THURSDAY, SEPTEMBER 19, 1889.

THE BRITISH ASSOCIATION.

NEWCASTLE-ON-TYNE, *Tuesday Evening*.

AMID much that has been stale and dull, the present meeting has not been without excitement. The discussions on heredity and Darwinism in D, Mr. Du Chaillu's attempt to make the Vikings responsible for the English-speaking peoples, and Dr. Nansen's appearance in several Sections, have all helped to keep the Sectional work alive, not to speak of the considerable number of papers of special scientific interest. Of the Sectional addresses, those of Prof. Geikie, Mr. Anderson, and Mr. Edgeworth are most spoken of for novelty and interest, the latter, curiously, for the touches of fun, if not humour, that the author succeeded in imparting into what seems at first sight a dry subject.

As to Newcastle itself, everybody is satisfied with the treatment this young city has accorded to the Association, and everybody is pleased to see the footing which science is getting in the place in the splendid new Science and Medical Schools. It is, moreover, gratifying that, in a district where the practical applications of science are everything, distinct acknowledgment should have been made of the value and indispensability of pure scientific research. This was done on two occasions: first, in the admirable speech of the Mayor (of whom all speak well) at the magnificent banquet given by Lord Armstrong; and again before an audience of over 3000 working men in the Drill Hall, when the various trades associations presented an address to Prof. Flower, before Mr. Baker gave his lecture on the Forth Bridge, and thus recognized that only as the result of long series of complicated researches, undertaken solely for themselves, has the accomplishment of such a stupendous undertaking been possible. It goes without saying, that Mr. Baker's lecture was a marked success; it is difficult to describe the enthusiasm evoked from an enormous audience keenly capable of appreciating the points. It was stated that over 8000 applications had been made for tickets, when only 3500 could be allotted.

The social arrangements have been all that could be wished. It would be difficult to convey any idea of the thoughtful arrangements that have been made in many cases to secure the comfort and enjoyment of the visitors, and especially for the hard-worked officials of the Association. The banquets, receptions, *conversaziones*, and other similar features, have been many and excellent; while no fault could be found with the excursions: that to Durham on Saturday gave pleasure to all who joined it, especially to those who were privileged to sit down to the substantial luncheon in the grand old Castle. At the Cathedral service afterwards, it was interesting to see the Sectional Secretaries sitting humbly in the chancel, and a few eminent Presidents of Sections exalted to the stalls of Canons and Prebends.

At the General Committee Meeting yesterday there was unusual excitement. These meetings are generally scantily attended; but, from the crowd which filled the room and hovered around the doors, it was evident that mischief was brewing. After other business was transacted, it was announced, to the evident regret and pain of everyone, that Mr. Atchison could no longer, owing to ill-health, continue to discharge his duties as Secretary, and that a successor would have to be appointed. Mr. Atchison's popularity with the Association was evident from the enthusiasm exhibited in the proposed votes of thanks accorded him. The name of the candidate selected by the Council for the approval of the meeting was then announced, and supported by Prof. Garnett and others. But opposition to the selection was immediately announced. It is unnecessary to report in detail what will probably be found in the daily papers; but it was evident that there was widespread dissatisfaction with the conduct of the Council. No one had any especial objection to this candidate, although he had never had any connection with the Association; but it was felt that the Council had acted unwisely in passing over certain other candidates who were of eminence in science and had had long experience of the workings of the Association. After many proposals had been made, and much excitement manifested, it was at last, almost unanimously, decided to reappoint Mr. Atchison, and authorize the Council to provide whatever assistance should be necessary. Happily, Mr. Atchison said he was willing to agree to this proposal, and so the incident ends at present. But it was expected that the matter might have gone further, and the whole subject of the constitution of the Association discussed, and the advisability of conforming it to existing conditions. Happily, such a discussion has been postponed: it would have been unwise to continue it in connection with so personal a matter.

At the same meeting, the invitation from Cardiff was accepted for 1891, while that from Edinburgh will no doubt be accepted for 1892. The meeting at Leeds, beginning on September 3 next year, will be presided over by Sir Frederick Abel.

It is impossible not to state, what all feel, that the success of the present meeting has been largely due to Prof. Merivale, who has been ably supported by Dr. Bedson, Dr. Dunn, and other members of the Local Committee. That the attendance (2431) is something like a thousand below the last Newcastle meeting, and still less than the Manchester meeting two years ago, is not surprising. Not only are the Paris Exhibition and the many Conferences being held in connection therewith a powerful counter-attraction, but there were in 1863 special local reasons for raising the attendance to so high a figure.

The following is the list of grants to be submitted to the meeting of the General Committee to-morrow:—

A.—*Mathematics and Physics*.

	£
Differential Gravity Meter ...	10
Electro-optics ...	50
Calculating Mathematical Tables ...	25
Seismological Phenomena of Japan ...	75
Pellian Equation Tables ...	15
Electrical Standards ...	50
Electrolysis ...	5

B.—*Chemistry*.

Properties of Solutions ...	10
Analysis of Iron and Steel ...	10
Isomeric Naphthalene Derivatives ...	15
Silent Discharge of Electricity ...	5
Methods of Teaching Chemistry ...	10
Absorption Spectra ...	30
Recording the Results of Water Analysis ...	10
Oxidation of Hydracids in Sunlight...	15

C.—*Geology*.

Erratic Blocks ...	10
Volcanic Phenomena of Vesuvius ...	20
Fossil Phyllopoda ...	20
Geological Record ...	100
Underground Waters ...	5
Excavations at Oldbury Hill ...	15
Cretaceous Polyzoa ...	10
Geological Photographs ...	10
Lias Beds of Northamptonshire ...	25

D.—*Biology.*

Botanical Station at Peradeniya	£50
Deep-sea Tow-net	10
Naples Zoological Station	100
West India Islands	100
Marine Biological Association	30

F.—*Economic Science and Statistics.*

Monetary Standard	10
Precious Metals in use as Money	15

G.—*Mechanical Science.*

Waves and Currents in Estuaries	150
Graphic Methods in Mechanical Science	15

H.—*Anthropology.*

North-Western Tribes of Canada	100
Effect of Occupations on Physical Development	20
Anthropological Notes and Queries	50
Anthropometric Calculations	10
Nomad Tribes of Asia Minor	25
Natives of India	10
Corresponding Societies	20
Total	£1265

SECTION C.

GEOLOGY.

OPENING ADDRESS BY PROF. JAMES GEIKIE, LL.D., F.R.SS. L. & E., F.G.S., PRESIDENT OF THE SECTION.

THE President of this Section must often have some difficulty in selecting a subject for his address. It is no longer possible to give an interesting and instructive summary of the work done by the devotees of our science during even one year. So numerous have the students of geological science become—so fertile are the fields they cultivate—so abundant the harvests they reap—that one in my present position may well despair of being able to take stock of the numerous additions to our knowledge which have accumulated within the last twelve months. Neither is there any burning question which at this time your President need feel called upon to discuss. True, there are controversies that are likely to remain unsettled for years to come—there are still not a few matters upon which we must agree to differ—we do not yet see eye to eye in all things geological. But experience has shown that as years advance truth is gradually evolved, and old controversies die out; and so doubtless it will continue to be. The day when controversies shall cease, however, is yet, I hope, far in the future; for should that dull and unhappy time ever arrive, it is quite certain that mineralogists, petrologists, palæontologists, and geologists shall have died out of the world. Following the example of many of my predecessors, I shall confine my remarks to certain questions in which I have been specially interested. And in doing so I shall endeavour to steer clear, as far as I can, of controversial matters. My purpose, then, is to give an outline of some of the results obtained during the last few years by Continental workers in the domain of glacial geology.

Those who are not geologists will probably smile when they hear one declare that wielders of the hammer are extremely conservative—that they are slow to accept novel views, and very tenacious of opinions which have once found favour in their eyes. Nevertheless, such is the case; and well for us that it is so. However captivating, however imposing, however strongly supported by evidence a new view may appear to be, we do well to criticize, to sift the evidence, and to call for more facts and experiments, if such are possible, until the proofs become so strong as to approach as near a demonstration as geologists can in most cases expect such proofs to go. The history of our science, and indeed of most sciences, affords abundant illustration of what I say. How many long years were the views of sub-aërial erosion, as taught by Hutton and Playfair, canvassed and controverted before they became accepted! And even after their general soundness had been established, how often have we heard nominal disciples of these fathers of physical geology refuse to go so far as to admit that the river-valleys of our islands have been excavated by epigene agents? If, as a rule, it takes some time for a novel view to gain acceptance, it is equally true that views which have long been held are only with difficulty discarded. Between the new and the old there is a constant struggle for existence, and if the latter should happen to survive, it is only in a modified form. I have often thought that a history of the evolution of geological theories would make a very entertaining and instructive work. We should learn from it, amongst other things, that the advance of our science has not always been continuous—now and again, indeed, it has almost seemed as if the movement had been retrograde. Knowledge has not come in like an overwhelming flood—like a broad majestic river—but rather like a gently-flowing tide, now advancing, now retiring, but ever, upon the whole, steadily gaining ground. The history I speak of would also teach us that many of the general views and hypotheses which have been from time to time abandoned as unworkable are hardly deserving of the reproach and ridicule which we in these latter days may be inclined to cast upon them. As the Scots proverb says, "It is easy to be wise behindhand." It could be readily shown that not a few discarded notions and opinions have frequently worked for good, and have rather stimulated than checked inquiry. Such reflections should be encouraging to every investigator, whether he be a defender of the old or an advocate of the new. Time tries all, and each worker may claim a share in the final establishment of the truth.

Perhaps there is no department of geological inquiry that has given rise to more controversy than that which I have selected for the subject of this address. Hardly a single step in advance has been taken without vehement opposition. But the din of contending sides is not so loud now—the dust of the conflict has to some extent cleared away, and the positions which have been lost or maintained, as the case may be, can be readily discerned. The glacialist who can look back over the last twenty-five years of wordy conflict has every reason to be jubilant and hopeful. Many of those who formerly opposed him have come over to his side. It is true he has not had everything his own way. Some extreme views have been abandoned in the struggle—that of a great Polar ice-sheet, for example, as conceived of by Agassiz. I am not aware, however, that many serious students of glacial geology ever adopted that view. But it was quite an excusable hypothesis, and has been abundantly suggestive. Had Agassiz lived to see the detailed work of these later days, he would doubtless have modified his notion, and come to accept the view of large continental glaciers which has taken its place.

The results obtained by geologists, who have been studying the peripheral areas of the drift-covered regions of our continent, are such as to satisfy us that the drifts of those regions are not iceberg-droppings, as we used to suppose, but true morainic matter and fluvio-glacial detritus. Geologists have not jumped to this conclusion—they have only accepted it after laborious investigation of the evidence. Since Dr. Otto Torell, in 1875, first stated his belief that the "diluvium" of North Germany was of glacial origin, a great literature on the subject has sprung up, a perusal of which will show that with our German friends glacial geology has passed through much the same succession of phases as with us. At first icebergs are appealed to as explaining everything; next we meet with sundry ingenious attempts at a compromise between floating ice and a continuous ice-sheet. As observations multiply, however, the element of floating ice is gradually eliminated, and all the phenomena are explained by means of land-ice and *Schmelz-wasser* alone. It is a remarkable fact that the iceberg hypothesis has always been most strenuously upheld by geologists whose labours have been largely confined to the peripheral areas of drift-covered countries. In the upland and mountainous tracts, on the other hand, that hypothesis has never been able to survive a moderate amount of accurate observation. Even in Switzerland—the land of glaciers—geologists at one time were of opinion that the boulder-clays of the low grounds had a different origin from those which occur in the mountain valleys. Thus it was supposed that at the close of the Pleistocene period the Alps were surrounded by great lakes or gulfs of some inland sea, into which the glaciers of the high valleys flowed and calved their icebergs—these latter scattering erratics and earthy *débris* over the drowned areas. Sartorius von Waltershausen[1] set forth this view in an elaborate and well-illustrated paper. Unfortunately for his hypothesis, no trace of the supposed great lakes or inland sea has ever been detected; on the contrary, the character of the morainic accumulations, and the symmetrical grouping and radiation of the erratics and perched blocks over the foot-hills and low grounds, show that these last have been invaded and overflowed by the glaciers themselves. Even the most strenuous upholders of the efficacy of icebergs as originators of some boulder-clays admit that the boulder-clay or till, of what we may call the inner or central region of a glaciated tract, is the product of land-ice. Under this category comes the boulder-clay of Norway, Sweden, and Finland, and of the Alpine lands of Central Europe, not to speak of the hilly parts of our own islands.

When we come to study the drifts of the peripheral areas it is not difficult to see why these should be considered to have had a different origin. They present certain features which, although not absent from the glacial deposits of the inner region, are not nearly so characteristic of such upland tracts. I refer especially to the frequent interstratification of boulder clays with well-bedded deposits of clay, sand, and gravel; and to the fact that these boulder-clays are often less compressed than those of the inner region, and have even occasionally a silt-like character. Such appearances do seem at first to be readily explained on the assumption that the deposits have been accumulated in water opposite the margin of a continental glacier or ice-sheet; and this was the view which several able investigators in Germany were for some time inclined to adopt.

But when the phenomena came to be studied in greater detail, and over a wider area, this preliminary hypothesis did not prove satisfactory. It was discovered, for example, that "giants' kettles"[2] were more or less commonly distributed under the glacial deposits, and such "kettles" could only have originated at the bottom of a glacier. Again, it was found that preglacial accumulations were plentifully developed in certain places below the drift, and were often involved with the latter in a remarkable way. The "brown-coal formation" in like manner was violently disturbed and displaced, to such a degree that frequently the boulder-clay is found to underlie it. Similar phenomena were encountered in regions where the drift overlies the Chalk—the latter presenting the appearance of having been smashed and shattered—the fragments having often been dragged some distance, so as to form a kind of friction-breccia underlying the drift, while large masses are often included in the clay itself. All the facts pointed to the conclusion that these disturbances were due to tangential thrusting or crushing, and were not the result of vertical displacements, such as are produced by normal faulting, for the disturbances in question die out from above downwards. Evidence of similar thrusting or crushing is seen in the remarkable faults and contortions that so often characterize the clays and sands that occur in the boulder-clay itself. The only agent that could produce the appearances now briefly referred to is land-ice, and we must therefore agree with German geologists that glacier-ice has overflowed all the drift-covered regions of the peripheral area. No evidence of marine action in the formation of the stony clays is forthcoming —not a trace of any sea-beach has been detected. And yet, if these clays had been laid down in the sea during the retreat of the ice-sheet from Germany, surely such evidence as I have indicated ought to be met with. To the best of my knowledge the only particular facts which have been appealed to, as proofs of marine action, are the appearance of bedded deposits in the boulder-clays, and the occasional occurrence in the clays themselves of a sea-shell. But other organic remains are also met with now and again in similar positions, such as mammalian bones and fresh-water shells. All these, however, have been shown to be derivative in their origin—they are just as much erratics as the stones and boulders with which they are associated. The only phenomena, therefore, that the glacialist has to account for are the bedded deposits which occur so frequently in the boulder-clays of the peripheral regions, and the occasional silty and uncompressed character of the clays themselves.

