not suited for testing uneducated persons. A similar instrument, introduced by Chibret and Meyer, of Paris, is to be found in ophthalmic hospitals.

I may further remark that I do not consider any test satisfactory unless made by an ophthalmic surgeon, as he alone is accustomed to deal with such people every day of the week, and can alone eliminate such errors as refraction-disease and stupidity.

D. D. REDMOND.

14 Harcourt Street, Dublin, May 3.

The Green Flash at Sunset.

YOUR correspondents (vol. xli. pp. 495, 538) seem to imply that this phenomenon is only seen at sea, but I observed it on May 17 while walking from east to west, near Worms Heath (Warlingham, Surrey). It had been an exceptionally fine day, since the morning, and about 8 p.m. there was not a cloud in the sky, except to westward, where strips of cloud were rapidly forming, and covering up the glow of sunset; the sun had sunk behind a hill, when, suddenly, my companion and I both saw a flash of green light against the thickest cloud; it lasted 1 or 2 seconds, just long enough for there to be no doubt about it. We compared it to the glare thrown by "green fire," extending over an area whose diameter appeared about four times that of the moon.

At 12 p.m. the same night it was raining.

I think this observation definitely negatives the sea-wave theory, while the appearance was seen at least in association with the condensation of aqueous vapour. Perhaps the reason it was not *bluish*-green was that this vapour absorbed the blue rays?

T. ARCHIBALD DUKES.

16 Wellesley Road, Croydon, June 2.

THE THEORY OF SCREWS.[1]

THE book before us, a large octavo volume of over 600 pages, gives in a connected form the results of Sir R. S. Ball's investigation in the theory of screws, as contained in his "Theory of Screws" and a series of publications in the Proceedings and Transactions of the Royal Irish Academy. But as its scheme is that of a text-book on theoretical mechanics, it begins with a chapter on the postulates and methods of mechanics; whilst chapter vii. is on the theory of moments of inertia; chapter viii. on impulsive forces capable of imparting to a rigid body a given state of velocity; and chapter x., on kinetic energy, contains a number of propositions from analytical dynamics. Here expressions for the kinetic energy, for its change in consequence of an impulse, Lagrange's equations of motion in generalized co-ordinates, Hamilton's principle of least action, and various other propositions, are developed in the usual form—that is to say, without the use of screws. The rest of the book relates to the theory of screws and its applications. This alone, as forming the characteristic feature of the book, concerns us here, and of it we shall try to give an outline.

In order not to be unintelligible to those who have no knowledge of Ball's creation, it will be necessary to begin with the very elements of the subject; and in order to form a just idea of the scope and importance of the new method, it will not be sufficient to give a sketch of the results obtained—it will be necessary to take a wider view of the subject. We shall then be able to form some idea of the inherent capabilities of the theory. These I believe to be very great—very great indeed. One of its peculiarities lies in this, that all the results obtained in modern algebra and geometry, as distinct from analysis, seem to be directly applicable to it.

Friends of synthetic geometry and of graphical methods, too, will find here a wide field for investigations. Grassmann's "Ausdehnungslehre" has already been pressed into its service, and the theory of vectors and quaternions is easily applicable. Clifford, in fact, has generalized the latter theory into that of biquaternions to embrace screws.

Mr. Cartesius, to make use of Sir Robert's personifications, has been dethroned, and Mr. Anharmonic together with Mr. One-to-one reign in his place.

Poinsot, whose investigations form the starting-point of the theory of screws, has proved that a rigid body can always be transferred from one position to any other by a rotation about a certain perfectly determined axis, together with a translation along this axis. These two motions combine to a motion identical with that of a nut on a screw. It is completely determined if the angle through which the rotation takes place, together with the ratio of the translation to the rotation, is given. This ratio—*the "pitch" of the screw*—characterizes the screw. As the motion does not at all depend upon the diameter of the screw, we may suppose this to become infinitely small, and then we have the notion of Sir R. Ball's *screw*.

A screw, therefore, is a line in space which has connected with it a certain pitch—*i.e.* a certain length, as the pitch is a linear magnitude. The compound motion considered is called a "twist" about a screw, and is known if the screw and the "amplitude" of the twist, *i.e.* the amount of rotation, is given. In the same way a system of forces can, according to Poinsot, always be reduced, and that in one way only, to a single resultant and a couple turning about the resultant; and these two dissimilar parts Ball combines to a "wrench on a screw," the line of action of the resultant force being the axis of the screw and the ratio of the moment of the couple to the force giving the "pitch," whilst the magnitude of the resultant force is called the "intensity" of the wrench.

We have thus a new entity—the screw—and its introduction forms the characteristic distinction of the theory. Connected with it is a kinematical and a kinetic entity—the twist about a screw, and the wrench on a screw.

