SUPPLEMENTARY METHODS

No reputation

We first analyse the case in which there is no reputation. Thus individuals know their own type but have no information on the type of their opponent. A strategy is a rule that specifies the effort of an individual as a function of their type. Our analysis is based on the quadratic form of payoff given in the Methods section of the main text.

*Best response to the resident strategy*

Suppose that almost all population members follow some given strategy (which we refer to as the resident strategy). Let the random variable \( Y \) denote the effort of a randomly selected population member. Let \( \hat{m} = E[Y] \) denote the mean effort in the population. Consider a mutant individual of type \( v \). If this individual expends effort \( x \) the expected payoff is \( E[W_v(x,Y)] \). By equations (13) of the main text

\[
E[W_v(x,Y)] = b_0 + b_1 (x + \hat{m}) - \frac{1}{2} B_2 (x^2 + E[Y^2]) + Jx\hat{m} - vx - \frac{1}{2} \alpha x^2. \tag{1}
\]

This is maximised at

\[
\hat{x}(v) = (b_1 + J\hat{m} - v)/(B_2 + \alpha). \tag{2}
\]

Note that this effort depends on the resident strategy only through the mean resident population effort \( \hat{m} \).

If this strategy coincides with the resident strategy, we must have \( \hat{m} = E[\hat{x}(V)] \), where the random variable \( V \) is the type of a randomly selected individual. Thus, averaging over \( v \) in equation (2), and using \( E[V] = \mu \) gives

\[
\hat{m} = (b_1 + J\hat{m} - \mu)/(B_2 + \alpha), \tag{3}
\]

so that

\[
\hat{m} = (b_1 - \mu)/(B_2 + \alpha - J). \tag{4}
\]

From the above analysis we see that if \( \hat{m} \) is given by equation (4) and the resident population adopts strategy (2) then the resident strategy is the unique best response itself. The strategy is therefore a Nash equilibrium strategy and is also an Evolutionarily Stable Strategy (ESS). It can also be seen that the strategy is the only Nash equilibrium strategy.

Note we can rewrite the ESS as

\[
\hat{x}(v) = \hat{m} - \hat{\delta}(v - \mu), \tag{5}
\]
where $\hat{\delta} = (B_2 + \alpha)^{-1}$.

**Response to reputation of partner**

**Reputation**

The reputation of an individual in the current round is taken to be a (possibly weighted) average effort of this individual in previous rounds. Specifically, we assume that there have previously been a large (essentially infinite) number of rounds. Let $Z_{-1}$ be the effort of the individual in the last round, and more generally let $Z_{-k}$ the effort $k$ rounds ago. Note that these efforts will typically be random variables, since $Z_{-k}$ is allowed to depend on the reputation of the randomly chosen partner $k$ rounds ago.

One form of reputation we analyse is simply the effort on the last round. That is the reputation of the individual is given by $R = Z_{-1}$. More generally, we may suppose that reputation is a discounted average of previous effort; i.e

$$R = (1 - \beta)[Z_{-1} + \beta Z_{-2} + \beta^2 Z_{-3} + \ldots],$$

(6)

where $\beta$ is a non-negative constant that satisfies $0 \leq \beta < 1$. Note that the reputation $R = Z_{-1}$ is a special case of (6) for which $\beta = 0$. We note that (6) is equivalent to

$$R = \beta R_{-1} + (1 - \beta)Z_{-1},$$

(7)

where $R_{-1}$ is the previous reputation of partner. This is equation (10) of the main text.

We also consider reputation to be average effort; i.e.

$$R = \lim_{k \to \infty} [Z_{-1} + Z_{-2} + Z_{-3} + \ldots + Z_{-k}] / k.$$

(8)

This can be thought of as a special case of (6) in which $\beta \to 1$.