The intercalated beds are, after all, not hard to explain. If we consider for a moment the geographical distribution of the boulder-clays, and their associated aqueous deposits, we shall find a clue to their origin. Speaking in general terms, the stony clays thicken out as they are followed from the mountainous and high-lying tracts to the low ground. Thus they are of inconsiderable thickness in Norway, the higher parts of Sweden, and in Finland, just as we find is the case in Scotland, Northern England, Wales, and the hilly parts of Ireland. Traced south from the uplands of Scandinavia and Finland, they gradually thicken out as the low grounds are approached. Thus in Southern Sweden they reach a thickness of 43 metres or thereabout, and of 80 metres in the northern parts of Prussia, while over the wide low-lying regions to the south they attain a much greater thickness—reaching in Holstein, Mecklenburg, Pomerania, and West Prussia a depth of 120 to 140 metres, and still greater depths in Hanover, Mark Brandenburg, and Saxony. In those regions, however, a considerable portion of the "diluvium" consists, as we shall see presently, of water-formed beds.

The geographical distribution of the aqueous deposits which are associated with the stony clays is somewhat similar. They are very sparingly developed in districts where the boulder-clays are thin. Thus they are either wanting, or only occur sporadically in thin irregular beds, in the high grounds of Northern Europe generally. Further south, however, they gradually acquire more importance until in the peripheral regions of the drift-covered tracts they come to equal and eventually to surpass the boulder-clays in prominence. These latter, in fact, at last cease to appear, and the whole bulk of the "diluvium" along the southern margin of the drift area appears to consist of aqueous accumulations alone.

The explanations of these facts advanced by German geologists are quite in accordance with the views which have long been held by glacialists elsewhere, and have been tersely summed up by Dr. Jentzsch (*Jahrb. d. königl. preuss. geologischen Landesanstalt für* 1884, p. 438). The northern regions, he says, were the feeding-grounds of the inland ice. In those regions melting was at a minimum, while the grinding action of the ice was most effective. Here, therefore, erosion reached its maximum—ground-moraine or boulder-clay being unable to accumulate to any thickness. Further south melting greatly increased, while ground-moraine at the same time tended to accumulate—the conjoint action of glacier-ice and sub-glacial water resulting in the complex drifts of the peripheral area. In the disposition and appearance of the aqueous deposits of the "diluvium" we have evidence of an extensive sub-glacial water-circulation—glacier-mills that gave rise to "giants' kettles"—chains of sub-glacial lakes in which fine clays gathered—streams and rivers that flowed in tunnels under the ice, and whose courses were paved with sand and gravel. Nowhere do German geologists find any evidence of marine action. On the contrary, the dovetailing and interosculation of boulder-clay with aqueous deposits are explained by the relation of the ice to the surface over which it flowed. Throughout the peripheral area it did not rest so continuously upon the ground as was the case in the inner region of maximum erosion. In many places it was tunnelled by rapid streams and rivers, and here and there it arched over sub-glacial lakes, so that accumulation of ground-moraine proceeded side by side with the formation of aqueous sediments. Much of that ground-moraine is of the usual tough and hard-pressed character, but here and there it is somewhat less coherent and even silt-like. Now a study of the ground-moraines of modern glaciers affords us a reasonable explanation of such differences. Dr. Brückner[1] has shown that in many places the ground-moraine of Alpine glaciers is included in the bottom of the ice itself. The ground-moraine, he says, frequently appears as an ice-stratum abundantly impregnated with silt and rock-fragments—it is like a conglomerate or breccia which has ice for its binding material. When this ground-moraine melts out of the ice—no running water being present—it forms a layer of unstratified silt or clay, with stones scattered irregularly through it. Such being the case in modern glaciers, we can hardly doubt that over the peripheral areas occupied by the old northern ice-sheet boulder-clay must frequently have been accumulated in the same way. Nay, when the ground-moraine melted out and dropped here and there into quietly-flowing water it might even acquire in part a bedded character.

The limits reached by the inland ice during its greatest extension are becoming more and more clearly defined, although its southern margin will probably never be so accurately determined

[1] "Untersuchungen über die Klimate der Gegenwart und der Vorwelt," &c. (*Natuurkundige Verhandelingen v. d. Holland. Maatsch. d. Wetensch. te Haarlem*, 1865).

[2] These appear to have been first detected by Prof. Berendt and Prof. E. Geinitz.

[1] "Die Vergletscherung des Salzachgebietes, &c.," *Geographische Abhandlungen herausgegeben v. A. Penck*, Band i. Heft 1.

as that of the latest epoch of general glaciation. The reasons for this are obvious. When the inland ice flowed south to the Harz and the hills of Saxony, it formed no great terminal moraines. Doubtless many erratics and much rock-rubbish were showered upon the surface of the ice from the higher mountains of Scandinavia, but owing to the fanning-out of the ice on its southward march, such superficial *débris* was necessarily spread over a constantly widening area. It may well be doubted, therefore, whether it ever reached the terminal front of the ice-sheet in sufficient bulk to form conspicuous moraines. It seems most probable that the terminal moraines of the great inland ice would consist of low banks of boulder-clay and aqueous materials—the latter, perhaps, strongly predominating, and containing here and there larger and smaller angular erratics which had travelled on the surface of the ice. However that may be, it is certain that the whole region in question has been considerably modified by subsequent denudation, and to a large extent is now concealed under deposits belonging to later stages of the Pleistocene period. The extreme limits reached by the ice are determined rather by the occasional presence of rock-striæ and *roches moutonnées*, of boulder-clay and northern erratics, than by recognizable terminal moraines. The southern limits reached by the old inland ice appear in this way to have been tolerably well ascertained over a considerable portion of Central Europe. Some years ago I published a small sketch-map ("Prehistoric Europe," 1881) showing the extent of surface formerly covered by ice. On this map I did not venture to draw the southern margin of the ice-sheet in Belgium further south than Antwerp, where northern erratics were known to occur; but the more recent researches of Belgian geologists show that the ice probably flowed south for some little distance beyond Brussels (see a paper by M. E. Delvaux, *Ann. de la Soc. géol. de Belg.*, t. xiii. p. 158). Here and there in other parts of the Continent the southern limits reached by the northern drift have also been more accurately determined, but, so far as I know, none of these later observations involves any serious modification of the sketch-map referred to.

I have now said enough, however, to show that the notion of a general ice-sheet having covered so large a part of Europe, which a few years ago was looked upon as a wild dream, has been amply justified by the labours of those who are so assiduously investigating the peripheral areas of the "great northern drift." And perhaps I may be allowed to express my own belief that the drifts of Middle and Southern England, which exhibit the same complexity as the "lower diluvium" of the Continent, will eventually be generally acknowledged to have had a similar origin. I have often thought that whilst politically we are happy in having the sea all round us, geologically we should have gained perhaps by its greater distance. At all events we should have been less ready to invoke its assistance to explain every puzzling appearance presented by our glacial accumulations.

I now pass on to review some of the general results obtained by Continental geologists as to the extent of area occupied by inland ice during the last great extension of glacier-ice in Europe. It is well-known that this latest ice-sheet did not overflow nearly so wide a region as that underneath which the lowest boulder-clay was accumulated. This is shown not only by the geographical distribution of the youngest boulder-clay, but by the direction of rock-striæ, the trend of erratics, and the position of well-marked moraines. Gerard de Geer has given a summary (*Zeitschrift d. deutsch. geolog. Ges.*, Bd. xxxvii. p. 177) of the general results obtained by himself and his fellow-workers in Sweden and Norway; and these have been supplemented by the labours of Berendt, Geinitz, Hunchecorne, Keilhack, Klockmann, Schröder, Wahnschaffe, and others in Germany, and by Sederholm in Finland.[1] From them we learn that the end-moraines of the ice circle round the southern coasts of Norway, from whence they sweep south-east by east across the province of Gottland in Sweden, passing through the lower ends of Lakes Wener and Wetter, while similar moraines mark out for us the terminal front of the inland ice in Finland—at least two parallel frontal moraines passing inland from Hango Head on the Gulf of Finland through the southern part of that province to the north of Lake Ladoga. Further north-east than this they have not been traced; but, from some observations by Helmersen, Sederholm thinks it probable that the terminal ice-front extended north-east by the north of Lake Onega to the eastern shores of the White Sea. Between Sweden and Finland lies the basin of the Baltic, which at the period in question was filled with ice, forming a great Baltic glacier, which overflowed the Åland Islands, Gottland, and Öland, and which, fanning-out as it passed towards the south-west, invaded, on the south side, the Baltic provinces of Germany, while, on the north, it crossed the southern part of Scania in Sweden and the Danish islands to enter upon Jutland.

The upper boulder-clay of those regions is now recognized as the ground-moraine of this latest ice-sheet. In many places it is separated from the older boulder-clay by interglacial deposits, some of which are marine, while others are of fresh-water and terrestrial origin. During interglacial times the sea that overflowed a considerable portion of North Germany was evidently continuous with the North Sea, as is shown not only by the geographical distribution of the interglacial marine deposits, but by their North Sea fauna. German geologists generally group all the interglacial deposits together, as if they belonged to one and the same interglacial epoch. This perhaps we must look upon as only a provisional arrangement. Certain it is that the fresh-water and terrestrial beds which frequently occur on the same or a lower level, and at no great distance from the marine deposits, cannot in all cases be contemporaneous with the latter. Possibly, however, such discordances may be accounted for by oscillations in the level of the interglacial sea—land and water having alternately prevailed over the same area. Two boulder-clays, as we have seen, have been recognized over a wide region in North Germany. In some places, however, three or more such boulder-clays have been observed overlying one another throughout considerable areas, and these clays are described as being distinctly separate and distinguishable the one from the other.[1] Whether they with their intercalated aqueous deposits indicate great oscillations of one and the same ice-sheet—now advancing, now retreating—or whether the stony clays may not be the ground-moraines of so many different ice-sheets, separated the one from the other by true interglacial conditions, future investigations must be left to decide.

The general conclusions arrived at by those who are at present investigating the glacial accumulations of Northern Europe may be summarized as follows:—

1. Before the invasion of Northern Germany by the inland ice the low grounds bordering on the Baltic were overflowed by a sea which contained a boreal and arctic fauna. These marine conditions are indicated by the presence under the lower boulder-clay of more or less well-bedded fossiliferous deposits. On the same horizon occur also beds of sand, containing fresh-water shells, and now and again mammalian remains, some of which imply cold and others temperate climatic conditions. Obviously all these deposits may pertain to one and the same period, or more properly to different stages of the same period—some dating back to a time when the climate was still temperate, while others clearly indicate the prevalence of cold conditions, and are therefore probably somewhat younger.

2. The next geological horizon in ascending order is that which is marked by the "lower diluvium"—the glacial and fluvio-glacial detritus of the great ice-sheet which flowed south to the foot of the Harz Mountains. The boulder clay on this horizon now and again contains marine, fresh-water, and terrestrial organic remains, derived undoubtedly from the so-called preglacial beds already referred to. These latter, it would appear, were ploughed up and largely incorporated with the old ground-moraine.

3. The interglacial beds which next succeed contain remains of a well-marked temperate fauna and flora, which point to something more than a mere partial or local retreat of the inland ice. The geographical distribution of the beds and the presence in these of such forms as *Elephas antiquus*, *Cervus elephas*, *C. megaceros*, and a flora comparable to that now existing in Northern Germany, justify geologists in concluding that the interglacial epoch was one of long duration, and characterized in Germany by climatic conditions apparently not less temperate than those that now obtain. One of the phases of that interglacial epoch, as we have seen, was the overflowing of the Baltic provinces by the waters of the North Sea.

4. To this well-marked interglacial epoch succeeded another epoch of arctic conditions, when the Scandinavian inland ice

[1] For papers by Berendt and his associates see especially the *Jahrbuch d. k. preuss. geol. Landesanstalt*, and the *Zeitschr. d. deutsch. geol. Ges.* for the past few years. Geinitz, *Forsch. z. d. Landes- u. Volkskunde*, i. 5; *Leopoldina*, xxii. p. 37; *I. Beitrag z. Geologie Mecklenburgs*, 1880, pp. 46, 56. Sederholm, *Fennia*, i. No. 7.

[1] H Schröder, *Jahrb. d. k. preuss. geol. Landesanstalt für* 1887, p. 360.

once more invaded Germany, ploughing through the interglacial deposits, and working these up in its ground-moraine. So far as I can learn, the prevalent belief among geologists in North Germany is that there was only one interglacial epoch; but, as already stated, doubt has been expressed whether all the facts can be thus accounted for. There must always be great difficulty in the correlation of widely-separated interglacial deposits, and the time does not seem to me to have yet come when we can definitely assert that all those interglacial beds belong to one and the same geological horizon.

