of these upper currents will be directed towards those regions where the air is coldest and driest near the surface of the earth, because being thereby densest, the great mass of the air will condense in the lower beds, thus leaving less air or a diminished pressure in the upper regions. Thus the high winter pressure of Asia will be maintained by air being poured upon it by upper currents from the regions of low pressure in the north of the Atlantic, the north of the Pacific, and the Equatorial regions of the south.

From these considerations the following *Theory of the Movements of the Earth's Atmosphere* necessarily follows. The winds on the surface of the earth are known from the isobaric lines, the direction being from regions of high towards regions of low pressures, subject to the changes produced in the direction of the currents by the earth's rotation; and the upper currents of the atmosphere may be inferred from the isobaric lines taken reversely together with the isothermal lines taken directly. In other words, the regions of lowest pressure, by giving the ascending currents, point out the sources or fountains whence the upper currents flow; and the isothermal lines, by showing where, on account of the low temperature, the greatest portion of the air is condensed in the lower beds and so diminishing the pressure in the upper beds, point out the regions towards and over which the upper currents diffuse themselves.

In this discussion of the prevailing winds and mean atmospheric pressure over the globe, there is contained the first approximate answer to the two questions:—

1. What are the motions of the earth's atmosphere?

2. What are the causes of these motions?

It has been shown by Dr. Balfour Stewart* that these questions form the first two stages in the development of meteorology. Considering the importance of the subjects of this discussion which enter deeply into Physical Geography, Geology, the Science of Navigation, and the General Physics of the globe, it is to be hoped that observation will be made and turned to account in rectifying the isobaric lines and filling up the blanks of the winds over the ocean, and portions of Africa and South America, data from these regions being in this as well as every similar discussion to a large extent wanting. ALEX. BUCHAN

SOCIETIES AND ACADEMIES

LONDON

Zoological Society, November 15.—Prof. W. H. Flower F.R.S., V.P., in the chair. An eighth letter was read from Mr. W. H. Hudson, containing further observations on the Ornithology of Buenos Ayres.—Mr. Selater exhibited a specimen

See NATURE of December 2, 1869, p. 129.