ON THE PERIODICITY OF THE SOLAR SPOTS

MOST of our readers are aware that the Kew observers, Messrs. De la Rue, Stewart, and Loewy, have for some time past been engaged in investigations, which, as far as they have already extended, go to show that there is an intimate and, as yet, unexplained connection between the configuration of the planets and the position and number of the spots on the sun. This result, which at once seems to land us in a sort of modern astrology, and which is so extraordinary, is, we suppose, on that ground, for we know of no other, questioned by many European astronomers. In this state of the case, no apology is needed, then, for reproducing, from the *Proceedings* of the American Philosophical Society, the results of a recent independent investigation of this subject by Dr. Kirkwood, together with some historical matter of some interest.

Dr. Kirkwood commences by reminding us that the most ancient observations of sun-spots of which we have any record, are those of the Chinese in the year 321 A.D.; the first notice of their detection by Europeans being found in the annals of the Frankish kings. A black spot, according to Adelmus, was seen on the sun's disc, March 17th, 807, and continued visible 8 days. Similar phenomena were again observed from the 28th of May to the 26th of August, A.D. 840. The year 1096 was also signalised by the appearance of spots so large as to be visible to the naked eye. The next date, in chronological order, is that of 1161, when a spot was seen by Averroës. Finally, on the 7th, 8th, and 16th of December, 1590, "a great blacke spot on the sunne," apparently "about the bignesse of a shilling," was observed at sea by those on board the ship "Richard of Arundell." The foregoing are, we believe, the only undoubted instances in which these phenomena were observed previous to the invention of the telescope.

From 1610 to 1750 the sun was frequently observed through instruments of various optical power, and the sparseness, or even the entire absence of spots, during considerable intervals of time, as well as their great number and magnitude at other epochs, were noticed by different astronomers. We come now to a most interesting and most remarkable epoch in the history of solar physics—an epoch in which the periodicity of the spots gradually come out.

The 11-Year Period of Schwabe.—In 1826, Hofrath Schwabe, of Dessau, commenced a series of sun-spot observations, which have been continued without interruption to the present time (1869).

Schwabe has shown a very marked periodicity in the spots; the interval between two consecutive maxima or minima being, according to him, about 10 years. Soon after the announcement of this interesting discovery, Dr. Lamont, of Munich, detected a corresponding decennial period in the variation of the magnetic needle; the epochs of maxima and minima in the latter coinciding with those in the former. These results have also been confirmed by other observers in places quite remote from each other; so that the decennial *magnetic* cycle may be regarded as well established. The equality of this period with that of the solar spots naturally suggested the hypothesis of their intimate relationship. Such a causal connection may be difficult of explanation; the fact, however, is placed beyond doubt by the researches of Wolf and Sabine.* The former, besides carefully observing the sun-spots since 1847, has discussed all accessible recorded observations, both solar and magnetic, bearing on the subject. He had thus ascertained a number of epochs of maxima and minima anterior to those observed by Schwabe,—from all of which he has determined the period of the spots to be 11·11 years. He undertakes to show, moreover, that this period coincides more exactly with that of the magnetic variation than the 10-year cycle of Lamont.

The 56-Year Period.—Besides Schwabe's period of 11 years, Wolf finds a larger cycle of 55 years, in which the solar activity passes through a series of changes. It is not, however, so distinctly marked as the cycle of Schwabe. Its last maximum was about 1837, and that preceding, about 1780.

The 233-Day Period.—Professor Wolf, after carefully discussing his own and Schwabe's observations, claims to have discovered two or three minor periods of solar activity. "By projecting all the results in a continuous curve, he finds in it a series of small undulations succeeding each other at an average interval of 7·65 months," or 233 days.

The 27-Day Period.—The same astronomer thinks he has detected a short period of variation corresponding to the sun's time of rotation with respect to the earth, or about 27 days.

The 584-Day Period.—De la Rue, Stewart, and Loewy, have found a period varying between 18 and 20 months; the mean being about 584 days. Other periods of maxima and minima will probably be detected; but those we have enumerated are perhaps the only ones sufficiently well established to justify any attempt at explanation.