I have dwelt upon the recent work of geologists in the peripheral areas of the drift-covered regions of Northern Europe, because I think the results obtained are of great interest to glacialists in this country. And for the same reason I wish next to call attention to what has been done of late years in elucidating the glacial geology of the Alpine lands of Central Europe—and more particularly of the low grounds that stretch out from the foot of the mountains. Any observations that tend to throw light upon the history of the complex drifts of our own peripheral areas cannot but be of service. It is quite impossible to do justice in this brief sketch to the labours of the many enthusiastic geologists who, within recent years, have increased our knowledge of the glaciation of the Alpine lands. At present, however, I am not so much concerned with the proofs of general glaciation as with the evidence that goes to show how the Alpine ground-moraines have been formed, and with the facts which have led certain observers to conclude that the Alps have endured several distinct glaciations within Pleistocene times. Swiss geologists are agreed that the ground-moraines which clothe the bottoms of the great Alpine valleys, and extend outwards sometimes for many miles upon the low grounds beyond, are of true glacial origin. Now these ground-moraines are closely similar to the boulder-clays of this country and Northern Europe. Like them, they are frequently tough and hard-pressed, but now and again somewhat looser and less firmly coherent. Frequently also they contain lenticular beds, and more or less thick sheets of aqueous deposits—in some places the stony clays even exhibiting a kind of stratification—and ever and anon such water-assorted materials are commingled with stony clay in the most complex manner. These latter appearances are, however, upon the whole best developed upon the low grounds that sweep out from the base of the Alps. The only question concerning the ground-moraines that has recently given rise to much discussion is the origin of the materials themselves. It is obvious that there are only three possible modes in which those materials could have been introduced to the ground-moraine: either they consist of superficial morainic *débris* which has found its way down to the bottom of the old glaciers by crevasses; or they may be made up of the rock-rubbish, shingle, gravel, &c., which doubtless strewed the valleys before these were occupied by ice; or, lastly, they may have been derived in chief measure from the underlying rocks themselves by the action of the ice that overflowed them. The investigations of Penck, Blaas, Böhm, and Brückner appear to me to have demonstrated that the ground-moraines are composed mostly of materials which have been detached from the underlying rocks by the erosive action of the glaciers themselves. Their observations show that the regions studied by them in great detail were almost completely buried under ice, so that the accumulation of superficial moraines was for the most part impossible; and they advance a number of facts which prove positively that the ground-moraines were formed and accumulated under ice. I cannot here recapitulate the evidence, but must content myself by a reference to the papers in which this is fully discussed.[1] These geologists do not deny that some of the material may occasionally have come from above, nor do they doubt that pre-existing masses of rock-rubbish and alluvial accumulations may have been incorporated with the ground-moraines; but the enormous extent of the latter, and the direction of transport and distribution of the erratics which they contain cannot be thus accounted for, while all the facts are readily explained by the action of the ice itself, which used its sub-glacial *débris* as tools with which to carry on the work of erosion.

Prof. Heim and others have frequently asserted that glaciers have little or no eroding power, since at the lower ends of existing glaciers we find no evidence of such erosion being in operation. But the chief work of a glacier cannot be carried on at its lower end, where motion is reduced to a minimum, and where the ice is perforated by sub-glacial tunnels and arches, underneath which no glacial erosion can possibly take place; and yet it is upon observations made in just such places that the principal arguments against the erosive action of glaciers have been based. If all that we could ever know of glacial action were confined to what we can learn from peering into the grottoes at the terminal fronts of existing glaciers, we should indeed come to the conclusion that glaciers do not erode their rocky beds to any appreciable extent. But as we do not look for the strongest evidence of fluviatile erosion at the mouth of a river, but in its valley- and mountain-tracks, so if we wish to learn what glacier-ice can accomplish, we must study in detail some wide region from which the ice has completely disappeared. When this plan has been followed, it has happened that some of the strongest opponents of glacial erosion have been compelled by the force of the evidence to go over to the other camp. Dr. Blaas, for example, has been led by his observations on the glacial formations of the Inn Valley to recant his former views, and to become a formidable advocate of the very theory which he formerly opposed. To his work and the memoirs by Penck, Brückner, and Böhm already cited, and especially to the admirable chapter on glacier-erosion by the last-named author, I would refer those who may be anxious to know the last word on this much-debated question.

The evidence of interglacial conditions within the Alpine lands continues to increase. These are represented by alluvial deposits of silt, sand, gravel, conglomerate, breccia, and lignites. Penck, Böhm, and Brückner find evidence of two interglacial epochs, and maintain that there have been three distinct and separate epochs of glaciation in the Alps. No mere temporary retreat and re-advance of the glaciers, according to them, will account for the phenomena presented by the interglacial deposits and associated morainic accumulations. During interglacial times the glaciers disappeared from the lower valleys of the Alps—the climate was temperate, and probably the snow-fields and glaciers approximated in extent to those of the present day. All the evidence conspires to show that an interglacial epoch was of prolonged duration. Dr. Brückner has observed that the moraines of the last glacial epoch rest here and there upon löss, and he confirms Penck's observations in South Bavaria that this remarkable formation never overlies the morainic accumulations of the latest glacial epoch. According to Penck and Brückner, therefore, the löss is of interglacial age. There can be little doubt, however, that löss does not belong to any one particular horizon. Wahnschaffe[1] and others have shown that throughout wide areas in North Germany it is the equivalent in age of the "upper diluvium," while Schumacher (*Hygienische Topographie von Strassburg i. E.*, 1885) points out that in the Rhine Valley it occurs on two separate and distinct horizons. Prof. Andreæ has likewise shown (*Abhandl. z. geol. Specialkarte v. Elsass-Lothringen*, Bd. vii. Heft 2) that there is an upper and lower löss in Alsace, each characterized by its own special fauna.

There is still considerable difference of opinion as to the mode of formation of this remarkable accumulation. By many it is considered to be an aqueous deposit; others, following Richthofen, are of opinion that it is a wind-blown accumulation; while some incline to the belief that it is partly the one and partly the other. Nor do the upholders of these various hypotheses agree amongst themselves as to the precise manner in which water or wind has worked to produce the observed results. Thus, amongst the supporters of the aqueous origin of the löss, we find this attributed to the action of heavy rains washing over and rearranging the material of the boulder-clays (Laspeyres, *Erläuterungen z. geol. Specialkarte v. Preussen*, &c., *Blatt Gröbzig, Zörbig, und Petersberg*). Many, again, have held it probable that the löss is simply the finest loam distributed over the low grounds by the flood-waters that escaped from the northern inland ice and the *mers de glace* of the Alpine lands of Central Europe. Another suggestion is that much of the material of the löss may have been derived from the denudation of the boulder-clays by flood-water, during the closing stages of the last cold period. It is pointed out that in some regions at least the löss is underlain by a layer of erratics, which are believed to be the residue of the denuded boulder-clay.

[1] Penck, "Die Vergletscherung der deutschen Alpen." Blaas, "Zeitschr. d. Ferdinandeums," 1885. Böhm, *Jahr. d. k. k. geol. Reichsanstalt.* 1885, Bd. xxxv. Heft 3. Brückner, "Die Vergletscherung d. Salzachgebietes," &c., 1886

[1] *Abhandl. z. geol. Specialkarte v. Preussen*, &c., Bd. vii. Heft 1; *Zeitschr. d. deutsch. geol. Gesellsch.* 1885, p. 904; 1886, p. 367.

We are reminded by Klockmann (*Jahrb. d. k. preuss. geol. Landesanstalt für* 1883, p. 262) and Wahnschaffe (*op. cit.*, and *Zeitschr. d. deutsch. geol. Ges.* 1886, p. 367) that the inland ice must have acted as a great dam, and that wide areas in Germany, &c., would be flooded, partly by water derived from the melting inland ice, and partly by waters flowing north from the hilly tracts of Middle Germany. In the great basins thus formed there would be a commingling of fine silt-material derived from north and south, which would necessarily come to form a deposit having much the same character throughout.

From what I have myself seen of the löss in various parts of Germany, and from all that I have gathered from reading and in conversation with those who have worked over löss-covered regions, I incline to the opinion that löss is for the most part of aqueous origin. In many cases this can be demonstrated, as by the occurrence of bedding and the intercalation of layers of stones, sand, gravel, &c., in the deposit; again, by the not infrequent appearance of fresh-water shells; but, perhaps, chiefly by the remarkable uniformity of character which the löss itself displays. It seems to me reasonable also to believe that the flood-waters of glacial times must needs have been highly charged with finely-divided sediment, and that such sediment would be spread over wide regions in the low grounds—in the slack-waters of the great rivers, and in the innumerable temporary lakes which occupied, or partly occupied, many of the valleys and depressions of the land. There are different kinds of löss or löss like deposits, however, and all need not have been formed in the same way. Probably some may have been derived, as Wahnschaffe has suggested, from the denudation of boulder-clay. Possibly, also, some löss may owe its origin to the action of rain upon the stony clays, producing what we in this country would call "rain-wash." There are other accumulations, however, which no aqueous theory will satisfactorily explain. Under this category comes much of the so-called *Berglöss*, with its abundant land-shells, and its generally unstratified character. It seems likely that such löss is simply the result of sub-aërial action, and owes its origin to rain, frost, and wind acting upon the superficial formations, and rearranging their finer-grained constituents. And it is quite possible that the upper portion of much of the löss of the lower grounds may have been re-worked in the same way. But I confess I cannot yet find in the facts adduced by German geologists any evidence of a dry-as-dust epoch having obtained in Europe during any stage of the Pleistocene period. The geographical position of our continent seems to me to forbid the possibility of such climatic conditions, while all the positive evidence we have points rather to humidity than dryness as the prevalent feature of Pleistocene climates. It is obvious, however, that after the flood-waters had disappeared from the low grounds of the Continent, sub-aërial action would come into play over the wide regions covered by glacial and fluvio-glacial deposits. Thus, in the course of time, these deposits would become modified,—just as similar accumulations in these islands have been top-dressed, as it were, and to some extent even rearranged. I am strengthened in these views by the conclusions arrived at by M. Falsan, the eminent French glacialist. Covering the plateaux of the Dombs, and widely spread throughout the valleys of the Rhone, the Ain, the Isère, &c., in France, there is a deposit of löss, he says, which has been derived from the washing of the ancient moraines. At the foot of the Alps, where black schists are largely developed, the löss is dark grey; but west of the secondary chain the same deposit is yellowish, and composed almost entirely of siliceous materials, with only a very little carbonate of lime. This *limon* or löss, however, is very generally modified towards the top by the chemical action of rain, the yellow löss acquiring a red colour. Sometimes it is crowded with calcareous concretions; at other times it has been deprived of its calcareous element and converted into a kind of pulverulent silica or quartz. This, the true löss, is distinguished from another *lehm*, which Falsan recognizes as the product of atmospheric action—formed, in fact, in place, from the disintegration and decomposition of the subjacent rocks. Even this *lehm* has been modified by running water—dispersed or accumulated locally, as the case may be (Falsan, "La Période glaciaire," p. 81).

All that we know of the löss and its fossils compels us to include this accumulation as a product of the Pleistocene period. It is not of postglacial age—even much of what one may call the "remodified löss" being of Late Glacial or Pleistocene age. I cannot attempt to give here a summary of what has been learned within recent years as to the fauna of the löss. The researches of Nehring and Liebe have familiarized us with the fact that at some particular stage in the Pleistocene period a fauna like that of the alpine steppe-lands of Western Asia was indigenous to Middle Europe, and the recent investigations of Woldřich have increased our knowledge of this fauna. At what horizon, then, does this steppe-fauna make its appearance? At Thiede, Dr. Nehring discovered in so-called löss three successive horizons, each characterized by a special fauna. The lowest of these faunas was decidedly arctic in type; above that came a steppe-fauna, which last was succeeded by a fauna comprising such forms as mammoth, woolly rhinoceros, *Bos*, *Cervus*, horse, hyæna, and lion. Now, if we compare this last fauna with the forms which have been obtained from true postglacial deposits—those deposits, namely, which overlie the younger boulder-clays and flood-accumulations of the latest glacial epoch—we find little in common. The lion, the mammoth, and the rhinoceros are conspicuous by their absence from the postglacial beds of Europe. In place of them we meet with a more or less arctic fauna, and a high-alpine and arctic flora, which, as we all know, eventually gave place to the flora and fauna with which Neolithic man was contemporaneous. As this is the case throughout North-Western and Central Europe, we seem justified in assigning the Thiede beds to the Pleistocene period, and to that interglacial stage which preceded and gradually merged into the last glacial epoch. That the steppe-fauna indicates relatively drier conditions of climate than obtained when perennial snow and ice covered wide areas of the low ground goes without saying; but I am unable to agree with those who maintain that it implies a dry-as-dust climate, like that of some of the steppe-regions of our own day. The remarkable commingling of arctic and steppe faunas discovered by Woldřich in the Böhmerwald (*Sitzungsb. d. kais. Akad. d. W. math. nat. Cl.*, 1880, p. 7; 1881, p. 177; 1883, p. 978) shows, I think, that the jerboas, marmots, and hamster-rats were not incapable of living in the same regions contemporaneously with lemmings, arctic hares, Siberian social voles, &c. But when a cold epoch was passing away the steppe-forms probably gradually replaced their arctic congeners, as these migrated northwards during the continuous amelioration of the climate.

If the student of the Pleistocene faunas has certain advantages in the fact that he has to deal with forms many of which are still living, he labours at the same time under disadvantages which are unknown to his colleagues who are engaged in the study of the life of far older periods. The Pleistocene period was distinguished above all things by its great oscillations of climate—the successive changes being repeated, and producing correlative migrations of floras and faunas. We know that arctic and temperate faunas and floras flourished during interglacial times, and a like succession of life-forms followed the final disappearance of glacial conditions. A study of the organic remains met with in any particular deposit will not necessarily, therefore, enable us to assign these to their proper horizon. The geographical position of the deposit, and its relation to Pleistocene accumulations elsewhere must clearly be taken into account. Already, however, much has been done in this direction, and it is probable that ere long we shall be able to arrive at a fair knowledge of the various modifications which the Pleistocene floras and faunas experienced during that protracted period of climatic changes of which I have been speaking. We shall even possibly learn how often the arctic, steppe-, prairie-, and forest-faunas, as they have been defined by Woldřich, replaced each other. Even now some approximation to this better knowledge has been made. Dr. Pohlig,[1] for example, has compared the remains of the Pleistocene faunas obtained at many different places in Europe, and has presented us with a classification which, although confessedly incomplete, yet serves to show the direction in which we must look for further advances in this department of inquiry.