**Form of the response**

During an interaction an individual chooses its effort in the interaction dependent on its own type and the reputation of partner. We restrict attention to rules for choosing effort that are of the form:

Effort $= d(v) + \lambda r$,  

(9)
where \( v \) is the state of the individual and \( r \) is the reputation of its current partner. Here \( d(.) \) is some (genetically determined) function of type and \( \lambda \) is a (genetically determined) constant satisfying \(-1 < \lambda < 1\).

**Reputation dynamics within a given resident population**

Suppose that almost all population members have the same response rule; i.e. use the same function \( d(.) \) and parameter \( \lambda \). Thus equation (9) specifies the resident population rule.

Consider the reputation of a randomly selected individual at a particular time. Denote the type of a randomly selected individual by the random variable \( V \) and set \( D = d(V) \). Let \( R_{-k} \) be the reputation that the partner \( k \) rounds ago had coming into that round. Then the effort of the focal individual \( k \) rounds ago was \( Z_{-k} = D + \lambda R_{-k} \). Thus when reputation is given by equation (6) the current reputation of the focal individual is

\[
R = D + \lambda (1 - \beta) [R_1 + \beta R_2 + \beta^2 R_3 + \ldots].
\]

(10)

This defines an autoregressive time series, where \( R_{-k} \) is the reputation of a randomly selected individual \( k \) rounds ago. By standard results\(^1\) this series achieves a stationary distribution over time. We assume that reputations are at this stationary distribution. Then we can assume that \( R_0, R_1, R_2, R_3, \ldots \) all have the same distribution, and that \( R_1, R_2, R_3, \ldots \) are independent since these reputations are from randomly and independently selected individuals in a large (infinite) population. Thus it is straightforward to show that the mean and variance of the reputation of a randomly selected population member are

\[
E[R] = \frac{E[D]}{1 - \lambda}
\]

(11)

\[
Var[R] = \frac{Var[D]}{1 - \kappa \lambda^2},
\]

(12)

where the constant \( \kappa \) is given by

\[
\kappa = \frac{1 - \beta}{1 + \beta}.
\]

(13)

It can also be verified that when reputation is given by equation (8), formulae (11) and (12) hold when \( \kappa = 0 \).

**Reputation of a mutant within the resident population**

We now consider a mutant individual that uses response rule

\[
\text{Effort} = \tilde{d} + \tilde{\lambda} r,
\]

(14)

\[
\text{Effort} = \tilde{d} + \tilde{\lambda} r,
\]
when the reputation of opponent is \( r \).

Following similar reasoning to that needed to derive equation (10), the reputation of this mutant can be written as

\[
\bar{R} = \bar{d} + \bar{\lambda}(1 - \beta)[R_1 + \beta R_2 + \beta^2 R_3 + \ldots],
\]

(15)

where \( R_1, R_2, R_3, \ldots \) are the reputations of the mutant’s previous partners. The mean and variance of the mutant’s reputation can then be expressed in terms of the resident quantities as

\[
E(\bar{R}) = \bar{d} + \bar{\lambda}E(R)
\]

(16)

and

\[
Var(\bar{R}) = \kappa \tilde{\lambda}^2 Var[R].
\]

(17)

**Efforts of mutant and partner**

Consider a round played by the above mutant with a randomly selected partner. Let \( \bar{R} \) and \( R \) be the current reputations of the mutant and resident respectively. Then the efforts are

\[
X = \bar{d} + \bar{\lambda}R
\]

(18)

and

\[
Y = D + \bar{\lambda}\bar{R}
\]

(19)

respectively, where \( D = d(V) \) and where \( V \) is the type of the opponent. Thus

\[
E(X) = \bar{d} + \bar{\lambda}E(R),
\]

(20)

and using equation (16) we have

\[
E(Y) = E(D) + \lambda(\bar{d} + \bar{\lambda}E(R)).
\]

(21)

We also have

\[
Var(X) = \bar{\lambda}^2 Var[R],
\]

(22)

and since \( D \) and \( \bar{R} \) are independent, using equation (17) we have

\[
Var(Y) = Var(D) + \kappa \lambda^2 \bar{\lambda}^2 Var[R].
\]

(23)

To find the covariance of \( X \) and \( Y \) we note that by equation (10)

\[
Cov(R, D) = Cov(D, D) = Var(D).
\]

Here we have used the fact that \( R_1, R_2, R_3, \ldots \) are all independent of \( D \). Thus by equations (18) and (19)
\[ \text{Cov}[X, Y] = \tilde{\lambda} \text{Var}[D], \]  

(24)

since \( R \) and \( \tilde{R} \) are independent.