That the solar spots are produced in some way by the planetary disturbance of the photosphere, is now generally admitted. As yet, however, the manner in which this influence is exerted can be little more than matter of conjecture. If the action is analogous to that of the moon on the earth, the relative disturbing power of the different members of the system will be as follows:*

Name.	Mass.	In Aph.	At M. Dist.	In Perih.
Mercury	$\frac{1}{4865751}$ (Encke)	63	111	219
	$\frac{1}{3000000}$ (Leverrier)	102	180	355
Venus	$\frac{1}{401211}$	203	207	211
Earth	$\frac{1}{314760}$	95	100	105
Mars	$\frac{1}{2994790}$	2	3	4
Jupiter	$\frac{1}{1047}$	194	214	236
Saturn	$\frac{1}{3496}$	8	10	12
Uranus	$\frac{1}{24899}$	0	0	0
Neptune	$\frac{1}{18780}$	0	0	0

The connection between the number of sun-spots and the positions of the planets was noticed by Wolf as long since as 1858. In the interesting memoir of De la Rue, Stewart, and Loewy, the causal connection between the positions of Venus and Jupiter and the behaviour of sun-spots seems to be clearly established. An inspection of the Table shows that writers generally have given undue weight to Saturn's influence. Again, although Mercury's action at aphelion is but feeble, and even, at his mean distance, less than that of Venus or Jupiter, his perturbing power at *perihelion* is the greatest of all planets—a fact which certainly demands consideration in any theory which refers the origin of solar spots to planetary agency. After giving the subject much study and attention, Dr. Kirkwood deems it impossible, *without the introduction of any modifying cause*, to establish a general correspondence between the different sun-spot periods and those of regularly recurring planetary configurations.

But the hypothesis that a particular portion of the sun's surface is more favourable to spot formation—or, in other words, more susceptible to planetary influence—than others, will, he believes, obviate all difficulty. Is

* These magnetic variations, which will not be discussed in the present paper, are mentioned to give completeness of view to the phenomena under consideration. It is also worthy of remark that the Aurora Borealis is believed to exhibit a corresponding periodicity. [We believe that Sir E. Sabine was the first to remark the connection between sun-spots and magnetic disturbances.—ED.]

* The table is derived from the formula $\delta = \frac{m}{a^3}$, where δ represents the disturbing power of a planet, m its mass, and a its distance.

there, then, any independent probability of the truth of this hypothesis? It is well known that the formation of spots occurs chiefly between particular parallels of *latitude*, and that the numbers are greater in the northern than in the southern hemisphere. It seems, therefore, at least not improbable that a like difference may exist in regard to *longitude*. "Sömmering directs attention to the fact, that there are certain meridian belts on the sun's disc, in which he had never observed a solar spot for many years together." * Buys-Ballot, of Utrecht, has found, from an elaborate discussion of a great number of meteorological observations, that there is a short period of variation in the amount of solar heat received by our planet; the period from maximum to maximum coinciding, at least approximately, with that of the sun's rotation with respect to the earth. Sir William Herschel also believed that one side of the sun, on account of some peculiarity in its physical constitution, was less adapted to radiate light and heat than the other.

On Dr. Kirkwood's hypothesis, the sun-spot period would be equal to the interval between two conjunctions of the disturbing planets on the heliographic meridian (designated by M) of that part of the surface most susceptible to their influence. It would depend, therefore, on the ratio of the sun's period of rotation to the interval between two consecutive conjunctions of such planets. Or, as Mercury's influence is extremely variable, a maximum would be produced by this planet's perihelion passage, when the most susceptible part of the sun's surface had the same, or nearly the same, heliocentric longitude. In order, then, to test this hypothesis, we must first inquire what is the most probable period of the sun's rotation?