During the last twenty years the evidence of interglacial conditions both in Europe and America has so increased that geologists generally no longer doubt that the Pleistocene period was characterized by great changes of climate. The occurrence at many different localities on the Continent of beds of lignite and fresh-water alluvia, containing remains of Pleistocene Mammalia, intercalated between separate and distinct boulder-clays, has

[1] Pohlig, *Sitzungsb. d. Niederrheinischen Gesellschaft zu Bonn*, 1884; *Zeitschr. d. deutsch. geolog. Ges.*, 1887, p. 798. For a very full account of the diluvial European and Northern Asiatic mammalian faunas by Woldrich, see *Mém. de l'Acad. des Sciences de St. Pétersbourg*, Sér. vii, t. xxxv., 1887.

left us no alternative. The interglacial beds of the Alpine lands of Central Europe are paralleled by similar deposits in Britain, Scandinavia, Germany, and France. But opinions differ as to the number of glacial and interglacial epochs, many holding that we have evidence of only two cold stages and one general interglacial stage. This, as I have said, is the view entertained by most geologists who are at work on the glacial accumulations of Scandinavia and North Germany. On the other hand, Dr. Penck and others, from a study of the drifts of the German Alpine lands, believe that they have met with evidence of three distinct epochs of glaciation, and two epochs of interglacial conditions. In France, while some observers are of opinion that there have been only two epochs of general glaciation, others, as, for example, M. Tardy, find what they consider to be evidence of several such epochs. Others, again, as M. Falsan, do not believe in the existence of any interglacial stages, although they readily admit that there were great advances and retreats of the ice during the Glacial period. M. Falsan, in short, believes in oscillations, but is of opinion that these were not so extensive as others have maintained. It is, therefore, simply a question of degree, and whether we speak of oscillations or of epochs, we must needs admit the fact that throughout all the glaciated tracts of Europe, fossiliferous deposits occur intercalated among glacial accumulations. The successive advance and retreat of the ice, therefore, was not a local phenomenon, but characterized all the glaciated areas. And the evidence shows that the oscillations referred to were on a gigantic scale.

The relation borne to the glacial accumulations by the old river alluvia which contain relics of palæolithic man early attracted attention. From the fact that these alluvia in some places overlie glacial deposits, the general opinion (still held by some) was that palæolithic man must needs be of postglacial age. But since we have learned that all boulder-clay does not belong to one and the same geological horizon—that, in short, there have been at least two, and probably more, epochs of glaciation—it is obvious that the mere occurrence of glacial deposits underneath palæolithic gravels does not prove these latter to be postglacial. All that we are entitled in such a case to say is simply that the implement-bearing beds are younger than the glacial accumulations upon which they rest. Their horizon must be determined by first ascertaining the relative position in the glacial series of the underlying deposits. Now, it is a remarkable fact that the boulder-clays which underlie such old alluvia belong, without exception, to the earlier stages of the Glacial period This has been proved again and again, not only for this country but for Europe generally. I am sorry to reflect that some twenty years have now elapsed since I was led to suspect that the palæolithic gravels and cave-deposits were not of postglacial but of glacial and interglacial age. In 1871–72 I published a series of papers in the *Geological Magazine*, in which were set forth the views I had come to form upon this interesting question. In these papers it was maintained that the alluvial and cave-deposits could not be of postglacial age, but must be assigned to preglacial and interglacial times, and in chief measure to the latter. Evidence was led to show that the latest great development of glacier-ice in Europe took place after the southern pachyderms and palæolithic man had vacated England ; that during this last stage of the Glacial period man lived contemporaneously with a northern and alpine fauna in such regions as Southern France ; and lastly, that palæolithic man and the southern Mammalia never revisited North-Western Europe after extreme glacial conditions had disappeared. These conclusions were arrived at after a somewhat detailed examination of all the evidence then available, the remarkable distribution of the palæolithic and ossiferous alluvia having, as I have said, particularly impressed me. I coloured a map to show at once the areas covered by the glacial and fluvio-glacial deposits of the last glacial epoch, and the regions in which the implement-bearing and ossiferous alluvia had been met with, when it became apparent that the latter never occurred at the surface within the regions occupied by the former. If ossiferous alluvia did here and there appear within the recently glaciated areas, it was always either in caves, or as infra- or interglacial deposits. Since the date of these researches our knowledge of the geographical distribution of Pleistocene deposits has greatly increased, and implements and other relics of palæolithic man have been recorded from many new localities throughout Europe. But none of this fresh evidence contradicts the conclusions I had previously arrived at ; on the contrary, it has greatly strengthened my general argument.

Prof. Penck was, I think, the first on the Continent to adopt the views referred to. He was among the earliest to recognize the evidence of interglacial conditions in the drift-covered regions of Northern Germany, and it was the reflections which those remarkable interglacial beds were so well calculated to suggest that led him into the same path as myself. Dr. Penck has published a map (*Archiv für Anthropologie*, Bd. xv., Heft 3, 1884) showing the areas covered by the earlier and later glacial deposits in Northern Europe and the Alpine lands, and indicating at the same time the various localities where palæolithic finds have occurred. And in not a single case do any of the latter appear within the areas covered by the accumulations of the last glacial epoch.

A glance at the papers which have been published in Germany within the last few years will show how greatly students of the Pleistocene ossiferous beds have been influenced by what is now known of the interglacial deposits and their organic remains Profs. Rothpletz (*Denkschrift d. schweizer. Ges. für d. gesammt. Nat.*, Bd. xxviii., 1881) and Andreæ (*Abhandl. z. geolog. Specialkarte v. Elsass-Lothringen*, Bd. iv., Heft 2, 1884), Dr. Pohlig (*op. cit.*) and others, do not now hesitate to correlate with those beds the old ossiferous and implement-bearing alluvia which lie altogether outside of glaciated regions.

The relation of the Pleistocene alluvia of France to the glacial deposits of that and other countries has been especially canvassed. Rothpletz, in the paper cited above, includes these alluvia amongst the interglacial deposits ; and in the present year we have an interesting essay on the same subject by the accomplished secretary of the Anthropological and Archæological Congress, which met last month in Paris. M. Boule correlates (*Revue d'Anthropologie*, 1889, t. i.) the palæolithic cave- and river-deposits of France with those of other countries, and shows that they must be of interglacial age. His classification, I am gratified to find, does not materially differ from that given by myself a number of years ago. He is satisfied that in France there is evidence of three glacial epochs and two well-marked interglacial horizons. The oldest of the palæolithic stages of Mortillet (CHELLÉENNE) culminated, according to Boule, during the last interglacial epoch, while the more recent palæolithic stages (MOUSTÉRIENNE, SOLUTRÉENNE, and MAGDALÉNIENNE) coincided with the last great development of glacier-ice. The palæolithic age, so far as Europe is concerned, came to a close during this last cold phase of the Glacial period.

There are many other points relating to glacial geology which have of late years been canvassed by Continental workers, but these I cannot discuss here. I have purposely, indeed, restricted my remarks to such parts of a wide subject as I thought might have interest for glacialists in this country, some of whom may not have had their attention directed to the results which have recently been attained by their fellow-labourers in other lands. Had time permitted I should gladly have dwelt upon the noteworthy advances made by our American brethren in the same department of inquiry. Especially should I have wished to direct attention to the remarkable evidence adduced in favour of the periodicity of glacial action. Thus Messrs. Chamberlin and Salisbury, after a general review of that evidence, maintain that the Ice Age was interrupted by one chief interglacial epoch, and by three interglacial sub-epochs or episodes of deglaciation. The same authors discuss at some length the origin of the löss, and come to the general conclusion that while deposits of this character may have been formed at different stages of the Glacial period, and under different conditions, yet that upon the whole they are best explained by aqueous action. Indeed a perusal of the recent geological literature of America shows a close accord between the theoretical opinions of many Trans-atlantic and European geologists.

Thus as years advance the picture of Pleistocene times becomes more and more clearly developed. The conditions under which our old palæolithic predecessors lived—the climatic and geographical changes of which they were the witnesses—are gradually being revealed with a precision that only a few years ago might well have seemed impossible. This of itself is extremely interesting, but I feel sure that I speak the conviction of many workers in this field of labour when I say that the clearing up of the history of Pleistocene times is not the only end which they have in view. One can hardly doubt that when the conditions of that period and the causes which gave rise to these have been more fully and definitely ascertained we shall have advanced some way towards the better understanding of the climatic conditions of still earlier periods. For it cannot be denied that our knowledge of Palæozoic, Mesozoic, and even early Cainozoic climates is unsatisfactory. But we may look forward to the time when much of this uncertainty will disappear. Meteorologists are every day acquiring a clearer conception of the distribution of atmospheric pressure and temperature, and the causes by which that distribution is determined, and the day is coming when we shall be better able than we are now to apply this extended meteorological knowledge to the explanation of the climates of former periods in the world's history. One of the chief factors in the present distribution of atmospheric temperature and pressure is doubtless the relative position of the great land and water areas; and if this be true of the present, it must be true also of the past. It would almost seem, then, as if all one had to do to ascertain the climatic condition of any particular period was to prepare a map, depicting with some approach to accuracy the former relative position of land and sea. With such a map could our meteorologists infer what the climatic conditions must have been? Yes, provided we could assure them that in other respects the physical conditions did not differ from the present. Now, there is no period in the past history of our globe the geographical conditions of which are better known than the Pleistocene. And yet when we have indicated these upon a map we find that they do not give the results which we might have expected. The climatic conditions which they seem to imply are not such as we know did actually obtain. It is obvious, therefore, that some additional and perhaps exceptional factor was at work to produce the recognized results. What was this disturbing element, and have we any evidence of its interference with the operation of the normal agents of climatic change in earlier periods of the world's history? We all know that various answers have been given to such questions. Whether amongst these the correct solution of the enigma is to be found time will show. Meanwhile, as all hypothesis and theory must starve without facts to feed on, it behoves us as working geologists to do our best to add to the supply. The success with which other problems have been attacked by geologists forbids us to doubt that ere long we shall have done much to dispel some of the mystery which still envelops the question of geological climates.

SECTION E.

GEOGRAPHY.

Opening Address by Colonel Sir Francis de Winton, K.C.M.G., F.R.G.S., President of the Section.

Geography has not inaptly been defined as "the science of distributions," and from whatever aspect we view it, whether from a large and comprehensive basis embracing all the conditions which surround it as a science, or from the narrower limits of simple physiography, we find certain well-defined principles, or one may term them natural laws, pervading everywhere, whose actions have, through their influences on the past, created the present, and according to the uses we now put them must largely govern the future.

The formation of our globe, unfolded to our vision by scientific discovery, brings us face to face with Nature in all her awful grandeur; and we learn how, under a beneficent and all-wise Providence, this world has been fashioned and made for the use of man during periods of time almost beyond man's calculations; and in the history of man upon earth—a mere drop in this ocean of time—we read of the rise and fall of nations, of great wars, of the discoveries of new routes (so ably described by my friend and talented predecessor in the address delivered by him in Section E last year), and we see what large and important developments have taken place as regards the commerce and trade of the world by the effect of these influences; and then, turning to more recent days, we enter upon the discovery of steam, and its application as a motive power,—a discovery which has given rise to extraordinary changes—changes by which the whole trade of the world and its industries have been stimulated and promoted. Add to this the inventions in electricity, by which almost instantaneous communication has been established to all parts of the globe, and we may well cease to wonder at the increase that has been manifested in what may be termed the motive power of the world, and the development of its larger activities.

Still the natural laws which govern this globe, in their relation to the science of geography, remain the same. It matters not how rapidly you travel from the pole to the equator, you will freeze at the one and perspire at the other; and while passing through the different zones of temperature lying between these regions—the frigid, temperate, and torrid zones—you will find each with its own products, varying with climate, soil, and peculiarity of position, and these variations pervade the whole realm of Nature. Take man as an example: with all his power of brain and reason, he is largely subject to his environment. Look at the toiling millions of the temperate zone, and the enormous activity they display, both mental and physical. Note their colour, form, nervous development; and then pass into the tropics, and the whole creature is changed: he is different in colour, and displays none of the energy or brain power of the white species of his kind. Why is this? It is chiefly due to the environment in which the creature is living.

The effect of climate upon race is somewhat remarkably illustrated in recent times by noticing the physique and nerve-power of the present race of Americans. The wonderful tide of emigration which has raised them to being a nation of 60,000,000 people may have exercised certain influences as regards this change; but there are many true Americans still in existence. Two hundred years ago they were the same race as ourselves, but the difference between us now is marked. The climate of America has given them an individual stamp, and a perceptible difference in outward semblance has shown itself even in this short space of time.

Similar changes are manifested throughout the whole animal and vegetable kingdom; and while the geologist, zoologist, botanist, ethnologist, and entomologist, each and all are separate branches of science, yet each and all have a common ground in geography and its application to the shape and form of land and sea; to the wrinkled folds of the earth's surface which we call mountains and valleys; to the mighty ocean with its currents of air and water, and the influences they exert; to the huge inland seas and lakes; to the great rivers and small streams; to the endless varieties in the animal and vegetable kingdoms; and we find these great elements of Nature contributing each in its own sphere to questions relating to the commerce of the world and the development of new countries.

In this brief introduction to my paper I have designedly, though very briefly, drawn your attention to the science of applied geography before passing in review the most recent explorations and discoveries of the present day; and while doing this, I shall endeavour to draw attention to the great necessity for a more thorough study of this science, and the influences it exerts upon trade and commerce, as we gain a better knowledge of the products of one country and the industries of another, as well as the importance of such knowledge to the great manufacturing centres of this nation as new countries are discovered and developed.

It must be remembered that we no longer enjoy a monopoly of trade. Other nations are exhibiting large commercial activities; and if we desire a continuance of the trade of Great Britain we must put our shoulders to the wheel with the same energies and creative power that have produced such astonishing results during the present century.

In the paper to which I have already alluded, it was clearly shown how largely the rise and fall of the great emporiums of commerce in past centuries were influenced by the struggle for the Eastern trade. This struggle is still going on. The Russians in Central Asia are steadily advancing as each year goes by, and developing that system of absorption which has characterized their policy, especially in that region. Central Asia is the chosen field of their explorers, and the recent decease of General Prjevalsky has been a great loss in the scientific world. A full account of his remarkable discoveries and explorations appeared in the Proceedings of the Royal Geographical Society.