**Game payoffs**

We now assume that the mutant individual has type \( v \). Following equation (13) of the main text, we write the payoff to an individual of type \( v \) that expends effort \( x \) when partner’s effort is \( y \) by

\[ W_v(x, y) = b_0 + b_1(x + y) - \frac{1}{2} B_2(x^2 + y^2) + Jxy - v(x - \frac{1}{2} \alpha x^2). \]  

(25)

As before we assume that \( 0 < \mu < b_1, B_2 > 0 \), and that \( |J| < B_2 \).

The mean payoff to the mutant is \( \text{E}[W_v(X, Y)] \). Let \( \bar{x} = \text{E}[X] \) denote the mean effort of the mutant and let \( \bar{y} = \text{E}[Y] \) denote the mean effort of the opponent. Then we can rewrite the mean payoff to the mutant as

\[ \text{E}[W_v(X, Y)] = W_v(\bar{x}, \bar{y}) - \frac{1}{2} [(B_2 + \alpha) \text{Var}[X] + B_2 \text{Var}[Y]] + J \text{Cov}[X, Y]. \]  

(26)

We are interested in the dependence of the payoff to a mutant on the parameters \( \tilde{d} \) and \( \tilde{\lambda} \) for fixed resident behaviour. Since \( \bar{x} \) and \( \bar{y} \) depend on both parameters, but the variance and covariance terms only depend on \( \tilde{\lambda} \) we can express the mean mutant payoff as

\[ \text{E}[W_v(X, Y)] = F(\tilde{d}, \tilde{\lambda}) + \frac{1}{2} G(\tilde{\lambda}), \]  

(27)

where

\[ F(\tilde{d}, \tilde{\lambda}) = W_v(\bar{x}(\tilde{d}, \tilde{\lambda}), \bar{y}(\tilde{d}, \tilde{\lambda})), \]  

(28)

and by equation (22) – (24)

\[ G(\tilde{\lambda}) = -\tilde{\lambda}^2 \text{Var}[R] [(B_2 + \alpha) + \kappa \tilde{\lambda}^2 B_2] - B_2 \text{Var}[D] + 2 \tilde{\lambda} J \text{Var}[D]. \]  

(29)

**Best mutant value of \( \tilde{\lambda} \)**

To find the best mutant response rule we set

\[ \frac{\partial \text{E}[W_v(X, Y)]}{\partial \tilde{d}} = 0, \]  

(30)

and

\[ \frac{\partial \text{E}[W_v(X, Y)]}{\partial \tilde{\lambda}} = 0. \]  

(31)
Since $G$ does not depend on $\tilde{d}$ equation (30) is equivalent to setting

$$\frac{\partial F}{\partial d} = 0.$$  \hfill (32)

We return to this equation below, after having found the best value of $\tilde{\lambda}$. Note that by equations (20) and (21) we have

$$\frac{\partial F}{\partial \lambda} = E[R] \frac{\partial F}{\partial d}.$$  \hfill (33)

Thus at the best response

$$\frac{\partial F}{\partial \lambda} = 0,$$  \hfill (34)

which by equation (31) implies that

$$G'(\tilde{\lambda}) = 0.$$  \hfill (35)

By equations (29) and (12) this gives

$$\tilde{\lambda} = \frac{J(1 - \kappa \lambda^3)}{B_z(1 + \kappa \lambda^3) + \alpha}.$$  \hfill (36)

We also have

$$G^*(\tilde{\lambda}) = -2\text{Var}[R][B_z(1 + \kappa \lambda^3) + \alpha] < 0 \quad \text{for all } \tilde{\lambda},$$  \hfill (37)

confirming that the turning point is the unique global maximum of $G(\tilde{\lambda})$.