On account of the *proper* motion of the solar spots, the time of the sun's rotation as determined by their *apparent* motion across the disc, varies from about 25 to 29 days. The *proper* motion of the spots has recently been discussed with great labour and ability by Professor Spörer, of Anclam, and Mr. Carrington, in England, who have shown conclusively that the rapidity of movement varies regularly with the latitude. The equatorial portions have the greatest angular velocity; in other words, the proper motion of the spots is in a direction contrary to that of the sun's rotation. The formulæ by which the astronomers named expressed the law for the dependence of the sun's apparent period of rotation on the latitude are as follows:—

According to Carrington, $\xi = 865' - 165' \sin \frac{7}{4} l$ . . . (1)

" " Spörer, $\xi = 16{\cdot}8475^\circ - 3{\cdot}3812^\circ \sin (41^\circ\ 13' + l)$ . . . . (2)

where ξ is the arc described in a solar day. The true time of rotation is supposed to be that indicated by an equatorial spot; and on this assumption, (1) gives

$$P = 24{\cdot}9711d. = 24d.\ 23h.\ 18m.\ 23s. \quad \ldots\ldots \quad (3)$$

or, (2) gives

$$P = 24{\cdot}62447d. = 24d.\ 14h.\ 59m.\ 0s. \quad \ldots\ldots \quad (4)$$

The true value is probably between the results here given.

But will this modifying element in the theory of planetary action afford a satisfactory explanation of the periodic recurrence of maxima and minima of solar spots? Let us consider.

(*a.*) *The* 11-*Year Cycle.*—The anomalistic period of Mercury is 87·9702*d.*, and

$$87{\cdot}9702d. \times 46 = 4046{\cdot}6292d. = 11{\cdot}077y. = T_1 \quad \ldots \quad (5)$$

This is very nearly equal to Wolf's value of the cycle, and agrees at least equally well with recorded facts.†

Again, $\frac{T_1}{163} = 24{\cdot}82594d. = 24d.\ 19h.\ 49m.\ 21s.$ . . . (6)

which is nearly a mean between Spörer's and Carrington's values of the sun's period of rotation. With this, therefore, as the time of the sun's axial revolution, we have 46 times the period of Mercury—equal to 163 times that of the sun's rotation. The recurrence of maxima at mean intervals of 11·077 years would thus be accounted for.* Again, the epochs at which sun-spots were seen before the invention of the telescope may be presumed, with much probability, to have been nearly coincident with the maxima epochs of Schwabe's cycle. Now, it is a remarkable fact that all of those dates, except perhaps the last, harmonise with the value which we have adopted for Schwabe's period of variation. Thus:

From 321, A.D. to 1860, we have 139 periods of 11·072 + yrs. each.

,,	321	,,	807	,,	44	,,	11·045	,,
,,	807·22	,,	840·5	,,	3	,,	11·093	,,
,,	840·5	,,	1096	,,	23	,,	11·109	,,
,,	1096	,,	1161	,,	6	,,	10·833	,,
,,	1161	,,	1590·9	,,	39	,,	11·024	,,
,,	1590·9	,,	1750·0	,,	14	,,	11·367(?)	,,
,,	1750·0	,,	1829·0	,,	7	,,	11·286	,,
,,	1829·0	,,	1860·5	,,	3	,,	10·500	,,

The variability of the period will be hereafter considered.

(*b.*) *Wolf's Cycle of* 56—57 *Years.*—The synodic revolution of Mercury is 115·87748*d.*, and

$$115{\cdot}87748d. \times 177 = 20510{\cdot}31396d. = 56{\cdot}15324y. = T_2 \ (7)$$

In this period the line of conjunction of Mercury and the earth advances 56·15324 revolutions. Now,

$$\frac{T_2}{826{\cdot}15324} = 24{\cdot}82628d. = 24d.\ 19h.\ 49m.\ 50s. \quad \ldots \quad (8)$$

This value of the sun's period of rotation differs from that in (6) by only 29 seconds. Adopting it, therefore, we find that Mercury and the earth will be in conjunction on the same heliographic meridian at regularly recurring epochs of 56 years and 56 days.