If we now consider a rigid body under the action of any forces, then the latter combine at every moment to a wrench on some screw, whilst the motion itself is always a twist about some other screw. If the body is constrained in any manner, then the reactions due to the constraint will also at every moment combine to a wrench about some screw.

The problem first to be solved is that of the combination of twists and wrenches. Let any two screws, α and β, be given, then wrenches on them constitute together a system of forces, and therefore combine to a wrench on some other screw, γ, which has to be determined. If the ratio of the intensities of the two given screws be varied whilst the screws themselves remain unaltered, then the screw, γ, of the resultant wrench also varies, and its axis describes a surface called the cylindroid. This is a ruled cubic surface which can be described as follows:—Let through a fixed line, l, a plane be drawn, and in it a circle be taken. Let a point, P, move uniformly in the circumference, whilst the plane itself turns uniformly about l, completing half a revolution whilst P describes the whole circumference. The perpendicular from P to l will then generate the cylindroid, and the screw on any generator will have a pitch equal to the length of the perpendicular from P to l. The line l is a nodal line of the surface and perpendicular to all screws on it. All cylindroids are similar, and through any two screws one cylindroid can always be drawn. The projections of all generators on a plane, perpendicular to the nodal line, form a flat pencil in which each ray corresponds to one screw. Also to each point on the circle corresponds one screw. We may here mention that this generation of the cylindroid stands in a very close relation to the plane representation of the cylindroid which is given in chapter xx. For if A, B are the ends of the diameter of the above circle which is perpendicular to the nodal line l, then to A and B correspond two generators of the cylin-

[1] "Theoretische Mechanik starrer Systeme auf Grund der Methoden und Arbeiten, und mit einem Vorworte von Sir Robert Ball, Royal Astronomer of Ireland." Herausgegeben von Harry Gravelius. (Berlin: Georg Reimer, 1889.)

droid which meet at right angles. Let the corresponding screws be α and β. Then if the circle when in a plane with α be turned about its diameter through a right angle it will be parallel to the plane of the pencil and may be taken to coincide with it. In this position we get the circle used in chapter xx. We recommend the reader to go through the first pages of this chapter when reading the third and fourth.

To combine two wrenches on two screws, α and β, we have to construct the cylindroid containing the screws and the flat pencil spoken of. If on the two rays in this pencil which are the projections of α and β the intensities of the wrenches be set off (they are the forces which together with couples constitute the wrenches), then their resultant gives not only the intensity of the resultant wrench, but it lies on the ray which is the projection of the screw of the resultant wrench. From this follows at once: Any two wrenches on screws of a cylindroid combine to a wrench whose screw lies again on the cylindroid; and conversely, a wrench on a screw belonging to a cylindroid can be decomposed into two wrenches on any two given screws on the cylindroid. Also, on any three screws of a cylindroid wrenches can be determined which are in equilibrium. It need scarcely be stated that the ratios only of their intensities are determined; but it is of importance to remember this.

The above results for the composition of wrenches hold also for twists about screws, provided that their amplitudes are very small, in conformity with the well-known fact that small rotations are combined in the same manner as forces. For this reason Sir R. Ball has limited his investigations to cases where the twist velocities have infinitely small amplitudes. These include equilibrium, beginnings of motion due to impulses and small oscillations. He also supposes the forces always to have a potential. Within these limits his results are of absolute generality.

The remarkable analogy between forces and rotations which appears in analytical mechanics rather as an accidental, though interesting, circumstance, is raised in the theory of screws to a principle of paramount importance.

If a rigid body acted on by a wrench receives a small twist, then the work done by the wrench is the product of the intensity of the wrench, of the amplitude of the twist, and of a geometrical factor which depends solely upon the two screws of the wrench and twist. Half this factor Ball calls "*the virtual coefficient of the two screws.*" If the screws meet it is proportional to the cosine of the angle between them; if the pitches of both screws vanish, or more generally if their sum vanishes, it becomes the moment of the two lines on which the screws lie. It partakes, therefore, of the nature of both these quantities, and its analogies to the cosine especially are, in many cases, very marked. If the virtual coefficient vanishes, then no work is done by the wrench in consequence of the twist. Now the virtual coefficient of two screws, α and β, depends symmetrically on both, hence if a wrench on α does no work when the body is displaced by a twist about β, then also a wrench on β does no work during a twist on α. For this reason two screws whose virtual coefficient vanishes are called *reciprocal*.

An immediate consequence of the definition of reciprocal screws is this, that a screw which is reciprocal to two screws, α, β, is reciprocal to all screws on the cylindroid determined by α, β. For a twist about any screw, γ, on the cylindroid can be decomposed into two about α and β; but the wrench can do no work against these, and therefore it can do no work against a twist about γ.