**Nash equilibrium value of $\tilde{\lambda}$**

A necessary condition for the resident strategy to be a Nash equilibrium is that $\tilde{\lambda} = \lambda$ in equation (36); i.e.

$$\kappa B_z \lambda^3 + \kappa J \lambda^2 + (B_z + \alpha) \lambda - J = 0.$$  \hfill (38)

To investigate the properties of solutions to this cubic equation we define $f : \mathbb{R} \to \mathbb{R}$ by

$$f(\lambda) = \kappa B_z \lambda^3 + \kappa J \lambda^2 + (B_z + \alpha) \lambda - J.$$  \hfill (39)

Then we can write the derivative of $f$ as

$$f'(\lambda) = 3\kappa B_z \lambda^2 + 2\kappa J \lambda + (B_z + \alpha) = 2\kappa B_z \lambda^2 + \kappa B_z(\lambda + \frac{J}{B_z})^2 + \frac{1}{B_z} [B_z^2 - \kappa J^2] + \alpha.$$  \hfill (40)

Since $|J| < B_z$ and $\kappa \leq 1$, we thus have
\[ f'(\lambda) > 0 \text{ for all } \lambda. \]  
(41)

It then follows that equation (38) has at most one root. We consider three cases:

**Case as \( J = 0 \).** Then from equation (38) \( \lambda^* = 0 \) is the unique root.

**Case \( 0 < J \leq 1 \).** Then since \( f(0) = -J < 0 \) and \( f(1) > B_2 - J > 0 \), equation (38) has a unique solution, \( \lambda^* \), satisfying \( 0 < \lambda^* < 1 \).

**Case \( -1 \leq J < 0 \).** Then by similar reasoning to the case above, equation (38) has a unique solution, \( \lambda^* \), satisfying \( -1 < \lambda^* < 0 \).

**Nash equilibrium value of \( \tilde{d} \)**

From now on we suppose that the resident strategy satisfies \( \lambda = \lambda^* \) and that the mutant strategy satisfies \( \tilde{\lambda} = \lambda^* \). It is convenient to set \( m^* \) as the mean effort in the resident population. Since the mean reputation equals mean effort, we then have \( E[R] = m^* \) and \( E[D] = (1 - \lambda^*)m^* \) by equation (11). Equations (20) and (21) then reduce to

\[ \tilde{x}(\tilde{d}, \lambda^*) \equiv E[X] = \tilde{d} + \lambda^* m^*, \]
and

\[ \tilde{y}(\tilde{d}, \lambda^*) \equiv E[Y] = (1 - \lambda^*)m^* + \lambda^* \tilde{d} + (\lambda^*)^2 m^*. \]
(43)

We can regard \( F \) as a function of \( \tilde{d} \) for fixed \( \lambda^* \); i.e. \( F(\tilde{d}) = W,(\tilde{x}(\tilde{d}, \lambda^*)), \tilde{y}(\tilde{d}, \lambda^*)) \). Then

\[ F'(\tilde{d}) = b_1 - (B_2 + \alpha) \tilde{x} + J \tilde{y} + \lambda^* (b_1 - B_2 \tilde{y} + J \tilde{x}) - \nu. \]
(44)

Also

\[ F^*(\tilde{d}) = -(B_2 + \alpha) + 2J\lambda^* -(\lambda^*)^2 B_2 < -B_2 + 2|J||\lambda^*| - (\lambda^*)^2 B_2 \]
\[ < -B_2 + 2B_2 |\lambda^*| - (\lambda^*)^2 B_2 = -(1 - |\lambda^*|)^2 < 0. \]
(45)