The principal work accomplished by the latest Russian explorers, Messrs. Grombchevski, Mr. Lidsky, and Mr. Grum-Grijmailo, in Central Asia have been in the region of the Pamir, and from thence across the Hindu Kush into Hunza. Also in Eastern Bokhara and in the upper waters of the Yarkand River, the Kalik Pass, and Kanjat. In the prosecution of these researches, which are all dangerously near our Indian frontier, very full reports are made, more especially as regards trade and commerce; and there is no doubt, since the completion of the Transcaspian railway to Samarcand, a great impetus has been given to Russian trade in Central Asia, even extending, by well-known routes, as far as the north-west provinces of China, where Russian goods are now found entering into competition with those of English manufacture.

By means of this railway, right into the heart of Asia, Russia has obtained the trade of a vast area, which formerly passed entirely through British hands. Both politically and commercially she is our rival in the East, and the question which nation is to be supreme must come sooner or later.

There is no more interesting country in the world than China. Her teeming and industrial population, her large mercantile centres, the geographical situation of her territory, her undeveloped mineral wealth, her individuality, and the magnitude of her trade with this country, all combine to invest her with a peculiar importance as regards our mercantile community. Coal has been discovered in all the seventeen provinces of the Chinese Empire, but the passive resistance offered by her rulers and her peoples to all attempts by foreign nations to obtain a footing in the interior have prevented any development of her resources. The day, however, cannot be far distant when railways, some of which are already projected, will open up the interior of China and make her better known; but we should be unworthy children of our forefathers if we permit the trade of this rich and widely-peopled country to pass from our hands, either from a want of energy, or from a departure from those principles of trade and commerce whose foundations are built upon the rocks of integrity and honest dealing. Nothing marks the individuality of the Chinese more than that, wherever you meet him, whatever his surroundings may be, he is John Chinaman still; he never adopts the dress, manners, or customs of other nations, but he remains constant to the pigtail, the quaint dress, and the umbrella; and if established in communities, you will find him with his joss-house, food, theatre, and his refreshment-places just as if he were in China.

Our knowledge of the latest acquisition in the East, Burmah, has been largely increased during the past eighteen months. Important surveys in North-Eastern Burmah by Colonel Woodthorpe, R.E., and Mr. Ogle have opened up an area of about 1500 square miles; and the fact of practicable routes between Assam and Burmah *viâ* the Palka Pass is now established. Burmah, with its large and intelligent population (numbering about 4,000,000), with its valuable minerals and precious stones, with its tropical products, is well worthy of the attention of the merchant adventurer; and as our knowledge of the physiography of the country is rapidly increasing, a study of its applied geography is strongly recommended to the student.

In our own territory of British India large and important surveys have been carried on under the able direction of Colonel Thuillier. These surveys are conducted in what is called the protected region; but very interesting additions, especially to the merchant, are made in the outlying territories bordering upon our Indian Empire, where no white man could go, by the employment of intelligent natives especially trained for the purpose. The information obtained by these men may be very profitably studied.

These Central Asian problems are full of deep significance to those desirous of developing and retaining the supremacy of the trade of this Empire in those regions; and I am happy to state that papers full of interest on these subjects will be presented to you during this meeting.

Turning to the northern parts of Asia, I feel some diffidence in speaking before a Newcastle audience on the subject of Siberia, for through your own townsmen, and Captain Wiggins, you are well acquainted with these regions. The exertions made by Captain Wiggins and those connected with him in this enterprise should receive the highest commendation; and that they have been so far successful is a matter for rejoicing. At the same time, I cannot but think that Russia, continuing the policy she has so steadily pursued for some years past, against the commercial development of Great Britain, would not object to the employment of British capital in opening up trade in her outlying dominions; for that trade, once fairly established on good business lines, would be absorbed on behalf of her own manufacturers. I do not attach any blame to Russia in this matter, but I am of opinion that more profits are to be gained when trade follows the British flag, for then British enterprise and money reap more certain reward. If the energy, talent, and perseverance which have been exhibited by Captain Wiggins and his partners had been utilized in the development of some of our own territories rather than in the territory of another nation, I feel sure they would command that success to which they are so justly entitled.

From the consideration of Siberia and the Northern Seas it is not a far step to Greenland, whose icy regions and eternal snows have been crossed for the first time in our history. The hero of this exploit, Dr. Frithjof Nansen, is a native of Norway, and the exploration which he has so recently conducted to a successful issue was rightly alluded to by the President of the Royal Geographical Society, in his annual address, as the most conspicuous achievement of the year.

Though young in years, Dr. Nansen proved himself to be a leader of men, and the account of his adventures will be found to be full of interest. The results of his expedition deal rather with the world of science than with commerce, as his discovery proved Greenland to be nothing more or less than a continent whose interior is a huge region of ice and snow. It, however, presents a most interesting study to those desirous of advancing our knowledge of glaciers and the glacial period. Dr. Nansen's description of this immense mass of frozen snow, forcing its way coastwards from the higher plateaus of the interior, by sheer weight and pressure, grinding, crushing, resistless in its slow but ever-moving power, gives one a faint idea of how the hills and valleys of the world were formed when, in remote periods of time, they too were under glacial influences.

Crossing from Greenland to North America, we still find ourselves in regions where ice and snow hold undisputed sway for a considerable portion of the year. The Canadian Government, with commendable activity, keep pushing forward their surveys into what is known as the old Hudson Bay Territory. The Mackenzie River has been found to be a far larger body of water than formerly supposed. More accurate surveys as regards the size of some of the great lakes of those regions are being made, and our knowledge of the climate and the isothermal variations of British North America is each year increasing.

Petroleum has been discovered, and, as the geological surveys advance, other discoveries of an important nature may reasonably be anticipated. I have been told of the existence of a huge bed of porous sandstone, saturated with mineral oil, which burns like coal.

Moving southwards, we pass through the prairie-lands of the North-West of Canada, traversed by the Canadian Pacific Railway. These rich lands are being rapidly developed, and should form a happy home for some of our surplus population. Colonization is a subject full of geographical considerations, but it demands a special paper, and I have neither space nor time to introduce it into this address. At the western edge of these prairie-lands are the Rocky Mountains, in whose foot-hills are now being reared large herds of cattle and horses, as well as flocks of sheep. Some cattle from these fertile regions were shipped last year to the English market, and no doubt a regular trade will soon follow this experiment.

Crossing the Rockies in a westward direction, you come to the Selkirk Range, then to the Gold Mountains, and lastly to the Cascades, whose wooded rocky sides plunge into the Pacific. Constant explorations are being carried on through these mountain ranges, chiefly in researches after gold and other precious metals, and our knowledge of their physiography is rapidly increasing. The Rev. Mr. Spotswood Green, in an interesting paper concerning these regions, tells us something of the configuration of the Selkirk Range, which offers alike to the mineralogist, sportsman, and Alpine explorer a field of great interest.

Continuing southward, we pass through the fertile plains and valleys of California, whose large industries in grape and orange culture are being fostered and developed. And from California you enter into Mexico, whose wonderful mineral resources are receiving a new impetus by the construction of railways, 4700 miles of which are now open to traffic. These railways will not only facilitate the transport of the wealth of Mexico from the coast to the sea, but they tend also to promote law and order among its restless and lawless population. As law and good government are established, so will trade and commerce and the natural riches of the country be promoted and encouraged.

Crossing over to South America, we find considerable progress in commercial activity, chiefly due to the increased means of communication.

In the smaller Republics upwards of 1500 miles of railway have been recently constructed; while in the larger States, Brazil has 6000 miles; Peru, 3000 miles; Chili, 1630; and the Argentine Republic, 4700—making a grand total in South America of 17,000 miles of railways. This allusion to railways may not be considered as bearing on the science of geography;

but railways are very important factors as regards the commerce and trade of the world, and by the facilities they afford they largely increase the power of exploration.

The southern portion of South America has been described by those who have visited and explored its savannahs and prairie-lands as possessing one of the richest grazing-lands of the world, and its development is only a question of time. In its present condition it offers a very interesting field of research to the explorer.

Time does not permit us to dwell long on the islands of the Pacific. Recent events concerning Samoa are fresh in your memories; and while some of these islands have developed commercially, it is when they lie in the great ocean tracks of the world that their real importance is manifested. Take for example the island of St. Vincent, of the Cape Verde Group. It is nothing but a barren rock, without any produce whatever; all its water has to be brought from a neighbouring island; yet it pays a large revenue to the Portuguese Government simply from coal dues, for it has a good harbour and lies directly in the line between Great Britain and the principal ports of South America; it has therefore become a most important coaling-station.

From the isles of the Pacific it is but a step to Australia, with its six great colonies of Queensland, Victoria, New South Wales, South Australia, Western Australia, and Tasmania, to which may be added New Zealand. Virgin fields untrodden by the foot of the white man are still awaiting the explorer to yield up their treasures to the science of applied geography; and when the marvellous progress that has been made in a few short years by our Australian colonies is weighed and considered, and as its vast interior is opened by exploration, and its mineral resources are developed, who could venture to predict the future that lies before it?

There are now nearly 11,000 miles of railway in operation, and many more miles are in course of construction throughout these various colonies—a sure and certain indication of their energy, wealth, material prosperity, and progress. Geographically speaking, some are not without their troubles. Take Queensland for instance. Her territory runs north and south for nearly 1500 miles, and lies both in the temperate and tropic zones. The Governments who during past years have administered her affairs have experienced some difficulties whilst endeavouring to reconcile the conflicting interests which arise out of her geographical position.

Laws relating to labour and capital in a temperate zone are not always in conformity with the industries and requirements of a tropical temperature, in which the white man is obliged to employ labour suitable to the climate. Hence we find a numerous section of the inhabitants of the northern part of this colony agitating in favour of separation. Australia has large coal measures, and abounds in precious metals as yet hardly developed.

Attached to Australia are the great and lesser islands forming the Australasian archipelago. The most important of these is New Guinea, and quite recently a successful exploration of its highest mountain range has been accomplished by the present administrator, Mr. Macgregor, who reached an elevation of about 14,000 feet. A very interesting paper was read before the Royal Geographical Society by Mr. Paul Thomson concerning the D'Entrecasteaux and Louisiade Groups, adjacent to New Guinea; and though many of these islands and their inhabitants are quite new to us, still the knowledge we gain from a study of their geographical position may be turned to practical uses by the merchant adventurer.

Last but not least in this record of geographical progress of the world is the vast continent of Africa.

As General Strachey, late President of the Royal Geographical Society, in his address of this year, remarks:—

"The reflection can hardly be avoided that, great as has been the advance of exploration in Africa during the last twenty or thirty years, the interest of geographers will, in the immediate future, be more and more centred in that continent. Excluding the polar regions, there is no considerable portion of the earth's surface, unless it is in Africa, the essential outlines of which have not been delineated."

These words are, I think, absolutely true. Whether we consider Africa in regard to the extraordinary explorations and developments since the commencement of the work of David Livingstone; or from the fact that vast areas of its tropical portion remain untouched as yet by exploration, and are therefore unknown; or from a contemplation of the teeming millions of its inhabitants, of which the larger portion have never seen a white man; or from the uncompleted work of the late General Gordon, and the re-establishment of the power of a civilized Government over the whole of the Nile basin; or from the slavery question, in which our nation has taken the most active and leading part; or from the spectacle of a white man, Emin Pasha, establishing a settled form of government in the heart of the continent, between the two great slave-dealing communities of the Bahr-el-Ghazal and that of the Upper Congo and Lake Tanganyika; or from the expedition sent to convey to him the succour he so much needs, under the leadership of Mr. H. M. Stanley; or from the intense interest recently exhibited by the nations of Europe in portioning out Africa between each other—an interest that has led on the west coast to the establishment of the Congo Free State, and the German protectorate in the Cameroons, France and Portugal adding largely to the possessions they already hold, and England contenting herself with strengthening her grip upon the Niger, and on the east coast by the formation of the British and German spheres of influence; or to the colonies which Great Britain possesses in the southern extremity of this great continent; or to the struggle which sooner or later must be fought out between Christianity and Mohammedanism as regards the native races of Central Africa, in which the River Congo will play an important part: I say when we consider all these and the many other problems of this continent, the vast interests they represent, and the varied influences they may yet exert on the future history of this earth, as well as the extraordinary part which Great Britain has been permitted to play in lifting the veil of mystery and doubt which up to our own times enveloped these regions, we are forced to acknowledge that the country in which the civilized world takes the most active and absorbing interest is Africa, and that the Dark Continent still maintains its supremacy.

As regards Africa two very remarkable journeys have recently been brought to a successful conclusion—that of Count Teleki, an Austrian, on the north, and that of Mr. Arnot in the regions south of the equator.

The former, entering Africa at Mombasa, at the head of a numerous and well-equipped caravan, passed through the Masai country by what is known as Thompson's route, and, pushing northwards, discovered Lake Rudolph, a large inland salt lake, and by following its shores he was enabled to trace with commendable accuracy its shape, size, and position. The existence of a large lake, called Samburu, in the direction of Count Teleki's journey, had for some time been spoken of by the Arabs who traded in that region, but nothing definite was known concerning it. Count Teleki also obtained much valuable information of the region between Mount Kenia and Lake Rudolph, its inhabitants, its rivers, and its products; and the details of his most interesting and successful journey have yet to be published.

Mr. Arnot, on the other hand, started in 1883 from Pietermaritzburg with a very slender equipment and hardly any following. His object was to prove the existence of healthy plateaus in the interior of Africa, where white men could live and prosecute the work of missionary civilization without being exposed to the malarial influences which exist in so many parts of Central Africa.

Taking a northerly course, he reaches the Zambesi, whose waters he follows as far as Lealui. From this point his route trends to the west as far as Robongo, the capital of the Bihé country. From Robongo he continues his march to Bailundu, and from thence he reaches Benguela, on the west coast. Thus he crossed Africa in the same direction as Livingstone's first journey, though somewhat to the south of Livingstone's route. While at Bailundu he meets some messengers from Msidi, the chief of the Garangenze country, who beseech him to visit their king; and having replenished his stores, he retraces his steps to the interior.