It is also proved that through every point in space there pass a single infinite number of screws, which are reciprocal to a cylindroid. These lie on the generators of a cone of the second order. Similarly, all screws in a plane which possess the property in question envelop a conic, and in chapter xxi. it is shown that this is always a parabola.

Two screws which meet can be reciprocal only if they meet at right angles or if the sum of their pitches vanishes. This gives rise to one of the most powerful methods for finding reciprocal screws. Thus, as every line meets a cylindroid in three points, and therefore cuts three screws on it, and as the cylindroid contains only two screws of equal pitch, it follows a screw, α, reciprocal to a cylindroid must cut one screw on it at right angles, and the two others which it meets must have equal pitches, viz. these must be equal and opposite to the pitch of α; and from this, again, it is easily deduced that every line which meets one screw on a cylindroid at right angles cuts, besides, two others which have equal pitch; for if on this line a screw be taken with a pitch equal to one of the two remaining screws which it cuts, it will be reciprocal to the cylindroid.

Just as two wrenches on screws α and β always combine to a wrench on a screw lying on a certain cylindroid, so three wrenches on screws α, β, γ, which do not lie on a cylindroid, combine to a wrench on a fourth screw which is connected with the three given ones, and which depends on the two ratios only of the intensities of the three given wrenches.

The entirety of all the screws which are got by varying these ratios forms a system of a double infinite number of screws, which has been called a screw-complex of the third order.

If any four screws belonging to such a complex are selected, then a wrench on one of them can be decomposed into three wrenches on the others. It is also always possible to determine wrenches on the four screws which are in equilibrium, and the ratios of their intensities alone are then determined. Similarly, five independent screws, *i.e.* screws which do not belong to a complex of lower order, give rise to a complex of order five, and six independent screws to a complex of order six. To this latter complex all screws in space belong, for in chapter v. it is shown that in general any wrench can be decomposed into six wrenches on six arbitrarily selected screws. A screw-complex of order two is nothing but a cylindroid, and a complex of order one consists of one single screw. That a complex of order six exhausts all screws in space, says only that the number of all screws is ∞^5, if ∞^1 denotes the number of points in a line, or the number of values which a single real variable, x, may assume. That the number of all screws is ∞^5 is also at once evident if we consider that the number of lines in space is ∞^4, and that on each line we have a single infinite number of screws which are obtained by giving its pitch all possible values from $-\infty$ to $+\infty$.

There is an important theorem that the screws which are reciprocal to all screws in a complex of order n form themselves a complex of order $6-n$.

One of the chief uses made of these results consists in the introduction of screw co-ordinates, viz. six independent screws are selected as co-ordinate screws. Then the intensities of the components of a wrench on these six screws are taken as the co-ordinates of the wrench. In the same way the co-ordinates of a twist are obtained. Lastly, by the co-ordinates of a screw are understood the co-ordinates of a wrench of unit intensity on the screw, or those of a twist of unit amplitude about it. To get, then, the co-ordinates of any wrench on, or a twist about, a screw, the co-ordinates of the latter have only to be multiplied by the intensity of the wrench or the amplitude of the twist. Between these screw co-ordinates exists, however, an equation of the second degree, just as between the ordinary homogeneous point co-ordinates there exists a linear equation. A screw is thus completely determined by the ratios of its six co-ordinates, *i.e.* by five numbers, which again shows that there are ∞^5 screws in existence. Having established the notion of these co-ordinates, there are next given, in chapter v., expressions in terms of the co-ordinates for the resultant of a number of wrenches or twists, for the work done by a wrench on one screw during a twist on another, and so on. These expressions are much simplified by selecting the screws of reference in a particular manner, viz. so that any two of them are reciprocal, and such a system of "co-reciprocal" screws is afterwards always used.

The expression for the virtual coefficient of two screws is in general a lineo-linear function of the co-ordinates of both screws. But this is simplified for the special system of co-ordinate screws just mentioned, in reducing to an expression of six terms only, each being the product of the co-ordinates of the two screws relating to the same co-ordinate screw into the parameter of this screw. This expression must vanish if the two screws shall be reciprocal. Hence the condition that a screw shall be reciprocal to a given screw is expressed as a linear equation between its co-ordinates, and it is important to add that every linear equation between its co-ordinates can be interpreted as meaning that the screw is reciprocal to some other screw. But one linear equation enables us to express one of the co-ordinates in terms of the others, so that all the co-ordinates of all screws which are reciprocal to a given screw can be expressed in terms of five co-ordinates, in other words, a screw in a complex of order five is determined by five co-ordinates. In the same way two linear equations limit a screw to a complex of order four, and so on, till we come to five equations as determining one single screw; which also shows that there is always one screw which is reciprocal to five given screws.