Thus the unique global maximum of \( F \) occurs when \( F'(\tilde{d}) = 0 \). That is

\[ (1 + \lambda^*)b_1 - (B_2 + \alpha - \lambda^* J)\tilde{x}^*(\nu) - (\lambda^* B_2 - J)\tilde{y}^*(\nu) - \nu = 0, \]
(46)

where the notation emphasises that the optimal mean efforts \( \tilde{x}^*(\nu) \) and \( \tilde{y}^*(\nu) \), depend on the value of \( \nu \). If the mutant strategy is a best response to the resident strategy then this equation must hold for all \( \nu \). Suppose that this best response equals the resident strategy. Then \( E[\tilde{x}^*(V)] = m^* = E[\tilde{y}^*(V)] \). Thus, averaging over \( \nu \) in equation (46) we have
\begin{align*}
(1 + \lambda^*)b_1 - (B_2 + \alpha - \lambda^* J)m^* - (\lambda^* B_2 - J)m^* - \mu &= 0, \\
\text{so that} \quad m^* &= \frac{(1 + \lambda^*)b_1 - \mu}{(1 + \lambda^*)(B_2 - J) + \alpha}. \\
\end{align*}

We now again focus on the mutant of type \( v \). If the population is at a Nash equilibrium we have \( \bar{x}(\bar{d}, \lambda^*) = \bar{x}^*(v) \) and \( \bar{y}(\bar{d}, \lambda^*) = \bar{y}^*(v) \). By equations (42), (43) and (46) we have

\[ \bar{d} = \gamma - \delta^* v, \]

where

\[ \gamma = \frac{(1 + \lambda^*)b_1 - \left[ \lambda^* \alpha + (2\lambda^* - (\lambda^*)^2 + (\lambda^*)^3)B_2 - (1 - \lambda^* + 2(\lambda^*)^2)J \right]m^*}{(1 + (\lambda^*)^2)B_2 + \alpha - 2\lambda^* J}, \]

and

\[ \delta^* = \frac{1}{(1 + (\lambda^*)^2)B_2 + \alpha - 2\lambda^* J}. \]

Making use of equation (48) it can be verified that

\[ \gamma = \delta^* \mu + (1 - \lambda^*)m^*. \]

Thus a convenient way to represent the effort of the mutant of type \( v \) when the opponent has reputation \( r \) is to express the effort as

\[ \text{Effort} = m^* - \delta^*(v - \mu) + \lambda^*(r - m^*). \]

Pulling the results together

We are now in a position to pull the above results together. Let \( S \) be the set of strategies which are of the form: if of type \( v \) and opponent’s reputation is \( r \) expend effort \( d(v) + \lambda r \) where \( d(\cdot) \) is any function of \( v \) and \( \lambda \) is a constant satisfying \( |\lambda| < 1 \). Then the above analysis shows that a necessary condition for a strategy to be a Nash equilibrium is that

(i) \( \lambda = \lambda^* \), where \( \lambda^* \) is the unique solution to equation (38)

(ii) The function \( d(\cdot) \) is given by \( d(v) = \gamma - \delta^* v \), where the constants \( \gamma \) and \( \delta^* \) are given by equation (50) and (51) respectively.

In other words the only candidate Nash equilibrium strategy is given by equation (53)
The analysis also shows that if this is the resident strategy, then this strategy is the unique best response to itself within the set $S$ of strategies. It follows that if we restrict attention to strategies within this class, the strategy given by equation (53) is an ESS.

