

BOSE–HUBBARD OPTICAL LATTICE

Peak on a peak

When is a condensate really a condensate? Calculations reveal that a ‘peak on a peak’ structure should be considered the true signature of the emergence of a Bose condensate in a Bose–Hubbard optical lattice.

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When Bose–Einstein condensation was first achieved in trapped atomic gases in 1995, things seemed much simpler in this system than in that of superfluid ^4He . However, studies of the superfluid–Mott insulator phase transition in optical lattices have indicated that the true signature of the superfluid phase is also controversial. Now, on page 617 of this issue, Kato and colleagues¹ report state-of-the-art quantum Monte Carlo calculations in the interacting Bose–Hubbard model for an optical lattice that clearly show that the unambiguous signature for a Bose condensate in this system is precisely the same as in liquid helium — namely, the appearance of a sharp peak in the momentum distribution $n(\mathbf{k})$ of the atoms, centred around $\mathbf{k} = 0$. Extending other recent work², Kato *et al.* have shown that, even in the presence of a strong on-site repulsion, periodic interference patterns in $n(\mathbf{k})$ cannot be used to identify the superfluid phase, because these patterns also occur well above the transition temperature, T_c .

Beginning with the striking experimental study of the superfluid–Mott insulator phase transition in optical lattices by Greiner *et al.*³, it has been generally assumed that the superfluid phase could be identified with the presence of well-defined interference patterns in the time-of-flight data associated with the reciprocal lattice of the optical lattice. The argument was that such a lattice of interference peaks was evidence of the coherent nature of the Bose-condensate wavefunction over many sites⁴. But this interpretation was called into question when model calculations² showed (albeit in the absence of on-site repulsion) that such patterns could arise well above T_c . In turn, this led to discussion of what the true signature of a Bose condensate in optical lattices is,

and how it might differ from the signature of superfluidity.

Kato *et al.*¹ start from the microscopic definitions of the Bose condensate and the superfluid fractions that are well known in studies of superfluid ^4He . The superfluid density is associated with the existence of persistent currents. These are very easy to measure in liquid ^4He , where superfluidity is a dramatic and visible phenomenon. The superfluid density can be defined formally in terms of a so-called winding number, which can be calculated using Monte Carlo techniques⁵. In contrast, the condensate fraction in a uniform system is the number of atoms with zero momentum and shows up as a delta function peak at $\mathbf{k} = 0$ in the momentum distribution $n(\mathbf{k})$ of atoms. The latter can also be calculated by Monte Carlo techniques and the condensate peak extracted. The definitive proof of the connection between superfluidity and an underlying Bose condensate in liquid ^4He came from numerical work⁵ that showed both become finite at the same temperature, $T_c = 2.18$ K. Similar calculations are now reported by Kato *et al.*¹ for the Bose–Hubbard optical lattice.

For the case of an interacting Bose–Hubbard optical lattice, Kato and colleagues have calculated the superfluid density and also extracted the condensate fraction from the momentum distribution $n(\mathbf{k})$. Their results are presented, as a function of temperature (both above and below T_c), for varying values of the ratio of the hopping to on-site repulsion — this ratio determines the essential physics of the model, including the superfluid–Mott insulator transition. The superfluid transition is calculated as the point where the superfluid density becomes finite. Experimentally, the accepted signature for superfluidity in trapped gases is the observation of a quantized vortex or a lattice of such vortices. Such a signature has not yet, however, been reported in Bose–Hubbard optical lattices.

Kato and colleagues’ plots of $n(\mathbf{k})$ clearly show that interference patterns

(peaks at the reciprocal lattice of the optical lattice) appear above as well as below T_c , even in the case of a strong on-site repulsion (see Fig. 2 of ref. 1). As these peaks appear on a broad background, they are described as a ‘bimodal’ structure. What is quite striking is the appearance of a growing narrow peak on the pedestal of the broader peak centred at $\mathbf{k} = 0$, which is the true signature of the formation of a Bose condensate. This ‘peak on a peak’ is described by the authors as a ‘trimodal’ structure. The delta-function condensate peak is studied as the hopping/on-site repulsion ratio decreases to the critical value corresponding to the transition to the Mott-insulator phase (where the superfluid transition temperature T_c vanishes).

As in the case of superfluid ^4He , the quantum Monte Carlo calculations reported by Kato *et al.* determine the superfluid and condensate fractions. Life is more difficult for experimentalists, who have to deal with instrumental broadening and also the errors arising from the assumption that the atom cloud expands ballistically after the optical lattice is switched off⁶. This ballistic assumption ignores interactions between atoms in the expansion, resulting in a final density distribution of atoms that gives a direct measure of the momentum distribution $n(\mathbf{k})$ of atoms before expansion. The equivalent assumption is made in extracting $n(\mathbf{k})$ from high-energy inelastic neutron scattering studies of superfluid ^4He . In that field, there has been extensive discussion of how to include the effects of ‘final states’, which need to be included to extract an accurate value of the condensate fraction⁷.