(*c.*) *The* 233-*Day Period.*—The mean interval between the consecutive conjunctions of Venus and Jupiter is 236·992*d.* The close agreement of these periods leaves little room to doubt that the latter is the true period of spot variation.

(*d.*) *The* 27-*Day Period.*—This is at once satisfactorily accounted for.

(*e.*) *The* 584-*Day Period.*—The identity of this period with that of the synodic revolution of Venus has already been indicated by De la Rue, Stewart, and Loewy.

Remarking that Dr. Kirkwood advances other facts in support of his argument, we pass at once to his conclusions.

1. A connection between the behaviour of sun-spots and the configurations of certain planets has been placed beyond reasonable doubt.

2. The theory, however, of spot formation by planetary influence is encumbered with anomalies and even inconsistencies, unless we admit the co-operation of a modifying cause.

3. The hypothesis that a particular part of the solar surface is more susceptible than others to planetary disturbance is rendered probable by the observations of different astronomers.

4. The 11-year cycle of spot variation is mainly dependent on the influence of Mercury.

5. The marked irregularity of this period from 1822 to

* Humboldt's Cosmos, vol. iv., p. 378.

† The following astronomical cycles are also nearly equal to this period of variation:—

1. 18 periods of Venus = 11·074*y.*
2. 35 syn. per. of Mer. = 11·104
3. 1 period of Jupiter = 11·860
4. $17t_1$ = 11·030*y.*
5. $28t_2$ = 11·082
6. $45t_3$ = 11·063

where t_1 = the syn. per. of Venus with respect to Jupiter; t_2 = syn. per. of Mercury with respect to Venus; and t_3 = that of Mercury with respect to Jupiter.

* It is not probable that Mercury is on the meridian M precisely at the epoch of perihelion passage. It is only necessary to suppose this coincidence to occur when the planet is *near* the perihelion point. Even at the distance of 30° the diminution of the disturbing power would be extremely small.

1867, is in a great measure due to the disturbing action of Venus.

6. Wolf's 56-year cycle is determined by the joint action of Mercury and the Earth. And,

Finally, *the hypothesis proposed accounts, as we have seen, for all the well-defined cycles of spot-variations.*

NOTE ON THE CORRELATION OF COLOUR AND MUSIC

WHILST engaged in the preparation of an article on the Analogy of Light and Sound for the current number of the *Quarterly Journal of Science*, I was led to examine the grounds of the frequently-assumed relationship between the colours of the solar spectrum and the notes of the musical scale. It is well known that Newton found a connection between the relative spaces occupied by each colour and the relative vibrations of the notes of the scale. But this, I presume, cannot be more than an accidental coincidence. The common basis of comparison is obviously the ratio of the wave-lengths in the two cases. Although according to the tables given in text-books no satisfactory connection can be found, yet many considerations appear to justify a stricter comparison of these natural scales of colour and sound.

The ratio of wave-lengths of the two extremes of the spectrum is usually taken as 1 : 0.57, or corresponding to the interval of a seventh in music.

But this statement is only true when a glass prism is employed; the ultra-violet rays are then suppressed. Substituting quartz for glass, light of higher refrangibility is seen: the limits of the spectrum can thus be extended from the solar line A to the solar line L.* Now, the wave-length of A (according to Ångström) is 760 millionths of a millimetre, and the wave-length of L (according to Mascart) is 381 millionths of a millimetre, or as the ratio of 1 : 0·50, exactly corresponding to the interval of an octave in music.

The ratios of the extreme colours of the spectrum and the extreme notes of an octave are coincident.

The next object is to compare the ratio of wave-lengths giving rise to the intermediate colours of the spectrum with the ratio of wave-lengths giving rise to the intermediate notes of the scale.

The most careful localisation of the colours of the spectrum with which I am acquainted is that by Prof. Listing.† In his recent memoir on the wave-lengths of the spectra of the metals, M. Thalén gives Prof. Listing's estimation of the extreme limits of each colour as follows :‡

TABLE I.