**Dependence of the ESS strategy on parameters**

We now consider how the parameters $\lambda^*$ and $\delta^*$ depend on the parameters $J$ and $\kappa$. To do so it is convenient to rewrite equation (39) as

$$f(\lambda; J, \kappa) = \kappa B_2 \lambda^3 + \kappa J^2 \lambda^2 + (B_2 + \alpha) \lambda - J,$$

and to explicitly express $\lambda^*$ as $\lambda^*(J, \kappa)$. In this notation $f(\lambda^*(J, \kappa); J, \kappa) = 0$. We note that by equation (41), $f(\lambda; J, \kappa)$ is a strictly increasing function of $\lambda$ for fixed $J$ and $\kappa$. Thus we have

$$\lambda^*(J, \kappa) > \lambda \iff f(\lambda; J, \kappa) < 0.$$  \hspace{1cm} (55)

**Dependence of $\lambda^*$ on $J$.**

Suppose that $-B_2 < J_1 < J_2 < B_2$. Then the analysis above shows that $-1 < \lambda^*(J_1, \kappa) < 1$. Thus since we also have $0 \leq \kappa \leq 1$, $f(\lambda^*(J_1, \kappa); J_2, \kappa) < f(\lambda^*(J_1, \kappa); J_1, \kappa)$ from equation (54). Hence $f(\lambda^*(J_1, \kappa); J_2, \kappa) < 0$ since $f(\lambda^*(J_1, \kappa); J_1, \kappa) = 0$. By criterion (55)

$$\lambda^*(J_2, \kappa) > \lambda^*(J_1, \kappa).$$  \hspace{1cm} (56)

This shows that:

- $\lambda^*(J, \kappa)$ is a strictly increasing function of $J$ for fixed $\kappa$.  \hspace{1cm} (57)

We also note that since $f(-\lambda; -J, \kappa) = -f(\lambda; J, \kappa)$ we have

$$f(-\lambda; -J, \kappa) = 0 \iff f(\lambda; J, \kappa) = 0.$$  \hspace{1cm} (58)

So that

$$\lambda^*(-J, \kappa) = -\lambda^*(J, \kappa).$$  \hspace{1cm} (59)

Finally, we note that for $J > 0$ we have $f(0; J, \kappa) < 0$ and $f(J / B_2; J, \kappa) > 0$. Thus by criterion (55) we have

$$0 < \lambda^*(J, \kappa) < J / B_2 \quad \text{for} \quad J > 0.$$  \hspace{1cm} (60)

Thus by equation (59)
\[ J / B_2 < \lambda^* (J, \kappa) < 0 \text{ for } J < 0. \] \hfill (61)

**Dependence of \( \lambda^* \) on \( \kappa \)**

To investigate the dependence on \( \kappa \), we first consider the case when \( J > 0 \). Suppose that \( 0 \leq \kappa_1 < \kappa_2 \leq 1 \). Then since \( \lambda^*(J, \kappa_2) > 0 \), we have \( f(\lambda^*(J, \kappa_2); J, \kappa_1) < f(\lambda^*(J, \kappa_2); J, \kappa_2) \). Thus since \( f(\lambda^*(J, \kappa_2); J, \kappa_2) = 0 \) we have \( f(\lambda^*(J, \kappa_2); J, \kappa_1) < 0 \) and hence

\[ \lambda^*(J, \kappa_2) < \lambda^*(J, \kappa_1) \] \hfill (62)

by criterion (55). In other words:

- \( \lambda^*(J, \kappa) \) is a strictly decreasing function of \( \kappa \) for fixed \( J > 0 \). \hfill (63)

Equation (59) then shows that

- \( \lambda^*(J, \kappa) \) is a strictly increasing function of \( \kappa \) for fixed \( J < 0 \). \hfill (64)

**Dependence of \( \delta^* \) on \( \kappa \)**

Fix \( J \) and suppose and consider the function

\[ h(\kappa) = [1 + (\lambda^*(J, \kappa))^2]B_2 + \alpha - 2J\lambda^*(J, \kappa). \] \hfill (65)

Then

\[ h'(\kappa) = 2[(\lambda^*(J, \kappa))B_2 - J] \frac{\partial \lambda^*}{\partial \kappa}. \] \hfill (66)

Thus by inequality (60) and property (63) we have \( h'(\kappa) > 0 \) for \( J > 0 \). Similarly, by inequality (61) and property (64) \( h'(\kappa) > 0 \) for \( J < 0 \). From equation (51) we deduce that

- \( \delta^*(J, \kappa) \) is a decreasing function of \( \kappa \) for all fixed \( J \). \hfill (67)

**Behaviour maximising mean population fitness when there is no reputation**

Suppose that there is no reputation. Thus individuals only respond to their type.