The momentum distribution of the non-condensate atoms is also strongly peaked at $\mathbf{k} = 0$ due to thermal and quantum effects. This causes problems in separating it from the sharp condensate peak when energy-resolution broadening is taken into account. As in the case of liquid ^4He experiments⁷, this will require

input from reliable calculations such as those reported by Kato *et al.* for the Bose–Hubbard optical lattice. This work is indeed an important milestone in the quantitative study of phase transitions in optical lattices.

References

1. Kato, Y., Zhou, Q., Kawashima, M. & Trivedi, N. *Nature Phys.* **4**, 617–621 (2008).
2. Diener, R. B., Zhou, Q., Zhai, H. & Ho, T.-L. *Phys. Rev. Lett.* **98**, 180404 (2007).
3. Greiner, M., Mandel, O., Esslinger, T., Hänsch, T. W. & Bloch, I. *Nature* **415**, 39–44 (2002).
4. Stoof, H. T. C. *Nature* **415**, 25–26 (2002).
5. Pollock, E. L. & Ceperley, D. M. *Phys. Rev. B* **36**, 8343–8352 (1987).
6. Spielman, I. B., Phillips, W. D. & Porto, J. V. *Phys. Rev. Lett.* **100**, 120402 (2008).
7. Sokol, P. E. in *Bose–Einstein Condensation* (eds Griffin, A., Snoke, D. W. & Stringari, S.) 51–85 (Cambridge Univ. Press, Cambridge, 1995).

SPACE EXPLORATION

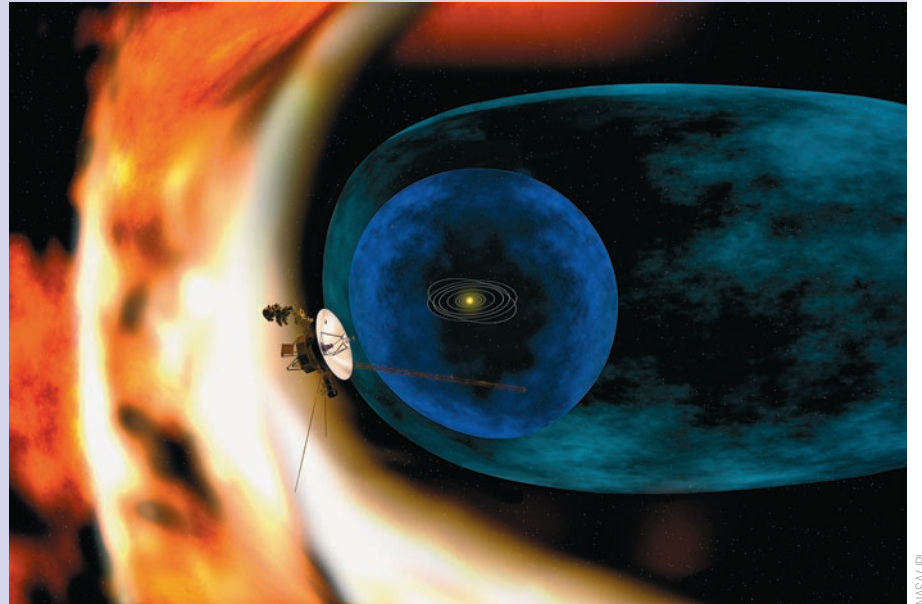
Boldly gone

Voyager 2 has left the Solar System. A series of papers in *Nature* documents its flight through the ‘termination shock’ of the heliosphere, as it heads out into the interstellar region (*Nature* **454**, 63–83; 2008).

Voyager 2 (shown in the illustration) and its twin, Voyager 1, embarked on their Grand Tour of the Solar System in 1977, flying past every one of the outer planets except Pluto. Although launched 16 days later than Voyager 2, but travelling in a different direction and at different speed, it was Voyager 1 that was first to reach the termination shock, in December 2004.

The shock marks the edge of the bubble known as the heliosphere, created by the solar wind of electrically charged particles (plasma). At the termination shock, the solar wind is slowed abruptly from supersonic to subsonic speeds by the pressure of gas and the magnetic field in interstellar space. Although it was initially expected that the heliosphere would be round, data from Voyager 1 suggested otherwise. Voyager 2, passing through the termination shock in 2007 at a point closer to the Sun and further south, confirmed the observation, indicating a possible local anisotropy of the interstellar magnetic field.

In fact, Voyager 2 passed the termination shock several times, owing to the oscillatory nature of the heliosphere, and was able to make detailed measurements of plasma waves, magnetic fields and energetic particles. Among other interesting findings, the data show



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unexpected temperature discrepancies that suggest a major role for ‘pick-up’ ions — interstellar neutral atoms that become ionized when entering the solar wind — in the shock’s dynamics.

The Voyager twins may remain operational for a decade or more, pushing further into the uncharted territory of interstellar space. In contrast, NASA’s Messenger spacecraft is probing the innermost regions of the Solar System, having completed the first flyby of Mercury for 33 years (*Science* **321**, 58–94; 2008).

Complementing and expanding on the analyses performed by Mariner 10 in the 1970s, Messenger has imaged more of the planet’s surface, and in different lighting conditions. The instruments on board have provided rich data on Mercury’s atmospheric and surface composition and its topography, as well as mapping the planet’s magnetic field and its interaction with the solar wind. More flybys are scheduled before Messenger moves into orbit around the planet in 2011.

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