We leave for the moment the line followed by Ball and Gravelius, in order to indulge in some very general speculations, in close connection with chapter xix., which seem best suited to give, in as short a compass as possible, a clear insight into the nature of the whole system of screws.

We are accustomed to express the fact that the number of points in a plane is ∞^2 by saying a plane, or in fact any surface, is of two dimensions if we consider the points as elements. Space is, in the same sense, of three dimensions, whilst it is of four dimensions if we consider the lines as elements.

We may extend this language, and say the aggregate of all screws forms a space of five dimensions, or as Clifford would have said, it is a five-way spread. If we now assume between the co-ordinates one equation, we may speak of the locus of screws whose co-ordinates satisfy this equation. It will be a four-way spread, or a space of four dimensions. This locus is called by Ball a screw-complex of order five and degree m, if m is the degree of the equation. The complexes spoken of before are of the first degree.

The geometrical theory of screws becomes thus identical with the geometry of a space of five dimensions, which latter we may call the screw-space.

Let us consider now two such complexes of 1st degree, one of order m, the other of order n. The first is determined by a set of $6 - m$ linear equations between the co-ordinates, the second by $6 - n$ such equations. All screws common to both have therefore to satisfy $12 - m - n$ equations, and in case that this number is not greater than six, they will constitute a complex of order $6 - (12 - m - n) = m + n - 6$. Thus a complex of order 4 and a complex of order 5 will have a complex of order 3 in common, whilst two complexes of order 3 will in general have no screw in common, though they may have a single screw or a whole cylindroid in common.

The geometrical theory of screws as the geometry of a particular space of five dimensions is not a mere extension of the ordinary Euclidian geometry. The six homogeneous co-ordinates of a screw are, as has already been mentioned, connected by an equation. This is of the form $R = 1$, where R is a quadratic expression of the co-ordinates. All elements at infinity in our screw-space are given by the equation $1 = 0$ or by $R = 0$. The *absolute* is thus a quadric locus, and therefore we have to deal with non-Euclidian geometry.

The advantage to the theory of screws to be derived from a study of this geometry are apparent at every step. We may in our screw-space conceive curves and surfaces of from 1 to 4 dimensions, by taking one or more equations between the co-ordinates. Of these, equations of the first degree determine the screw-complexes. But equations of the second degree, which determine quadric complexes, or as Ball calls them screw-complexes of second degree, are also constantly of use. Such an equation may be taken in a complex of order n. In the treatise before us they appear in congruences of the 3rd and 6th order. We will give here one illustration.

Let $p_1, p_2, \ldots p_n$ be the pitches of the n co-reciprocal co-ordinate screws, and let $a_1, a_2, \ldots a_n$ be the co-ordinates of a screw a with pitch p_a. Then is p_a given by the equation

$$p_a = p_1a_1^2 + p_2a_2^2 + \ldots + p_na_n^2.$$

This equation can be made homogeneous by aid of $R = 1$, and becomes

$$Rp_a = p_1a_1^2 + p_2a_2^2 + \ldots + p_na_n^2,$$

where R also is supposed to contain n of the a only, the others being replaced by aid of the linear equations which determine the complex of order n. It follows that the absolute $R = 0$ is the locus of screws of infinite pitch, whilst

$$p_1a_1^2 + p_2a_2^2 + \ldots + p_na_n^2 = 0$$

is the locus of screws of zero-pitch. Both are quadrics.

If we take a screw β, we may form its polar with regard to any quadric. If we select the last quadric mentioned, the polar is

$$p_1a_1\beta_1 + p_2a_2\beta_2 + \ldots + p_na_n\beta_n = 0.$$

But this equation is also the condition that a and β are reciprocal screws. In each complex the quadric of zero-pitch becomes thus of special importance, reciprocal screws being conjugate poles with regard to them.

As we cannot directly realize a space of more than three dimensions, it becomes of importance to represent the elements in such a space by other elements in ordinary space, and, when possible, by elements in a plane. That this is always possible is clear.

For instance, as all conics in a plane are ∞^5 in number, we have as many conics in a plane as there are screws in space, and we may therefore represent each screw by a conic in a plane. To screws on a cylindroid would then correspond all conics in a pencil. We might then speak of the cross-ratio of four screws as given by the cross-ratio of the corresponding conics in the pencil. All screws belonging to a complex of order 3 would be represented by conics forming a net, *i.e.* by conics having a common polar triangle.

