This section describes the Monte-Carlo simulations we used to compare the observed distribution of visual displacement index (DI) to two others: 1- the predicted distribution if all neurons were either head-centered or eye-centered, and 2- the distribution predicted by our model.

**Eye-centered/head-centered bimodal visual DI distribution**

We first asked whether the observed distribution of visual DIs is significantly different from the expected distribution if all visual receptive fields (RFs) were either eye-centered or head-centered. The expected distribution was obtained through Monte-Carlo simulations involving the following steps:

1. We first estimated the noise in our neural recordings. We found that the variance of the spike count had a power law dependence on the mean with an exponent of 0.99 (rounded to 1) and a Fano factor (or multiplier) of 1.34. This value of Fano factor is within the range reported in other studies (Gershon et al., 1998).

2. We used this estimate of noise to generate an artificial dataset in which all receptive fields are either eye-centered or head-centered. Because we did not want to make prior assumptions about the shape of the receptive fields, we used the RFs of our 53 visual cells mapped when the animal looked at the central fixation point. For each cell, we first label the cell as head-centered with probability \( q \) or eye-centered with probability \( 1-q \). The value of \( q \) was adjusted to optimize the fit to the observed distribution (see below for details). For eye-centered cells, we generated the RFs for fixations to the right and to the left by shifting the central RF by the full extent of the eye displacement (DI=1, Supplementary Fig. 1). Those RFs were then corrupted with noise with the same statistics as the one observed in the data using 16 samples per position as was done in the experiment (i.e., each position on the screen was tested 16 times).

   We then used our covariance method (see Methods) to compute the DI for each cell from the artificial data. The same procedure was used for cells labeled as head-centered, except that the RFs were kept at the same location on the screen across eye fixation (DI=0). This procedure was repeated 500 times to obtain the expected distribution of DIs. Finally, we used a Kolmogorov-Smirnov test to ask whether the observed distribution of DI was significantly different from the expected one. The value of the probability \( q \) was chosen to maximize the P value of the Kolmogorov-Smirnov test, i.e. to minimize the probability of rejecting the null hypothesis that the two distributions of DI are the same. In other words, we placed ourselves in the worse-case scenario and optimized the fit between the expected and observed DI distributions. Even in this case, we found that the observed distribution is significantly different from the expected distribution if all receptive fields were either eye-centered or head-centered (\( P<0.009 \), Supplementary Fig. 2a).
Supplementary Figure 1: Monte Carlo simulation for generating receptive fields for variable displacement indices. The central receptive field of all 53 cells were interpolated linearly (using steps 1/10<sup>th</sup> of the distance between two adjacent visual stimulus locations). Then, for each cell, we shifted the central RF to the right and left by an amount proportional to the DI to generate RFs for rightward and leftward fixations. Next, the RFs (red dots) were corrupted by noise with the same statistics as what we observed in our data. Finally, the displacement index was estimated using our covariance method.

Supplementary Figure 2: Distribution of visual and tactile displacement indices. (a) Visual DI. The observed distribution of DI (black bars) is not significantly different from the distribution predicted by the model (dashed line) but is significantly different from the distribution predicted from a mixture of eye-centered and head-centered receptive fields (dotted line). (b) Same as in (a) for tactile DI. As for the visual DIs, the observed distribution of DI (black bars) is not significantly different from the distribution predicted by the model (dash line) but is significantly different from the distribution predicted by head-centered receptive fields (dotted line).
Visual DI distribution of the model

We used the same procedure as before except that the DIs were not set to 0 or 1, but were drawn from the DI distribution predicted by the model under the assumption that the visual tactile ratios were uniformly distributed over the interval [0.3, 1.2] (see Supplementary Fig. 3). This particular range of visuo-tactile ratio was chosen to optimize the fit to the observed distribution, i.e., to maximize the P value of the Kolmogorov-Smirnov test. Note that for a uniform distribution of visual tactile ratio, the corresponding distribution of DI is not uniform. The visual DI distribution has two modes, one around 0 and another around 0.9. This is due to the shape of the curves relating visual tactile ratio and DI, as illustrated on supplementary figure 3. This procedure was repeated 500 times to obtain the expected distribution of DIs for our model (Supplementary Fig. 2a). We found that the predicted and observed distributions are not significantly different (P>0.3).

Our analyses rely on the assumption that neural noise distribution is similar in all the cells, i.e. that the Fano factor is the same for all neurons. Clearly, this is not true for biological neurons, since Fano factors vary from cell to cell. However, these results are robust and do not seem to depend critically on the assumed level of neural noise: if we double the Fano factor to 2.68 for all neurons (rather than 1.34) the conclusions remains the same. The bimodal eye-centered/head-centered distributions is significantly different from the observed distribution (P<0.03) and the distribution predicted by the model is not significantly different from the observed distribution (P>0.4).

DIs are insensitive to gain changes with eye position, and thus, partial DIs cannot be explained by a gain modulation by posture, as observed in our sample and in many other brain areas. However, the Monte Carlo method cannot rule out eye position effects other than RFs shifts that would also affect DIs. For exemple, if RFs are wide enough we could imagine an overall change in the shape of the RFs without a shift of its boundary, that would result in partial DIs. We did not observe any systematic eye position effects other than gain changes and RFs shifts in our sample.

Distribution of tactile DIs

We have applied the same procedure as above to the DI distribution for the tactile receptive fields (Supplementary Fig 2b). This time, the DI were either all set to 0 (purely head-centered cell) or drawn from the unimodal distribution centered on 0.02, as predicted by the model (see supplementary Fig 3). The distribution of tactile DI is significantly different from the distribution expected from purely head-centered tactile receptive fields, P<0.009. This is difficult to see on Fig R3b which uses the same bin size as for the visual DI distribution, but it is explained by the fact that tactile RF mapping was performed at a higher resolution thereby allowing to measure small shifts quite reliably.

This result is quite surprising since the observed distribution of tactile DIs suggest that the tactile RFs are purely head-centered. Yet, our Monte-Carlo simulations suggest that the distribution is in fact slightly shifted, by an amount consistent with what our model predicts (on average by only 2%, but up to 10% for some cells).

Once again, these results are robust and do not depend critically on the assumed level of neural noise: if we double the Fano factor to 2.68 (rather than 1.34) the conclusions remains the same. The head-centered distributions is significantly different from the observed distribution (P<0.04) and the distribution predicted by the model is not significantly different from the observed distribution (P>0.3).
Supplementary Figure 3: Distribution of visual and tactile displacement indices predicted by a uniform distribution of visuo-tactile ratio. Central plot is reproduced from Fig. 8 in main text: solid line = visual DIs as a function of the visuo-tactile ratio; dashed line = same as the solid line but for tactile DIs. Bottom plot: uniform distribution of visuo-tactile ratio. Left plot: corresponding distribution of visual DIs. Right plot: corresponding distribution of tactile DIs.

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