Supplementary Notes: Derivation of the powerlaw spectrum

In the absence of the back-pressure from energetic electrons an approximate solution to the transport equation for energetic particles (Eq. (3)) can be obtained for the reconnection geometry shown in Fig. 4(a). The calculation follows along the lines of the 1-D shock calculations of the late 1970's. For simplicity we ignore the thermal conduction and take the width Δ_y of the multi-island region to be small enough to treat the drive term dc_{Ax}/dy as a discontinuity as in the shock case. It can't be a true discontinuity because we require $\nabla \cdot u = 0$. In any case integrate Eq. (3) along y from just upstream of the island region to the symmetry line where $u_y = 0$ and discard the back-pressure of the energetic particles. This gives

$$\Delta_y \frac{\partial}{\partial x} u_x f + \frac{A}{3} c_{Ax0} \frac{\partial}{\partial v} v f = u_{in} f_{in}, \tag{1}$$

where u_{in} is the inflow velocity and f_{in} and c_{Ax0} are the upstream distribution function and Alfvén speed. Now integrate this equation in the x direction from the symmetry line at $x = 0$ where $u_x = 0$ to some nonzero value of x. This gives

$$\Delta_y u_x f + \frac{A}{3} c_{Ax0} \frac{\partial}{\partial v} v \int_0^x dx f = x u_{in} f_{in}. \tag{2}$$

In this equation u_x increases linearly with x until $x = \Delta_x$ when it reaches its maximum value of c_{Ax0}, the Alfvénic outflow velocity. The solution of this equation is that f is independent of x for $|x| < \Delta_x$, which results in all of the terms of the equation varying linearly with x. We can then simply replace x by Δ_x in Eq. (2),

$$f + \frac{A\Delta_x}{3\Delta_y} \frac{\partial}{\partial v} v f = f_{in}, \tag{3}$$

where we have used $u_{in} = c_{Ax0}\Delta_y/\Delta_x$. This is readily integrated to produce Eq. (4) in the paper. We have found that the analytic powerlaw given in Eq. (4) is in surprisingly good agreement with the numerical solutions to Eq. (3) in the limit $\beta_0 = 0$, which is the limit in which the back-pressure is not important.