We thus get a graphical representation in a plane, and can obtain our results by constructions in a plane. But the geometry of conics in a plane has scarcely been far enough developed to make general use of them, and for screw-complexes of lower order simpler representations may be found. Thus the screws on a cylindroid can be represented most conveniently by the points on a circle which stands in close relation to the cylindroid and gives rise to a graphical solution of problems relating to a body with two degrees of freedom. This is done in chapter xx., full of interesting detail. Again, screws in a complex of order 3, whose number is ∞^2, can be represented by points in a plane. This has been worked out in chapters xxi. and xxii. In fact, here the screw co-ordinates, three in number, are simply taken as tri-linear co-ordinates of a point. It follows at once that the locus of points with equal pitch must be a conic, the "absolute" being the locus of pitch ∞, and one conic relates to zero-pitch. This latter may, without loss of generality, be made a circle.

It is of interest to notice that for a screw-complex of order 3 the screws which have a given pitch form themselves a quadric surface, viz. they form one set of generators on a hyperboloid, the other set of generators having pitch $-p$, and containing thus screws in the complex reciprocal to the others.

Other quadrics enter the theory, especially one containing the locus of screws about which a body may twist without receiving kinetic energy, and which is, of course, imaginary; and one connected with the potential. These last two determine the principal screws of inertia, of which more later on.

For screw-complexes of order 4 no graphical representation is given. The difficulty lies here in this—that the dynamics require constantly metrical relations, and these are not very simple in the plane representation, by conics for instance. It is here that the non-Euclidian character of the geometry comes out.

These speculations are in close connection with the contents of chapter xix., where projective relations between two congruences of the same order are investigated. It is here that Herr Gravelius has more particularly introduced original work of his own in bringing Sir R. Ball's Mr. One-to-one more prominently to the foreground.

Up to this we have considered chiefly the geometry of systems of screws. It is now time to consider the kinematics of a rigid body and the action of forces on it.

If a body is perfectly free it can twist about every screw in space. As these can be decomposed into six twists about the co-ordinate screws, the body is said to have six degrees of freedom. If the body is constrained in any manner—and here the generality of the nature of the constraint has to be noticed—then it will not be able any longer to twist about all screws. But we have seen already if it can twist about n screws it can twist about all screws belonging to the complex of order n derived from them. The freedom of a body is therefore fully characterized by the complex which contains all screws about which the body can twist. If this is of order n, then the body has n degrees of freedom. An attempt to twist the body about any other screw will evoke a reaction due to the constraint which will reduce to a wrench upon some screw. Such a wrench cannot do any work against a possible twist of the body, hence the screws on which wrenches of constraint are possible must be reciprocal to the screws which determine the freedom of the body; they form, therefore, the reciprocal complex. We thus get the very general theorem about the equilibrium of a body. If a body has n degrees of freedom then it will be in equilibrium under the action of all wrenches on screws of a certain complex of order $6 - n$. This complex may be called the complex of constraint.

Again, if a body is subjected to an impulsive wrench upon a screw, η, not belonging to the complex of constraint, it will begin to turn about some screw, α, called the instantaneous screw. At the same time an impulsive wrench of constraint will be evoked. Conversely, in order to produce a twist on α as instantaneous screw we may apply an impulsive wrench on η, but with this we may combine a wrench on any one of the screws belonging to the complex of constraint. As the latter is of order $6 - n$, all screws derivable from these, together with the screw η will form a complex of order $7 - n$ This complex of order $7 - n$ and the complex of order n which determine the freedom have $7 - n + n - 6 = 1$ screw in common (see above). This screw is called the reduced impulsive wrench.

We thus have proved if a body has freedom of order n, then there is always one and only one screw, η, in the complex which determines the freedom, such that an impulsive wrench on it makes any given screw, α, the instantaneous screw. The converse, also, is evidently true. Between the impulsive and instantaneous screw in the complex exists, therefore, a one-one correspondence, or, to express this differently, the complex of instantaneous and that of impulsive screws are projective. They are also coincident. But if we have two coincident projective spaces of $n - 1$ dimensions, then there are always n screws in one which coincide with their correspondents. This proves if a body has n degrees of freedom, then there exist n screws, and in general only n, such that an impulsive wrench on one of them produces a twist on the same screw. These n screws—and the discovery is one of the triumphs of the theory—are called the principal screws of inertia, as they depend on the distribution of matter in the body. These screws are also co-reciprocal, and may therefore be taken as co-ordinate screws. They are a generalization of the principal axes of inertia in the ordinary theory; and to show their importance it is sufficient to point to the importance of the principal axes of inertia in the ordinary theory of a free body, or of a body of which one point is fixed, and to remember the simplification obtained by taking them as axes of reference.

For a free body the screws of inertia lie on the principal axes of the body which pass through the mass-centre, two on each, with pitches equal to the corresponding radius of gyration, taken positive for the one and negative for the other. The ordinary theory has no analogon to this if the body is constrained, excepting in the few cases where a point or an axis of the body is fixed, or where the body has plane motion only.