Consider a population in which the effort of an individual of type \( v \) is \( x(v) \). Let

\[ m_v = E\{x(V)\} \] denote the mean population effort. Consider a contest in which two randomly selected population members meet. Let \( U \) and \( V \) denote the types of these individuals. Then the mean population payoff can be expressed as

\[ \overline{W} = E\{b_0 + b_1(x(U) + x(V)) - \frac{1}{2}B_2(x^2(U) + x^2(V)) + Jx(U)x(V) - Vx(V) - \frac{1}{2}\alpha x(V)^2\} \] \hfill (68)
(cf. Equation (1). Since $U$ and $V$ are independent this can be written as

$$
\bar{W} = b_0 + (2b_1 - \mu)m_c + [J - B_2 - \frac{1}{2} \alpha]m_c^2 - [B_2 + \frac{1}{2} \alpha] Var\{x(V)\} - Cov\{V, x(V)\}.
$$

(69)

We investigate the function $x(v)$ that maximises this mean payoff.

For given mean and variance of $x(V)$, and given distribution of $V$, $Cov\{V, x(V)\}$ is minimised when $x(v)$ depends linearly on $v$ with negative slope. Thus we can express $x(v)$ as

$$
x(v) = m_c - b(v - \mu),
$$

(70)

where $m_c$ and $b \geq 0$ are constants. Substituting this form of $x(v)$ into expression (69) gives

$$
\bar{W} = b_0 + (2b_1 - \mu)m_c + \frac{1}{2} [2J - 2B_2 - \alpha]m_c^2 - \frac{1}{4} [2B_2 + \alpha]b^2 \sigma^2 + b \sigma^2.
$$

This is maximised when

$$
b = \frac{1}{2B_2 + \alpha},
$$

(71)

and

$$
m_c = \frac{2b_1 - \mu}{2B_2 + \alpha - 2J}.
$$

(72)

### Mean population effort

Define

$$
m(\lambda) = \frac{(1 + \lambda)b_1 - \mu}{(1 + \lambda)(B_2 - J) + \alpha}.
$$

(73)

Note that:

- $m(0) = \hat{m}$, where $\hat{m}$ is the mean population effort when there is no reputation (see equation (4)).
- $m(\lambda^*) = m^*$, where $\lambda^*$ is the responsiveness to partner and $m^*$ is the mean population effort when there is reputation (see equation (48)).
- $m(1) = m_c$, where $m(1) = m_c$ is the mean population effort when population members cooperate optimally (see equation (72)).

It can easily be verified that since $|J| < B_2$ we have $m'(\lambda) > 0$ for all $\lambda$. It therefore follows that when $J > 0$ (and hence $\lambda^*(J, \kappa) > 0$) we have
\[ \hat{m} < m^* < m_c, \]  

and when \( J < 0 \) (and hence \( \lambda^*(J, \kappa) < 0 \)) we have

\[ m^* < \hat{m} < m_c. \]

We now investigate how mean population effort depends on \( \kappa \) for fixed \( J \) when there is reputation; i.e. we investigate the dependence of \( m^*(J, \kappa) = m(\lambda^*(J, \kappa)) \) on \( \kappa \). For fixed \( J > 0 \), \( m^*(J, \kappa) \) is a strictly decreasing function of \( \kappa \) since \( \lambda^*(J, \kappa) \) is a strictly decreasing function of \( \kappa \). Similarly, for fixed \( J < 0 \), \( m^*(J, \kappa) \) is a strictly increasing function of \( \kappa \).

**Supplementary references**