From January 1885 to February 1886 he perseveres in his attempt to reach the capital of Msidi's country, and his efforts are at length crowned with success. After a sojourn among these people for two years, during which time he thoroughly succeeded in obtaining their confidence and that of their ruler Msidi, he returned to Europe in the latter part of last year, but not before he had established two other white missionaries at Mukururu to continue the work he had begun.

He also made several small expeditions during his residence at Mukururu, the most interesting of which was to the cave-dwellers of Urua, mentioned by Livingstone. This kingdom of Garangenze is situated to the east of Lake Moero; and Mr.

Arnot has recently published a book of his travels, giving a very clear and interesting account of these people, their manners, and their customs. Of all Livingstone's followers, Mr. Arnot very closely resembles the great leader in the patient earnestness, the quiet energy, and the scanty resources with which he prosecuted his remarkable journeys.

He has quite recently returned to the west coast of Africa with the intention of rejoining his friends at Garangenze.

The events which attended the expedition under Mr. H. M. Stanley to succour and relieve Emin Pasha are so well known to you all that I shall only attempt a brief recapitulation here.

We have learned from his own pen how, after much suffering and great hardships, he eventually overcame all the difficulties and obstacles which had to be encountered while conducting his caravan from the head waters of the Congo to the lake Albert Nyanza; that on reaching that lake he met Emin Pasha.

The value of Mr. Stanley's journey and the remarkable energy and courage he displayed, his high scientific attainments, and the information that will result from his labours, are, from a geographical point of view, of the highest interest. The desiccation of the lake Albert Nyanza, and its influences on the rise and fall of the Nile, are not the least remarkable of these problems. For my own part, I am of opinion that this rise and fall is mainly caused by the rapid growth of tropical water-plants. During the dry season this vegetation increases enormously, and at the first rains large masses of aquatic growth are loosened by the rising of the waters. These masses, in the form of floating islands, pass downwards on the bosom of the flowing waters, and on reaching a wide and shallow part of the river, such as we find at the Bahr-el-Ghazal, they gradually but quickly collect till they form a dam of sufficient density to obstruct the progress of the river; and the water thus arrested finds a temporary lodgment in the lake Albert Nyanza, causing it to overflow its normal boundaries. At length the vegetable dam can no longer withstand the weight and pressure of the water bearing upon it; a portion gives way; a channel is opened; and the river, hurrying on to the sea, overflows the banks of the Lower Nile and drains the lake to a lower level. This is what happens to the Albert Nyanza, which is nothing more than a huge backwater of the Upper Nile basin, and it accounts for the lake being seen at two different levels by those two distinguished explorers Mr. H. M. Stanley and Sir Samuel Baker, and hence the difference of opinion as to its true extent and size that has arisen between them. We know that this phenomenon takes place on Lake Tanganyika, as Stanley found a marked difference in its level on the two occasions he rested upon its shores. He also followed the Lukuga River from the Tanganyika Lake to its junction with the Congo; and there is no doubt that a vegetable dam, such as I have described, forms at the point of departure of this river from the lake, and prevents its regular flow till the weight and pressure behind it sweeps all away. During the second year that I was on the Congo we had an unusually heavy flood at the time of the first rains. The river rose several feet in one night, and some months afterwards news came from the Upper Congo that the waters of the big lake had broken through, and this no doubt had reference to the Lukuga River and Lake Tanganyika.

Now, as regards the countries through which we have been passing, there are certain points of great interest connected with the science of applied geography, to which I desire to draw your special attention.

The first of these points is the study of the great railway systems of the world, and the application of railways to the development of new countries. Take our Indian possessions for example. What a change has been wrought, not only as regards the commerce of the country, but also with reference to the social condition of its inhabitants and their manners and customs! The introduction of Indian wheat, by means of these railways, into the markets of Europe has caused a revolution in the trade in that commodity. We find this especially in America, where it has upset the calculations of those gigantic combinations or rings which sought to obtain a monopoly in the supply of this universal article of food. Thus the construction of railways in the East exercises commanding influences over the markets of the West.

Consider also the traffic from China and Japan to America, with its 60,000,000 people, by means of the great Atlantic and Pacific railways, in tea and raw materials. Now, although railways cannot compete with direct traffic by sea, when the necessity for more rapid conveyance of certain goods arises, we find that a combination of sea and land transport is often adopted in preference to the longer route by sea alone.

The development of any country, no matter what its geographical position may be, is enormously increased by the construction of railways. Take the Congo Free State as an instance (which is undoubtedly the finest property in Central Africa). So long as the Upper Congo region, with its miles—measured by thousands—of navigable tributaries, was separated from the Lower Congo by the rapids extending from Stanley Pool to Matadi, this magnificent territory was practically shut to trade and commerce. Every piece of goods in the interior had to be carried on men's heads for more than 200 miles, and all ivory and other products were brought to the coast in the same way. Roughly speaking, such transport costs about £40 per ton.

The Congo Free State has wisely determined to build a railway, of some 250 miles in length, to cross this cataract region; and the moment it is completed the future of that country is assured.

H.M. the King of the Belgians has kindly given permission for a Belgian officer of distinction, Captain Thys, to read a paper at this meeting on this railway, which will afford a more detailed account of this wise and patriotic undertaking.

I have mentioned railways as the first point of interest because they are creations of our own time, and have therefore a special interest of their own; but the most important factor in the early history of the science of applied geography, and to which the establishment of our great colonial empire is mainly due, is the record of the merchant adventurers.

Their voyages and exploits, extending to every part of the globe, began at the end of the fourteenth century, in the reign of Henry VIII., when the Cabots (Venetians) sailed from England to Newfoundland, and afterwards to Florida. This expedition and those which followed it were fitted out at the expense of corporations of merchants, with the object of extending the commerce of the country by a search after trade in new and foreign lands. They were placed under the command of some well-known leader, and the results obtained were extraordinary.

In 1530 the merchant adventurers of England attempted the North-West Passage, as it is called, to China, and between 1550 and 1578, Sir H. Willoughby, Frobisher, and Sir H. Gibbon all made remarkable voyages.

Between 1585 and 1615, Davis, Hudson, and Baffin were sent by merchant companies to the polar seas, and their discoveries are handed down by the straits and bays which they discovered, and which bear their names.

In 1580 Drake took the first English vessels into the Pacific Ocean. Drake was not only a bold and successful navigator, but he was also a commander of men, in which he showed rare tact and ability.

In 1588 the merchants of Exeter established a trade with the West African coast, and the Senegal Company was formed.

In 1553 the first effort to reach India was made *viâ* the Cape of Good Hope. It was not, however, till the year 1660 that any progress was made in the East. In that year the East India Company was formed, and it is to the establishment of this Company that we owe our great Indian Empire. The year 1669 saw the formation of the Hudson Bay Company—a Company which exists at the present day. And so the record goes on down to our own times. Not the least amongst the trading corporations of Great Britain were the merchant adventurers of this city in which we are now assembled; and they, too, contributed in no small degree, not only in the past but in the present, to the extension of our geographical knowledge and its application as a science. No doubt the spirit and energy of our Scandinavian forefathers has been fostered and encouraged until it has now found its development in the enterprise and prosperity of this great mercantile centre of the north of England. And the old churches of Jarrow and Monkwearmouth bear further testimomy to the fact that, as commerce drew together communities which became centres of maritime energy and progress, religion was not forgotten, and the seed of knowledge and truth thus sown in the early history of the past has spread itself throughout the length and breadth of the great colonial empire of Greater Britain.

Following on the discoveries of the sixteenth and seventeenth centuries, and the marvellous results to which they have given birth, the story of our own times, from a geographical point of view, is quite as wonderful. As I remarked at the beginning of this paper, the discovery of steam as a motive power has brought the world into an extraordinary condition of contactiveness, and quite recently several new companies have been formed in the same spirit and on the same lines as those followed by the old merchant adventurers. These later creations are being started under more favourable conditions than their predecessors. for they have all the advantages which modern science and modern appliances can afford. The English Government have wisely encouraged and promoted the formation of these trading corporations. In countries where climate and circumstances of environment are not favourable to colonization by white men, our colonial system of government progresses somewhat slowly. It has not the elasticity, nor the adaptability, to provide for the many contingencies which must naturally arise when a few white men maintain the position of rulers over large areas, peopled by savage and uncivilized races.

In the Island of Borneo there is the North Borneo Company trading, governing, and civilizing a large portion of territory with marked success.

On the west coast of Africa, the Royal Niger Company is developing the great natural resources of that magnificent river, and its tributary the Binué.

On the east coast there is the Imperial British East African Company, operating in what is known as the British sphere of influence north of Zanzibar. Though not a twelvemonth has passed since they commenced their work, their initiatory proceedings have been remarkably successful, and there is every prospect of an early and rapid development of the territory committed to their charge. In the south-eastern portion of Central Africa, the African Lakes Company have fairly established themselves; and a new company is now being formed to open up and civilize a further portion of that section of the African continent.

The establishment of these great trading and governing centres is likely to exercise most important influences. They are, as I have before pointed out, from their organization and objects, better adapted at the outset to compete with and overcome the obstacles which present themselves to established forms of bureaucratic government; at the same time the Government of this country can interfere in cases of necessity, by the grants that have been made to them of Royal Charters, under which they carry on their operations.

A wise control and judicious administration combined with the introduction of commerce and civilization will, at no distant date, open these territories to the markets of the world, to the missionary, and to the scientific explorer. The commercial element of geography also enters very largely into their promotion and prosperity because of the fields they open to our home manufactures. It is important here to observe that, if these territories had passed into the hands of other nationalities, but a very limited quantity of British goods would ever have entered into them, and their value, as a market for the industries of the nation, would have been lost.

The establishment of a Geographical Society in this city is of real importance. Its objects should be the collection of information, and the study of applied geography in all its varied branches and aspects. It should aim to furnish complete information concerning the geography of all parts of the globe. In Chambers of Commerce our large trade centres have, no doubt, means of guiding and controlling some of our most important mercantile operations, but they afford no opportunities to the student, they are not teaching bodies; and there are instances where considerable risks have been incurred and heavy losses sustained in some of their ventures, simply from a want of knowledge of geographical data.

I should like to see a Geographical Society in every large city of this Empire, conducted on the lines I have briefly suggested, because the study of, and interest in, the commercial geography of this great Empire and the world is too much neglected amongst us. Past prosperity, and a tendency to run in the same groove, narrow our commercial horizon. Slowly, but surely, other nations, competing with us in many parts of the world, are doing so successfully because of the study they make of commercial geography.

It is for this reason I have in my address dwelt strongly upon the question and study of geography as an applied science, and it is for a greater reason I urge its importance, viz. that we may hand down to our children unimpaired the heritage bequeathed us by our forefathers; a heritage gained by courage, energy, perseverance, and patriotism—qualities which, under God's blessing, have made this nation the head of the commerce of the world.

SECTION F.

ECONOMIC SCIENCE AND STATISTICS.

OPENING ADDRESS BY PROF. F. Y. EDGEWORTH, M.A., F.S.S., PRESIDENT OF THE SECTION.

Points at which mathematical reasoning is applicable to political economy—

A.—Perfect competition—
1. Simplest type of market.
2. Complex system of markets; simplified by certain abstractions.
3. The more concrete problem of an exchange and distribution.

B.—Monopoly—
1. Transactions between a single monopolist and a competing public.
2. Transactions between two monopolists or combinations.

The use of these applications of mathematics to political economy illustrated by comparison with—
1. Applied mathematics generally.
2. The mathematical theory of statistics.

Conclusion.

AT the meeting of the British Association which was held at Cambridge about a quarter of a century ago, Jevons submitted to this Section a "general mathematical theory of political economy," which, as he himself records, was "received without a word of interest or belief." I propose to consider the justice of the unfavourable verdict which our predecessors appear to have passed on the mathematical method introduced by Jevons.

There is some difficulty in discussing so abstruse a subject in this place. It is as if one should discourse on the advantages of classical education on an occasion on which it might seem pedantic to cite the learned languages. I shall evade this difficulty by addressing to students some appended notes,[1] which, like the boy of proverb, are to be seen, not heard.

The cardinal article of Jevons's theory is that the value in exchange of a commodity measures, or corresponds to, the utility of the least useful portion of that commodity. What a person pays per month or year for a sack or ton of coal is not what he would be willing to give for the same rather than be without fuel altogether. Rather the price is proportioned to the advantage which the consumer expects from the portion which he could best dispense with—to the "final utility," in Jevons's happy phrase.

I shall not be expected here to dwell on a subject which has been elucidated in treatises of world-wide reputation, such as those of Profs. Marshall, Sidgwick, Walker, and I would add Prof. Nicholson's article on "Value" in the "Encyclopædia Britannica." Those writers seem to present what I may call the economical kernel of Jevons's theory divested of the mathematical shell in which it was originally inclosed; whereas my object is to consider the use of that shell—whether it is to be regarded as a protection or an encumbrance.

I may begin by removing an objection which the mere statement of the question raises. The idea of reducing human actions to mathematical rule may present itself to common-sense as absurd. One is reminded of Swift's "Laputa," where the beef was cut into rhomboids and the pudding into a cycloid, and the tailor constructed a very ill-fitting suit of clothes by means of rule and compasses. It should be understood, however, that the new method of economical reasoning does not claim more precision than what has long been conceded to another department of science applied to human affairs—namely, statistics. It is now a commonplace that actions such as suicide or marriage, springing from the most capricious motives, and in respect of which the conduct of individuals most defies prediction, may yet, when taken in the aggregate, be regarded as constant and uni-

[1] The appended notes are referred to by letters of the alphabet, thus: (a).

form. The advantage of what has been called the law of large numbers may equally be enjoyed by a theory which deals with markets and combinations.

But, indeed, even the limited degree of arithmetical precision which is proper to statistical generalizations need not be claimed by our mathematical method rightly understood. It is concerned with quantity, indeed, but not necessarily with number. It is not so much a political arithmetic as a sort of economical algebra, in which the problem is not to find x and y in terms of given quantities, but rather to discover loose quantitative relations of the form: x is greater or less than y, and increases or decreases with the increase of z.