Name.	Limiting Wave-lengths in ten-millionths of a millimetre.
Red	7234 to 6472
Orange	6472 to 5856
Yellow	5856 to 5347
Green	5347 to 4919
Blue	4919 to 4555
Indigo	4555 to 4241
Violet	4241 to 3967

Taking the *mean* of the two limits to represent the average wave-length of each colour, we have the following series :—

TABLE II.

Name.	Mean wave-lengths in ten-millionths of a millimeter.	Ratio.
Red	6853	100
Orange	6164	89
Yellow	5601	81
Green	5133	75
Blue	4737	69
Indigo	4395	64
Violet	4104	60

* Mr. Crookes informs me that on favourable occasions he has even seen beyond L.

† Poggendorff's "Annalen," 1868, vol. 131, p. 564.

‡ Trans. Roy. Soc. Upsal, third series, vol. vi.: also Annales de Chimie et de Physique, October 1869, and NATURE, No. 2.

Calling the wave-length of the mean red 100, the numbers in the third column express the corresponding ratios of the mean wave-lengths of the other colours.

In the next table is given the similar data as regards sound. The *first* column contains the names of the musical notes; the *second* their actual wave-lengths starting from the middle C; the *third* column gives the relative wave-lengths in fractions of C; and the *fourth*, the ratio without fraction, C being taken as 100.

TABLE III.

Name.	Wave-length in inches.	Ratio of wave-lengths.		
C	52	1	or	100
D	$46\frac{1}{3}$	$\frac{8}{9}$		89
E	42	$\frac{4}{5}$		80
F	39	$\frac{3}{4}$		75
G	35	$\frac{2}{3}$		67
A	31	$\frac{3}{5}$		60
B	$27\frac{1}{2}$	$\frac{8}{15}$		53
C_2	26	$\frac{1}{2}$		50

Placing together the ratio given in the last columns of Tables II. and III., the following remarkable correspondence comes out :—

TABLE IV.

Colour.	Ratio.	Notes.	Ratio.
Red	100	C	100
Orange	89	D	89
Yellow	81	E	80
Green	75	F	75
Blue . 69, Indigo 64 } mean,	67	G	67
Violet	60	A	60
[Ultra-Violet	53]	B	53
[Obscure	50]	C_2	50

Assuming the colour *red* to correspond to the note C, then we find *orange* exactly corresponds to D; *yellow* is almost exactly the same as E; and if we take the wave length of E from observation and not from theory, we have 52 : 42 = 100 : 80·8, a still closer approximation to yellow. The ratio of green is identical with that of F. *Blue*, however, does not correspond to G, nor *Indigo* to A; but blue and indigo are practically one colour in the spectrum,—the line of demarcation, difficult to fix between any other colours, is impossible to be established here. I think, therefore, I am justified in putting them together, and if we do so we find their mean ratio exactly corresponds to G. Violet now exactly corresponds to the ratio given by A. Here all distinct colour ends. But beyond this region Sir John Herschel detected a lavender colour, which finally shades away into a dusky grey. The wave-length of this ultra-violet region is not given by Prof. Listing; hence the ideal position is calculated and inserted in the table within brackets. As the lower C is placed at the *mean* red, the upper C would then correspond to a region in the spectrum altogether obscure: viz., at the solar line O. But as already stated above, if we place the lower C at the *extreme* red, then its higher octave would fall on the line L, or within the range of vision.* The great difference of position thus produced at the violet extremity by a slight movement at the other end of the spectrum, is caused by the crowding together of the colours at the red end. This is shown, together with the correspondence of the ratios of sound and colour, in the accompanying diagram.

The musical scale is thus literally a rainbow of sound. Harmony in colour and music may thus, probably, be found to have a common physical basis. There are many indications that this is the case. For example, the juxtaposition of two colours nearly alike is bad, and so also two adjacent notes of the scale sounded together produce discord. The succession of colours in the spectrum

* A suggestion, made, I believe, by Sir J. Herschel, that the colours of the spectrum would probably repeat themselves if we could see beyond the lavender, both supports, and gains support from, this analogy.