It is in such generalizations that the theory of screws excels. It has given us here the best and simplest co-ordinates for all cases of the motion of a single rigid body acted on by any forces and constrained in any manner conceivable.

We will now suppose that the co-ordinates thus pointed out are used, and find the instantaneous screw corresponding to any given impulsive wrench. Each component wrench produces a twist about its own screw, whose amplitude depends in a very simple manner on the intensity of the impulsive wrench; so that the intensities of the component twists are known, and these give the resultant twist.

We next consider the kinetic energy, T, of the body due to a twist on a screw, α. Let $\alpha_1, \alpha_2, \ldots$ be its components, $p_1, p_2, \ldots$ the pitches of the co-ordinate screws, and $\dot{\alpha}$ the twist velocity. It is then shown that, M being the mass of the body,

$$T = M\dot{\alpha}^2(p_1^2\alpha_1^2 + p_2^2\alpha_2^2 + \ldots + p_n^2\alpha_n^2).$$

Denoting the expression in the brackets by u_α^2, we have $T = M\dot{\alpha}^2 u_\alpha^2$. The quantity u_α is a length; the expression for T is therefore of the same form as that for the rotation of a body about an axis with angular velocity $\dot{\alpha}$, the radius of gyration being replaced by $u_\alpha/\sqrt{2}$. This last expression deserves a name. If we adopt Clifford's word "spin-radius," instead of radius of gyration, the name twist-radius suggests itself as suitable for u_α or $u_\alpha/\sqrt{2}$.

We now come to consider the problem of small oscillations. Let there then be a body of n degrees of freedom in a position of equilibrium under a system of forces which have a potential V. Let A denote the complex defining the freedom. If the body be displaced by a small twist about a screw, α, belonging to the complex A, then the forces are not any longer in equilibrium; hence they will give rise to a wrench on some screw λ. This wrench may be combined with any wrench of constraint; but just as in case of impulsive wrenches there is one single screw

λ belonging to the complex A, hence now also we have in the complex A a one-one correspondence between the screws α and the screws λ. There are therefore, again, n screws α, which coincide with their corresponding screws λ. These have got the name of "principal screws of potential." They depend on the system of forces or on the potential V, just as the screws of inertia depend on the distribution of matter. These n screws, again, are co-reciprocal. They have the property that a twist about one of them evokes a wrench on the same screw, the wrench being due to the applied forces. To show the importance of these principal screws of potential it will be sufficient to remark that the potential is, under the circumstance explained, a homogeneous function of the second degree of the n co-ordinates by which the displacement is defined. This function becomes the sum of n terms containing the squares only of the co-ordinates if the principal screws of potential are taken as co-ordinate screws.

Now, suppose that the body has been displaced by a twist about a screw α, this could be done by a wrench upon the screw η, which as impulsive screw corresponds to α as instantaneous screw. At the same time this displacement calls a wrench on a screw λ into play due to the potential V. To every screw α corresponds thus one screw η and one screw λ. Hence the latter are also connected by a one-one correspondence, and there are therefore n screws α such that the corresponding screws η and λ coincide. The screws α thus obtained are called "harmonic screws." They possess this property: A twist about a harmonic screw evokes a wrench which in its turn tends to produce a twist on the original harmonic screw. Hence if the equilibrium is stable this wrench will tend to twist the body back to the position of equilibrium, and thus produce small oscillations about the harmonic screw. From this we get the following theorem, distinguished again by its great generality:—

If a rigid body having n degrees of freedom is in a position of stable equilibrium under the action of a system of conservative forces, then it can, on being disturbed, perform n distinct oscillations, which consist each of a twisting about a single screw. Every other oscillation is a combination of these.

These are the chief results which so far have been obtained by the theory of screws as applied to a single rigid body. They form the contents of chapters vi. to xii. These general results are, in the next six chapters, applied and considered more in detail for each of the six possible cases of degrees of freedom which a rigid body may have. Then there follow four chapters on graphical methods, already referred to.

All the former investigations relate to one single rigid body. But Sir R. Ball, in 1881, published a paper in which he extends his theory to systems of rigid bodies by a method as beautiful as it is suitable to the purpose.

The bodies, of which we suppose there are μ, are taken in a definite order. Every body of the system will at every moment twist about some screw. We thus get a et of μ screws, about which at any moment the bodies twist. If we take two consecutive twists, then their resultant depends only on the ratios of the two amplitudes, and conversely the screw of the resultant determines this ratio. If the screws about which two consecutive bodies twist are given, and also the screw on which their resultant lies, then the amplitude of the first twist determines that of the other. If, therefore, the screws about which the μ bodies twist at any moment are given, and besides the $\mu - 1$ screws on which the resultant twists of consecutive bodies lie, then the amplitude of the first determines that of every other twist. The set of $2\mu - 1$ screws thus obtained is called a screw-chain, and it is said that the system of bodies twists at any moment about a certain screw-chain.

