Supplementary Methods

1 Single neuron model

Each neuron is simulated by the leaky integrate-and-fire model. The membrane potential $V(t)$ obeys the following equation:

$$C_m \frac{dV(t)}{dt} = -g_L(V(t) - V_L) - I_{syn}(t),$$

where $C_m$ is the capacitance, $g_L$ is the leak conductance, $V_L$ is the resting potential and $I_{syn}$ is the total synaptic current in the cell.

When the membrane potential $V(t)$ of each neuron reaches a threshold $V_{\text{threshold}} = -50\text{mV}$, a spike is emitted and $V(t)$ is set to the reset potential $V_{\text{reset}} = -55\text{mV}$ for a refractory period $T_r = 2\text{ms}$. For inhibitory neurons in the superior colliculus and the cortical network models, we used the following parameters: $C_m = 0.2\text{nF}$, $g_L = 20\text{nS}$ and $V_L = -70\text{mV}$. For all other neurons in the model, we used $C_m = 0.5\text{nF}$, $g_L = 25\text{nS}$ and $V_L = -70\text{mV}$.

The synaptic current $I_{syn}(t)$ is described by:

$$I_{syn}(t) = g_{\text{AMP}A}s_{\text{AMP}A}(t)(V(t) - V_E) + \frac{g_{\text{NMD}A}s_{\text{NMD}A}(t)(V(t) - V_E)}{1 + [\text{Mg}^{2+}]e^{-0.062V(t)/3.57}} + g_{\text{GABA}_A}s_{\text{GABA}_A}(t)(V(t) - V_I),$$

where $V_E (=0)$ and $V_I (=70\text{ mV})$ are the reversal potentials, $[\text{Mg}^{2+}] (=1.0\text{mM})$ is the extracellular magnesium concentration and $g$ is the synaptic efficacy. The gating variable $s$ obeys

$$\frac{ds(t)}{dt} = \sum_k \delta(t - t^k) - \frac{s}{\tau}$$

for AMPA and GABA_A receptor mediated currents and

$$\frac{ds(t)}{dt} = \alpha(1 - s(t))\sum_k \delta(t - t^k) - \frac{s}{\tau}$$

for NMDA receptor mediated current, with $\alpha = 0.63$. The decay constant $\tau$ is 2ms for AMPA, 100ms for NMDA and 5ms for GABA_A. $\delta(t - t^k)$ is the delta function and $t^k$ is the time of the $k$th presynaptic spike.

2 Short-term facilitation

We implemented short-term facilitation (STF) at the SCe-to-SCI synapses in the superior colliculus. The gating variable $s$ is multiplied by the STF factor $F$, which obeys the following dynamics [1]:

$$\frac{dF}{dt} = \alpha_F(1 - F)\sum_k \delta(t - t_k) - F/\tau_F,$$

where the dimensionless factor $\alpha_F$ equals 0.15 and the decay constant $\tau_F$ equals 1000 ms.
3 Synaptic connections and background inputs

All synaptic connections between neural populations or within a neural population are all-to-all. The values of synaptic efficacy $g$, unless otherwise specified, are:

for SC,

$$
\begin{align*}
&g_{N_{\text{NMDA}}}^{SCe^R-SCe^R} = g_{N_{\text{NMDA}}}^{SCe^L-SCe^L} = 1.5, \\
&g_{N_{\text{NMDA}}}^{SCe^R-SCi} = g_{N_{\text{NMDA}}}^{SCe^L-SCi} = 0.7, \\
&g_{N_{\text{NMDA}}}^{SCe^R-Cxi} = g_{N_{\text{NMDA}}}^{SCe^L-Cxi} = 0.11, \\
&g_{GABA}^{SCe^R-SCe^R} = g_{GABA}^{SCi-SCe^L} = 2.5,
\end{align*}
$$

for BG,

$$
\begin{align*}
&g_{GABA}^{SNr^R-SCe^R} = g_{GABA}^{SNr^L-SCe^L} = 2.5, \\
&g_{GABA}^{CD^R-SNr^R} = g_{GABA}^{CD^L-SNr^L} = 0.6,
\end{align*}
$$

and for Cx,

$$
\begin{align*}
&g_{\text{AMPA}}^{Cxe^R-SCe^R} = g_{\text{AMPA}}^{Cxe^L-SCe^L} = 3.5, \\
&g_{\text{AMPA}}^{Cxe^R-CD^R} = g_{\text{AMPA}}^{Cxe^L-CD^L} = 0.8 - 4.6, \\
&g_{N_{\text{NMDA}}}^{Cxe^R-Cxe^R} = g_{N_{\text{NMDA}}}^{Cxe^L-Cxe^L} = 0.085, \\
&g_{N_{\text{NMDA}}}^{Cxe^R-Cxe^R} = g_{N_{\text{NMDA}}}^{Cxe^L-Cxe^R} = 0.2805, \\
&g_{N_{\text{NMDA}}}^{Cxe^R-Cxe^L} = g_{N_{\text{NMDA}}}^{Cxe^L-Cxe^R} = 0.043825, \\
&g_{N_{\text{NMDA}}}^{Cxe^R-Cxe^R} = g_{N_{\text{NMDA}}}^{Cxe^L-Cxe^R} = 0.14462, \\
&g_{\text{AMPA}}^{Cxe^R-Cxi} = g_{\text{AMPA}}^{Cxe^L-Cxi} = 0.04, \\
&g_{N_{\text{NMDA}}}^{Cxe^R-Cxi} = g_{N_{\text{NMDA}}}^{Cxe^L-Cxi} = 0.13, \\
&g_{GABA}^{Cxe^R-Cxe^R} = g_{GABA}^{Cxe^L-Cxe^L} = 1.3, \\
&g_{GABA}^{Cxe^R-Cxi} = g_{GABA}^{Cxe^L-Cxi} = 1.0.
\end{align*}
$$

All values are in nS. Superscripts denote the presynaptic and postsynaptic populations. For example, $SCe^R-SCi$ represents $SCe^R$-to-$SCi$ connection, and $SCe^R-SCe^R$ represents the recurrent connection of $SCe^R$.

All neurons receive background Poisson inputs with mean conductance equal to 0.4864 nS for SCe neurons, 5.12 nS for SCi neurons, 13.76 nS for SNr neurons and 1.6 nS for CD neurons. We follow Ref. [2] for all the parameters, including numbers of neurons, values of synaptic strength and background Poisson inputs in the Cx network.

References
