IN RETROSPECT

On the Six-Cornered Snowflake

Philip Ball celebrates the fourth centenary of Johannes Kepler's ice-crystal analysis.

Did anyone ever receive a more exquisite New Year's gift than was given to the German scholar Johannes Matthäus Wackher von Wackenfels, 400 years ago? A booklet of just 24 pages, it was written in 1611 by his friend Johannes Kepler, court mathematician to the Holy Roman Emperor Rudolf II in Prague. In *De nive sexangula* (*On the Six-Cornered Snowflake*), Kepler attempts to explain why snowflakes have their striking hexagonal symmetry.

In his introduction, Kepler writes that he had noticed a snowflake on the lapel of his coat as he crossed the Charles Bridge in Prague, and so came to ponder on its remarkable geometry. This charming, witty work seeded the notion from which all of crystallography blossomed: that the geometric shapes of crystals can be explained in terms of the packing of their constituent particles.

Kepler's analysis of the snowflake comes at an interesting juncture. It unites the older, Neoplatonic idea of a geometrically ordered Universe that reflects God's design with the newly emerging mechanistic philosophy, in which natural phenomena are explained by proximate causes that may be hidden, or 'occult' (like gravity), but are not mystical.

Kepler was not the first to notice that the snowflake is six-sided. That is recorded in Chinese documents dating back to the second century BC, and in the West the snowflake's 'star-like' forms were noted by philosopher and theologian Albertus Magnus in the thirteenth century. René Descartes later explored Kepler's topic, drawing six-fold stars and ice 'flowers' in his meteorological book *Les Météores* (1637), and Robert Hooke's microscopic studies recorded in *Micrographia* (1665) revealed elaborate, hierarchical branching patterns.

"There must be a cause why snow has the shape of a six-cornered starlet," Kepler wrote in *De nive sexangula*. "It cannot be chance. Why always six? The cause is not to be looked for in the material, for vapour is formless and flows, but in an agent." This 'agent', he suspected, might be mechanical: the orderly stacking of frozen 'globules' that represent "the smallest natural unit of a liquid like water" — not explicitly atoms, but as good as.

Here he was indebted to the English mathematician Thomas Harriot, who acted as a navigator for Walter Raleigh's voyages to the New World in 1584–5. Raleigh sought Harriot's advice on the most efficient way to stack cannonballs on the ship's deck, prompting the ingenious Harriot to theorize about the close-packing of spheres. Around 1606–8, Harriot communicated his thoughts to Kepler, who returned to the issue in *De nive sexangula*.

Kepler asserted that hexagonal packing "will be the tightest possible, so that in no other arrangement could more pellets be stuffed into the same container". This assertion about maximal close-packing is known as Kepler's conjecture. It was proved using computational methods only in 1998, by US mathematician Thomas C. Hales, whose proof was published 7 years later (*Ann. Math.* **162,** 1065–1185; 2005). Kepler's vision of crystals as stackings of particles also informed the eighteenth-century mineralogical theory of René-Just Haüy, which forms the basis of all crystallographic understanding today.

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Mathematician Johannes Kepler explored the snowflake's six-fold geometry in 1611.

Less commonly acknowledged as a source of inspiration for Kepler is the seventeenth-century enthusiasm for cabinets of curiosities (*Wunderkammern*), collections of rare and marvellous objects from nature and art presented as microcosms of the universe. Rudolf II had one of the most extensive cabinets, to which Kepler would have had privileged access. The forerunners of museum collections, the cabinets have rarely been recognized as having any real influence on the nascent experimental science of the age. But Kepler mentions in his booklet having seen in the palace of the Elector of Saxony in Dresden "a panel inlaid with silver ore, from which a dodecahedron, like a small hazelnut in size, projected to half its depth, as if in flower" — a showy example of the metalsmith's craft that set Kepler thinking about how an emergent order gives crystals their facets.

De nive sexangula
JOHANNES KEPLER
First published 1611.

Yet in the end, Kepler is defeated by the snowflake's ornate form and plate-like shape. He realizes that although the packing of spheres creates regular patterns, they are not necessarily hexagonal or flat, let alone as ramified and ornamented as that of the snowflake. His failure to explain the regularity of the snowflake is no disgrace, for not until the 1980s was this seen to be a consequence of branching growth instabilities biased by the hexagonal crystal symmetry of ice.

Kepler instead fell back on Neoplatonic occult forces. God, he suggests, has imbued the water vapour with a "formative faculty" that guides its form. There is no apparent purpose to the flake's shape, he observes: the "formative reason" must be purely aesthetic or frivolous, nature being "in the habit of playing with the passing moment". That delightful image, which touches on the late Renaissance debate about nature's autonomy, remains resonant today in questions about the adaptive value (or not) of some complex patterns and forms in biological growth.

Kepler signs off his inconclusive tract with an incomparably beautiful variant of 'more research is needed': "As I write it has again begun to snow, and more thickly than a moment ago. I have been busily examining the little flakes."

But the influence of *De nive sexangula* goes further. It was in homage that British crystallographer Alan Mackay called his seminal 1981 paper on quasicrystals *De nive quinquangula*. Here, three years before the experimental work that won Dan Shechtman this year's chemistry Nobel, Mackay showed that a Penrose tiling could, if considered as the basis of an atomic 'quasi-lattice', produce five-fold diffraction patterns.

Quasicrystals showed up in metal alloys, not snow. But Mackay has indicated privately that it might indeed be possible to induce water molecules to pack this way, and quasi-crystalline ice was recently reported in computer simulations of water confined between plates (J. C. Johnston *et al. J. Chem. Phys.* **133,** 154516; 2010). Whether it can furnish five-cornered snowflakes remains to be seen. ■

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