In case of systems of rigid bodies, the screw-chain has to be considered as the fundamental entity, which takes the place of a screw in case of a single body.

In a finite number of bodies we get a screw-chain of a finite number of screws. These will, in the screw-space of five dimensions, be represented by a finite group of points (elements). If, however, the number of bodies increases and becomes infinite, as in the case of the molecules of a fluid, this group of points may form a continuous locus of one or more dimensions. We may thus get, instead of screw-chains, continuous curves and surfaces of screws, and their geometry will be that of a group of points in five-dimensional non-Euclidian space.

This suggests an enormous field for investigation, and it is of interest to see that every progress in the algebra and geometry of such a space must indicate also progress in dynamics.

But these are speculations far beyond the contents of the book under review.

All results obtained for twists can at once be transferred to wrenches. Accordingly a system of forces acting on a system of bodies can be reduced to a wrench upon a screw-chain.

There are reciprocal screw-chains, screw-chains of inertia, complexes of screw-chains, complexes of freedom and of constraint, and complexes reciprocal to them. In fact, the screw-chain seems now to take in every respect the place of the screw in the theory of a single body. These screw-chains in their kinematical and dynamical applications to systems of rigid bodies form the contents of the chapters xxiii. and xxiv.

The last two chapters in the book give Sir R. Ball's theory of content, in which the author tries successfully to overcome the difficulty which offers itself in the determination of metrical relations without any reference to measuring a length. By "content" is understood the aggregate of all elements in what Clifford called a three-way spread. The investigation is carried on quite algebraically by aid of the methods of Grassmann's "Ausdehnungslehre." In the book before us this is worked out, partly with reference to Clifford's theory of biquaternions, and ends with the introduction of Clifford's vectors in non-Euclidian space.

It will be asked what progress in the science of dynamics, and through dynamics in natural philosophy, has been made by Ball's creation. The theory of screws is a mathematical speculation full of life, full of interest and charm for the mathematician who likes to find new physical interpretations for geometrical and algebraical results and methods. The physicist, however, may say that the theory does not increase our power over Nature. But I am inclined to think that when further developed it will be a great, perhaps a very great, help to progress. Does not every molecule of a fluid having rotational motion twist about some screw? And does not a vortex-line suggest a screw-chain containing an infinite number of elements?

The theory of screw-chains, containing a finite number of elements belonging to a system of bodies with one degree of freedom, seems to indicate a truly scientific classification of mechanisms, and may conceivably render great aid in the invention of mechanisms which answer a given purpose.

The essentially geometrical character of the new method seems particularly well adapted to give graphical solutions of dynamical problems, and thus a "graphical dynamics" appears to find here a sound foundation. In this direction much has been done already, but much remains to be done. Also the restrictions of infinitely small amplitudes of the twists has to be broken through, and the infinitesimal calculus has to be pressed into the service.

Meanwhile, we congratulate Sir Robert Ball on the results which his persevering labours have achieved, and Herr Gravelius on the courage which led him to under-take the task of writing a text-book on this subject, and on the success with which he has accomplished it. The book ought to give a great impulse to the study of this theory, and to enlist many friends in its service.

O. HENRICI.

THE SIXTH SCIENTIFIC CRUISE OF THE STEAMER "HYÆNA" WITH THE LIVERPOOL MARINE BIOLOGY COMMITTEE.

THE Liverpool Salvage Association having kindly placed their s.s. *Hyæna* once more at the disposal of the Liverpool Marine Biology Committee, a four-days' dredging cruise was arranged and successfully carried out at Whitsuntide. The old gunboat left the Mersey on Friday, May 23, and steamed to the Menai Straits. Some of the party spent the afternoon and evening collecting on the shore at Puffin Island, off which the *Hyæna* was anchored for the night. On the following morning, after a few hauls of the dredge near Puffin Island, and between Penmon Point and Beaumaris, and again off Port Dinorwic, the steamer went through the straits to Carnarvon Bay, and commenced working along the southern coast of Anglesey.

The dredges and various kinds of tow-nets, surface and bottom, were used at intervals. Mr. W. E. Hoyle's deep-water closing net, which has now been modified so that its movements of opening and closing are effected by the passage of an electric current, was experimented with frequently during the cruise—not so much with the object of collecting specimens, as for the purpose of detecting and remedying any possible defects in the construction, and of guarding against conditions which might interfere with the proper action of the apparatus. On the whole the net worked satisfactorily, the causes of occasional failures were discovered, and when the improved form of frame used by the Germans has been adopted, the apparatus will no doubt be a most useful addition to the implements of the marine biologist.