Such is the character of what may be called perhaps the leading proposition in this calculus—namely, the mathematical theory of supply and demand. The use of a curve introduced by Cournot to represent the amount of a commodity offered, or demanded, at any particular price, supplemented by Jevons's theory of final utility (*a*), does not indeed determine what price will rule in any market. But it assists us in conjecturing the direction and general character of the effect which changes in the condition or requirements of the parties will produce. For example, in the case of international trade the various effects of a tax or other impediment, which most students find it so difficult to trace in Mill's laborious chapters, are visible almost at a glance by the aid of the mathematical instrument (*b*). It takes Prof. Sidgwick a good many words to convey by way of a particular instance that it is possible for a nation by a judiciously regulated tariff to benefit itself at the expense of the foreigner. The truth in its generality is more clearly contemplated by the aid of diagrams such as those employed by the eminent mathematical economists Messrs. Auspitz and Lieben (*c*).

There seems to be a natural affinity between the phenomena of supply and demand and some of the fundamental conceptions of mathematics, such as the relation between function and variable,[1] between the ordinate of a curve and the corresponding abscissa,[2] and the first principle of the differential calculus; especially in its application to the determination of *maxima* and *minima*. The principle of equilibrium is almost as dominant in what Jevons called the mechanics of utility as in natural philosophy itself. In so many instances does mathematical science supply to political economy what Whewell would have called "appropriate and clear" conceptions. Their use might, perhaps, be illustrated by comparing—however fancifully, and *si parva licet componere magnis*—the advance in economics which Jevons initiated or continued to the advance in mathematics which the new and sublime method invented by Sir William Hamilton appears to have effected. Algebra and geometry are to ordinary language in political economy somewhat as quaternions are to ordinary algebraic geometry in mathematical physics, if we accept the view of the latter relation which has been given by a very competent judge, Clerk Maxwell. "I am convinced," he says, "that the introduction of the ideas as distinguished from the operations and methods of quaternions will be of great use in the study of all parts of our subject, and especially . . . . where we have to deal with a number of physical quantities, the relations of which to each other can be expressed far more simply by a few expressions of Hamilton's than by the ordinary equations."[3] This is the spirit in which the economist should employ mathematics—"the ideas as distinguished from the operations and methods."

In considering the above-given, and indeed any concrete instances, it is hardly possible to keep to what may be called the simplest type of supply and demand, the ideal market in which we contemplate only two groups of competitors and only two articles of exchange; say, gold for corn, or any other *quid pro quo*. In general, and especially when considering what rates of exchange tend to rule in an average of transactions, it is proper to take into account that the dealings in one market will affect those in another. If the *entrepreneur* has less to pay for machinery, *cæteris paribus*, he will be able to offer more on the labour market. Thus we obtain the idea of a system of markets mutually dependent. In a general view of this correlation it is not necessary to distinguish whether the state of one part is connected as cause or effect with the other parts of the system. As Prof. Marshall[1] says:—"Just as the motion of every body in the solar system affects and is affected by the motion of every other, so it is with the elements of the problem of political economy" (*e*).

This conception of mutually dependent positions is one in which minds disciplined in mathematical physics seem peculiarly apt to acquiesce. In other quarters there may be observed a restless anxiety to determine which of the variables in a system of markets is to be regarded as determining or regulating the others. In one of the principal economic journals there has lately been a pretty stiff controversy on the question which of the parties in the distribution of the national produce may be regarded as "residual claimants upon the product of industry" (*Quarterly Journal of Economics*, 1887, p. 287; 1888, p. 9); whether it is the working class which occupies this preferential position, or if the "real keystone of the arch" is interest. Such questions certainly admit of a meaning, and probably of an answer. But they will probably appear of secondary importance to those who accept, as the first approximation to a correct view of the subject, the principle of mutual dependence—what may be called the Copernican theory of distribution, in which one variable is not more determined by another than the other is by that one (*f*).

Among the factors of this economic equilibrium I have not as yet explicitly included cost of production. Rather, the system of markets which so far I have had in view is that which would arise if the articles of exchange were periodically rained down like manna upon the several proprietors, and each individual sought to maximize his advantage according to the law of final utility. But now we must observe that self-interest does not operate in this fashion. We must take account of efforts and sacrifices.

Here, again, the language of symbol and diagram is better suited than the popular terminology to express the general idea that all things are in flux, and that the fluxions are interdependent. In Prof. Marshall's words, "As a rule, the cost of production of a thing is not fixed; the amount produced and its normal value are to be regarded as determined simultaneously under the action of economic laws. It, then, is incorrect to say, as Ricardo did, that cost of production alone determines value; but it is no less incorrect to make utility alone, as others have done, the basis of value" ("Economics of Industry," p. 148). Among those who may have gone astray in the latter sense, who, in their recoil from Scylla, are at least sailing dangerously near Charybdis, may be placed the important Austrian school who have rediscovered and restated the theory of final utility without the aid of mathematical expression. To amplify a figure suggested by one of them,[2] let us figure the hard conditions of industrial life by the austerity of a schoolmaster, who, in order to cultivate patience and fortitude in his scholars, should distribute among them certain rewards—it might be toys and sweets—in return for certain amounts of fatigue and pain endured. Thus, the cost of procuring a marble might be writing out twenty lines; the cost of a top, standing half an hour in the stocks. Supposing exchange to be set up among the members of the youthful population, free competition being assumed, there would theoretically arise an equilibrium of trade in which the value of each article would correspond to its final utility. That is, if a top exchanged for ten marbles, it might be expected that each boy would prize the last top about as highly as the last decade of marbles which he thought fit to purchase. So far, final utility may be regarded as the regulating principle.

But it is equally true that the final *dis*-utilities of the exchanged articles will be equal. If a top is worth ten marbles, we are entitled to expect such an adjustment of trade that each and every boy would as soon stand in the stocks half an hour as

[1] The treating as constant what is variable—e.g. *supply*, *margin*, *wages fund*, is the source of most of the fallacies in political economy.

[2] For instance, the two meanings of increased demand—which Mr. Sidgwick has contrasted as the *rise* and the extension of demand—are most easily and with least liability to logomachy distinguished as the variation of an ordinate (1) due to the displacement of the curve, the abscissa not varying, or (2) corresponding to an increment of the abscissa, the curve being undisturbed.

[3] Clerk Maxwell, "Electricity and Magnetism," Art. He says in the context: "As the methods of Descartes are still the most familiar to students of science, and as they are really the most useful for purposes of calculation, we shall express all our results in the Cartesian form." Compare Prof. Marshall's dictum with respect to the use of the vulgar tongue in "Economic Reasonings," cited below, p. 9.

[1] In a remarkable review of Jevons's theory in the *Academy* of April 1, 1872.

[2] Cf. Prof. Böhm Bawerk: "Es kann ein Erzieher einem Knaben, um ihn gegen Weheleidigkeit abzuhärten, für die tapfere, freiwillige Erduldung von Schmerzen ein sehnlich begehrtes Spielzeug in Aussicht stellen. So untergeordnet das Vorkommen solcher Fälle auch sein mag, so wichtig ist es für die Theorie festzustellen, dass Arbeit und Arbeitsplage doch nicht der einzige Umstand ist, auf den sich . . . die Wertschätzung gründen kann."—*Konrad's Jahrbuch*, 1886, p. 43.

Y 2

write out two hundred lines—the cost of ten marbles at twenty lines per marble.

To be sure, final utility may be conceived as operating by itself without reference to cost of production, as we tacitly assumed in our first paragraphs. Whereas the converse conception of a traffic in discommodities has less place in real life.

But it is not worth while weighing the two principles against each other, *in vacuo*, so to speak, and abstracting the real circumstances by which each is differently modified. As these are introduced, the balance will oscillate now in favour of one side, now of the other; perhaps leaving it ultimately uncertain whether cost of production or final utility is the more helpful in the explanation of economic phenomena.

For instance, in our allegory let us introduce the supposition that there is only one variety of cost—say, the common labour of writing out verses. If, now, the authorities fix twenty lines as the cost of a marble, and 200 as the cost of a top, it is predictable that a top will be worth ten marbles. It is equally true indeed, now as before, that the final utility of a top will be equal to the final utility of ten marbles. But the latter proposition, though equally true, is not equally useful. For it does not afford the simple and exact method of prediction which is obtained by the Ricardian view upon the supposition made. But then the supposition that there is only one variety of sacrifice is not always appropriate. And even if that were appropriate, it might not be helpful when we introduce the condition that the cost of procuring each article is not fixed definitely, but varies increasingly or decreasingly with the amount procured. Thus, the cost of the first marble given out might be twenty lines; of the next marble, twenty-one lines; with an equally varying scale for tops. Upon this supposition the two propositions that value corresponds to final utility and also final disutility might be equally true, but equally useless for the purpose of prediction.

Again, it may be that a man is freer to vary the extent of his expenditure than the duration of his work (*g*). The final disutility experienced by the Secretary of this Association during its meetings must be fearful. For it is not open to him to terminate at pleasure his day's work, as if he were employed by the piece. He would not, however, have accepted the office unless the advantages, less by all the trouble, were at least as great as in any other position open to him. Now this equation of the net advantages in different occupations is—co-ordinately and (in a mathematical sense) *simultaneously* with the equation of final utility for different kinds of expenditure—a condition of normal economic equilibrium (*h*). Yet again, the free play of this tendency is impeded by the existence of "non-competing groups."

I cannot be expected here to enumerate all the conditions of economic equilibrium. For a complete exposition of the complexities at which I have thought it necessary to glance, I must refer to the second book of Prof. Sidgwick's "Political Economy." It will be evident to his readers [2] that what may be called the general economic problem of several trading bodies distributing and exchanging *inter se* under the influence of self-interest and in a *régime* of competition is much more hopelessly difficult than the as yet imperfectly solved dynamical problem of several material bodies acting on each other *in vacuo*. When Gossen, the predecessor of Jevons as an exponent of the law of final utility, compares that principle to the law of gravitation, and the character of our science to that of astronomy, he betrays a too parental partiality. A truer, though still too flattering, comparison would be afforded by some very immature and imperfect specimens of physics—say the theory of fluid motion applied to the problems of house ventilation.

There is a certain resemblance between the uniformity of pressure to which the jostling particles of a gas tend and the unity of price which is apt to result from the play of competition. As the architect is guided by studying the laws according to which air flows, so it will help the builder of economic theory to have mastered the principle of movement towards equilibrium. But even in the material constructions practice is apt to lag far behind theory, as every reader in the British Museum knows. Much less are we able to predict what currents will flow between the different compartments of the industrial system. We know so imperfectly the coefficient of fluid friction, and the other conditions of the general problem: what compartments may be regarded as completely isolated and hermetically sealed, which partitions are porous and permeable.

Moreover, there is one operation of competition, which it does not seem easy or helpful to represent by physical analogies—the transference from one occupation to another, the equation of net advantages or total utilities in different employments; industrial as distinguished by Cairnes from commercial competition. The latter operation appears to me to admit much better of mathematical expression than the former, which is not so well represented by the equilibrium of a physical system.[1] Accordingly, the equation of net advantages has been judiciously omitted by Jevons in his formulation of the cost of production. And the Helvetian Jevons, as we may call Prof. Walras, appears to have altogether made abstraction of the cost of production considered as importing sacrifice and effort.

Prof. Walras, illustrating the operation of a simple market, supposes each dealer, before going to market, to write down his scale of requirements—how much he would be willing to buy or to sell at each price. From these data it would be easy to calculate beforehand the rate of exchange which would prevail in the market formed by those individuals. But, when we advance from the simplest type of market to the complexities introduced by division of labour, it is seen to be no longer a straightforward problem in algebra or geometry, given the natures of all the parties, to find the terms to which they will come. Here, even if we imagine ourselves in possession of numerical data for the motives acting on each individual, we could hardly conceive it possible to deduce *a priori* the position of equilibrium towards which a system so complicated tends.

Accordingly, it may be doubted whether the direct use of mathematical formulæ extends into the region of concrete phenomena much below the height of abstraction to which Jevons has confined himself. However, the formulation of more complicated problems has still a negative use, as teaching the Socratic lesson that no exact science is attainable. As Dupuit, one of the greatest of Jevons's mathematical predecessors, points out, "Quand on ne peut savoir une chose, c'est déjà beaucoup que de savoir qu'on ne sait rien" (*Annales des Ponts et Chaussées*, 1844, p. 372). If, he says, the early theorists, instead of formulating the balance of trade, had confined themselves to declaring the question above their powers, they would probably have done a greater service than the successors who refuted them. So Cournot, referring to his own mathematical treatment of economics, "Aussi nos modestes prétensions étaient-elles non d'accroître de beaucoup la domaine de la science proprement dite, mais plutôt de montrer (ce qui a bien aussi son utilité) tout ce que nous manque pour donner la solution vraiment scientifique de questions que la polémique quotidienne tranche hardiment" ("Revue Sommaire"). Similarly Jevons says ("Theory of Political Economy," p. 157, second edition), "One advantage of the theory of economics, carefully studied, will be to make us very careful in our conclusions when the matter is not of the simplest possible nature."

In the vineyard of science, to perform the part of a pruning-hook is an honourable function; and a very necessary one in this age of luxuriant speculation, when novel theories teem in so

[1] Suppose our allegorical schoolmaster should discontinue the system of rewards, and prefer to cultivate diligence by requiring each boy from time to time to bring up a certain number of lines, written out—whether by himself or another would not be scrutinized—or to be responsible for the cleaning of a window, after the manner of Mr. Squeers's practical method. In the traffic of discommodities which would be set up on this supposition, the (negative) value of each article of exchange would be measured solely by its disutility. However, it must be admitted, I think, that this latter hypothesis is rather more absurd than the former abstraction—with reference to real life at least; for, as it happens, the traffic in impositions more nearly resembles what is said to occur in actual schools.

[2] There occurs to me only one point at which the use of mathematical illustrations more complicated than those which I have referred to in my first two headings would conduce to the apprehension of Mr. Sidgwick's theorems. I allude to his repeated statement that, not only in international trade, as Mill pointed out, but also in trade in general, there may be several rates of exchange at which the supply just takes off the demand. This statement, taken without reservation, goes the length of destroying the prestige which is now attached to competition. Prof. Marshall, in an important passage, recommends arbitrators and combinations to imitate the method of a celebrated engineer, who, in order to make a breakwater, first ascertained the slope at which a bank of stones would naturally be arranged under the action of the waves, and then let down stones so as to form such a slope ("Economics of Industry," p. 215). Now, if gravitation acted sometimes vertically and sometimes at an angle of 45°, if the forces of competition tended to two distinct positions of equilibrium, the construction of the economic breakwater would become arbitrary. It is important, therefore, to show the limits of Prof. Sidgwick's theory. See the appended note (*j*).