The *Hyæna* anchored for the night in a small rocky bay, Porth Dafarth, on the south side of Holyhead Island, Anglesey, and half the party of over twenty biologists were landed to sleep on shore. After dark those who remained on board commenced tow-netting by electric light, and repeated with some modifications the experiments which had been made during the last two cruises of the *Hyæna* at the Isle of Man (NATURE, vol. xxxviii. p. 130, and vol. xl. p. 47) in 1888 and 1889. The large arc lamp was hoisted over the side of the ship so as to throw a strong glare on the water, and Edison-Swan incandescent lamps were sent down to the bottom in tow-nets which were hauled up at intervals. Comparatively few Cumacea, Amphipoda, and Schizopoda were obtained this time, but shrimps and young fishes were—for the first time in our experience—attracted by the light to the surface, and some of them were caught and preserved. One of the ship's boats was kept in the area illuminated by the arc lamp, and by leaning over her side the small objects in the surface-layer of water could be most distinctly seen, and particular animals picked out and captured with a hand-net as they darted about in the neighbourhood of the light.

Two of the party got up at 3 a.m., and took a surface tow-netting about dawn, which was afterwards found to contain a much greater number of Copepoda, and more variety, than any of the other tow-nettings, either day or electric light, surface or bottom. Amongst other interesting things it contained a large number of *Peltidium depressum*, which had not been taken at all during the day, and only in very small numbers with the electric light bottom net. This same species has recently been taken in quantity at Puffin Island by leaving a tow-net out all night attached to a buoy. It is usually found sticking on *Laminaria* in the day-time, but evidently comes to the surface in abundance late at night or early in the morning.

The following day was spent in steaming slowly about off the southern coast of Anglesey, dredging and tow-netting at frequent intervals. The surface life was found to be very poor—comparatively few Copepoda and almost no representatives of other free-swimming groups being obtained; but Mr. Thompson noticed the relative abundance in all the tow-nettings, both surface and bottom, during the day, and also with the electric light, and at dawn, of unusually large specimens of *Dias longiremis*, and also the prevalence of the somewhat uncommon *Isias clavipes* in all the surface gatherings, though none were taken in the bottom ones.

The dredging results were fairly good: some very fine sponges were obtained, and Ascidians were plentiful. One patch of rich ground was discovered near Rhoscolyn Beacon, where *Comatula* was brought up in abundance along with various Tunicata, Holothurians, Nudibranchs, Zoophytes, Polyzoa, and large sponges. After dark, in Porth Dafarth, the electric lights were again used for a couple of hours. This time the large arc lamp was taken to the stern and suspended close to the surface of the water, but as it was not working steadily one of the incandescent submarine lamps was lowered over the side and kept a few inches under water, and this proved most effective in attracting animals to a stationary tow-net or a hand-net beside it. On the fourth day the *Hyæna* returned through the Menai Straits to Liverpool. As usual the specimens collected have been distributed to specialists, and the detailed reports upon the various groups will appear in the next volume of the "Fauna of the Liverpool District."

W. A. HERDMAN.

W. S. DALLAS.

THE death of this genial and accomplished man will awaken feelings of no ordinary regret, not only among geologists, but among naturalists all over the country. For two-and-twenty years his tall, handsome person has been the most familiar figure at the rooms of the Geological Society in Burlington House. Always at his post, with a pleasant smile of welcome, ever ready with assistance from his large treasures of knowledge and experience, knowing more intimately than anyone else the affairs and traditions of the Society, proud of its history and keenly sensitive for its scientific reputation, he had come to be looked upon as a kind of *genius loci*—the living embodiment of the Society's aims and work.

Of those who knew Mr. Dallas only in his later years, and saw his whole-hearted devotion to the geological labours intrusted to him, probably few were aware that he was not always a geologist. He began life with zoological inquiries, and devoted his attention more especially to insects. His early papers appeared in the Transactions of the Entomological Society, but he prepared also a Catalogue of the Hemipterous Insects in the British Museum, which was published as far back as the years 1851–52. Yet he did not confine himself to one branch of zoology; on the contrary, his reading and knowledge ranged over a wide domain in natural history. In the year 1856 he published his "Natural History of the Animal Kingdom," by far the best work of the kind in its day, which rendered important service to biology, in making the study of living forms more attractive, and in providing for that study a much more accurate groundwork than had ever before been obtainable. The value of his labours was recognized not long afterwards by his being appointed Curator of the Yorkshire Philosophical Society's Museum at York—an office which he held for ten years, until in 1868 he obtained the post which he held up to the last—that of Assistant Secretary, Librarian, and Curator to the Geological Society of London.