[1] Commercial competition might be likened to a system of lakes flowing into each other; industrial competition to a system of vessels so communicating by means of valves, that when the level of one exceeded that of another to a certain extent, then *per saltum* a considerable portion of the contents of that one (a finite difference as compared with the differentials of the open system) is discharged into the other.

many new economic journals. I give in the appended notes an example of this corrective process applied to a theory of great worth and authority, and concerning the most vital interests, such as the relations of employer and employed, and the Socialist attack on capital (*i*). In directing this weapon of criticism against Prof. Walker, I act upon the Miltonic rule for selecting an adversary—

> " Best with the best, more glory will be won,
> Or less be lost."

In the preceding remarks I have had in view, as presumably most favourable to computation, the case of bargains in which there is competition on both sides. It is now to be added that the mathematical method is nearly as applicable to a *régime* of monopoly. Here Cournot, rather than Jevons, is our guide. Cournot's masterly analysis of the dealings between a monopolist seller and a number of buyers competing against each other has been copied out of mathematics into the vulgar tongue by many well-known writers, and need not here be repeated (*k*).

It is in this department, perhaps, that we can best answer Cairnes's challenge to Jevons to produce any proposition discovered by the mathematical method which is not discoverable by ordinary reasoning. Not, indeed, that the economist is bound to answer that challenge; any more than, in order to prove the advantages of international trade, he is concerned to deny that claret may be produced in Scotland.

The following proposition is a particular case of a more general theorem given by Cournot. Let there be a railway and a line of steamers, each forming part of a certain through journey, and separately useless; the fares will be lower when both means of transport belong to a single company than where there is less monopoly, the two services being in the hands of two companies, each seeking its own gain independently of the other.

The *rationale* of this somewhat paradoxical proposition is not easily discerned without the aid of symbols. Cournot, in a popular redaction (" Revue Sommaire ") of the theories which he first conceived in a mathematical form, suggests, as a generally intelligible explanation, that it is better to be at the mercy of a single master than of several petty tyrants. But this seems to be a commonplace of the sort which, in the absence of rigid reasoning, has so often deceived the amateur economist. Might it not be applied to the case of monopoly in general?

It would be hard to say how much this remarkable proposition may add to the arguments in favour of the Government monopolizing railways. Nor would I undertake to estimate the practical significance of Cournot's numerous mathematical theorems on the taxation of monopolists. We might perhaps compare the function of the sovereign science with respect to the theory of monopolies to the duty of Government as to their management—to exercise a general supervision without attempting to control details.

We have in the last few paragraphs been supposing monopoly on one side of the market, on the other side a public competing with each other. Let us now consider the bargain between two monopolists, whether individuals, or rather corporate trading-bodies, combinations in the most general sense of the term. The mathematical analysis of this case brings very clearly into view the important property, which is not very prominent in writings of the pre-Jevonian era, that the bargain between two self-interested co-contractors is not determinate in the same sense as in a *régime* of perfect competition.

No doubt, if we take a very simple case—such as that imagined by De Quincey, of the bargain between the owner of a musical-box and a colonist already on his way to a distant region where no luxuries can be purchased—it is easy to see that the bargain may settle down at any point between certain limits. But where both the amount of commodity to be sold and money to be paid are variable, as in the momentous case of a bargain between a combination of employers on the one hand and *employés* on the other, it is a less familiar truth that the terms of the contract are in general to some extent indeterminate. For instance, the bargain may be either all in the interest of the one party, say long hours and small pay, or, on the other hand, high wages with much leisure.

The significance of this proposition has been missed by many of those who have treated the subject without the aid of the appropriate apparatus. Some fail to see that there is any peculiarity in the bargain between isolated units. Another discerns the indeterminateness of the bargain only in the special case in which the article exchanged is a large indivisible object, like a house. Another limits the difficulty to the case of a single negotiation as distinguished from a contract which, as in the actual labour market, may be modified from time to time. Another tells us that in such a bargain the most anxious party gains least.

All these phrases seem to obscure the cardinal distinction that perfect competition tends to a determinate settlement, whereas in a *régime* of combination a principle of adjustment is still to seek. What is that principle?

At a former meeting of the British Association, on the occasion of a discussion on sliding scales, I stated the difficulty which there might be, in the absence of competition, in defining fair wages and reasonable terms, and I asked the eminent Professor who introduced the subject in what direction one should look for a solution of this difficulty. His reply imported, as I understood, that no other general rule can be given but this: to obtain a full knowledge of, and bring a candid judgment to bear on, all the circumstances relevant to each case. To which I would add that one circumstance relevant to this whole class of cases is just the fact that there is in the abstract such a marked difference between combination and competition.

Possibly the dry light of abstract science may enable us to see a little further into this difficulty. Analysis strongly suggests that the right solution is what may be called the utilitarian arrangement, that which is productive of the greatest sum total of advantage for all concerned. The utilitarian determination is clearly discerned to be by no means necessarily coincident with the settlement towards which competition tends. For instance, the *vrai prix*, in Condillac's sense, as determined by the play of supply and demand in the labour market, might be such that the *entrepreneur* class should take the lion's share, leaving the labourer a bare and painful subsistence; but there is no ground to believe that this is the best possible arrangement. From an abstract point of view it is by no means evident that a free labour market "is the only way to equity, that any interference with it must involve injustice" (Danson). Nor need it appear "a great fundamental principle—as inevitable in its action as gravitation—that a fair day's wages for a fair day's labour is determined by the proportion which the supply in the market bears to the demand" (Rupert Kettle on "Arbitration"). It may be true indeed, in a practical sense, that perfect competition is "not less harmonious and beneficent in its operation than gravity" (Walker, " Political Economy "); but theoretically it is tenable that there is an adjustment of contracts more beneficent than that which the mechanical play of competition tends to establish (*l*).

To introduce these philosophical conceptions of utilitarianism will doubtless seem irrelevant to those who are immersed in the details of business. But the practical man should be reminded that in other spheres of action, politics and morals, the principle of utility, however badly received at first, has exercised a great influence—though doubtless not so great as was anticipated by some theorists, and requiring to be largely tempered with common-sense.

Such, I think, are the principal points at which mathematical reasoning is capable of being applied to political economy. In estimating the use of this method it is natural to take as our standard the helpfulness of mathematics in other departments of science.

As compared with mathematical physics, the mathematical theory of political economy shows many deficiencies. First, there is the want of numerical data, which has been already noticed. It is true that there is a faint hope of obtaining what Jevons too confidently expected—statistical data for the relations between supply and price. It is true also that in the higher mathematics conclusions which are quantitative without being numerical are more frequent than is usually supposed. Some political economy is as exact as some mathematical physics. The fields cultivated by Section A and Section F may overlap, but it must be admitted that the best part of our domain corresponds to what is the worst part of theirs. If you inquire as to the products of our inferior soils, we must confess, if we do not wish to conceal the nakedness of the land, that over a large portion of our territory no crop is produced. We are employed only in rooting out the tares which an enemy has planted. Much of our reasoning is directed to the refutation of fallacies, and a great part of our science only raises us to the zero point of nescience from the negative position of error. "*Sapientia prima stultitiâ caruisse.*" In this introductory portion of political economy we have seen that the mathematical method is likely to be serviceable (see above, p. 498).

It is not to be supposed, however, that the work of our Section is wholly destructive; that like the islanders of whom it was said that they earned a precarious livelihood by washing one another's clothes, so we are occupied only in mangling each other's theories. Like imprudent sectaries, by our mutual recriminations we have obscured the virtues common to our profession. What Jevons said of Cairnes, that his own opinions were much more valuable than his objections against other people's opinions, is true of Jevons himself and other controversial economists. Now, this possibility of mutual misunderstanding by persons who are both in the right is connected with a circumstance which it is not irrelevant here to notice. It is that in our subject, unlike physics, it is often not clear what is the prime factor, what elements may be omitted in a first approximation. One writer on rent may emphasize distance from the centres of population as the main attribute, and introduce fertility of soil as a perturbation of the abstract result given by the first view. Another fixes attention on the powers of the soil, and allows for other elements, as for friction. So in the theory of money, the state of credit or the quantity of metal have each been regarded as the prime variable.[1] It need not be pointed out how unfavourable to exact science is such a state of the subject-matter. Imagine an astronomer hesitating whether in the determination of Jupiter's movements the sun or the planet Saturn played the most important part. That is the condition of many of our speculations.

It will not be expected that from such materials any very elaborate piece of reasoning can be constructed. Accordingly another point of contrast with mathematical physics is the brevity of our calculations. The whole difficulty is in the statement of our problems. The purely computative part of the work is inconsiderable. Scarcely has the powerful engine of symbolic language been applied when the train of reasoning comes to a stop. The case is like that of the swell in *Punch*, who, about to enter a hansom, inquires solicitously of the driver whether he has got a good horse. "Yes, sir; very good 'oss." "Aw—then dwive to next door." However, our road, though short, is so slippery as to require every precaution.

It follows that in economics, unlike physics, the use of symbols may perhaps be dispensed with by native intelligence. It must be admitted that the correct theory of value has been rediscovered by Menger, and restated by his follower, Böhm-Bawerk, without the explicit use of mathematics. Without the law, they have done by nature the things contained under the law. Still, under a higher dispensation, they might have attained greater perfection. Nor can equal accuracy be ascribed to all the followers of Menger. Nor is the terseness which comes of mathematical study a characteristic of this Austrian school (*m*).

Another point of contrast between the mathematical science of the physicist and the economist is that the former appeals to a larger public. Mathematics is as it were the universal language of the physical sciences. It is for physicists what Latin used to be for scholars; but it is unfortunately Greek to many economists. Hence the writer who wishes to be widely read—who does not say, with the French author, "*J'imprime pour moi*"—will do well not to multiply mathematical technicalities beyond the indispensable minimum, which we have seen reason to suppose is not very large. The parsimony of symbols, which is often an elegance in the physicist, is a necessity for the economist. Indeed, it is tenable that our mathematical constructions should be treated as a sort of scaffolding, to be removed when the edifice of science is completed. As Prof. Marshall, one of the highest authorities on this subject, says: "When a man has cleared up his mind about a difficult economic question by mathematical reasoning, he generally finds it best to throw aside his mathematics and express what he has to say in language that is understanded of the people" (*Academy*, June 1881). Upon this view mathematical discipline might be compared to grammar or to the study of classical literature, which it is profitable to have learnt thoroughly, while it is pedantic to obtrude one's learning.

From these considerations it may appear that our little branch of science is of quite a rudimentary form. The solid structure and regular ramifications of the more developed mathematical sciences are wanting. A less unfavourable contrast would be presented if we compared our method, not with applied mathematics generally, but with that particular branch of it which comes nearest to ours in its proximity to human interests—the use of the calculus of probabilities in social statistics.

There is really only one theorem in the higher part of the calculus, but it is a very difficult one, the theory of errors, or deviation from averages. The direct applications of this theory to human affairs are not very considerable. Perhaps the most conspicuous example is afforded by an investigation to which, if I had undertaken to review the work done in our subjects during the past year, I ought to have directed particular attention—Mr. Galton's rigid proof of the fact and amount of *regression*, or reversion, in children compared with parents, and other relationships.

But, beyond the isolated instances in which the theory of deviations is applied in social statistics with the same strictness and cogency as in physics, there is a wide zone of cases in which the abstract theory is of use as giving us some idea of the value to be attached to statistical results. Mr. Galton justly complains of the statisticians who "limit their inquiries to averages, and do not revel in the more comprehensive views" of the deviations from averages. "Their souls seem as dull to the charm of variety as that of the native of one of our flat English counties, whose retrospect of Switzerland was that, if its mountains could be thrown into its lakes, two nuisances would be got rid of at once." But great caution is required in transferring the theory of errors to human affairs; and the calculus of probabilities may easily be made, in Mill's phrase, the "opprobrium of mathematics."

Now, in all these respects there is a considerable resemblance between the higher parts of the two branches of science which are cultivated in this Section. It may be said that in pure economics there is only one fundamental theorem, but that is a very difficult one: the theory of bargain in a wide sense. The direct application of mathematical reasoning is, as we have seen, limited—more limited, I think, than the corresponding function of the higher statistics. But, on the other hand, the regulative effect, the educational influence, of studies like those of Cournot and Jevons are probably very extensive.

How extensive, it would be difficult to decide without defining the limits of a province within which our special subject is included—the use of abstract reasoning in political economy. Now, on this vexed question, and with reference to the heated controversy between the historical and the deductive schools, the mathematical economist as such is not committed to any side. It may be dangerous to take wide general views; it may be better to creep from one particular to another rather than ascend to speculative heights. Our only question here is whether, if that ascent is to be made, it is better to proceed by the steep but solid steps of mathematical reasoning, or to beguile the severity of the ascent by the zigzag windings of the flowery path of literature. It is tenable that the former course is safest, as not allowing us to forget at what a dangerous height of abstraction we proceed. As Prof. Foxwell has well said,[1] with reference to the mathematical methods in the hands of Jevons and Prof. Marshall, "It has made it impossible for the educated economist to mistake the limits of theory and practice, or to repeat the confusions which brought the study into discredit and almost arrested its growth."

I trust that I have succeeded in distinguishing the question what is the worth of abstract reasoning in political economy from the much more easily answered question whether, if it is worth doing, it is worth doing well.[2] The mathematical economist is concerned to separate his method from that mathematical and metaphysical reasoning which Burke repudiates as inapplicable to human affairs; from that abstract method which he has in view when he says:—"Nothing can be more hard than the heart of a thoroughbred metaphysician. It comes nearer to the cold malignity of a wicked spirit than to the frailty and passion of a man" ("Letters on a Regicide Peace"). Burke is referring to the Jacobin philosophers; but our withers are unwrung, if similar words should be applied to some of the "sophisters and economists" of a later generation. Just as a political party, if

[1] Compare Cournot: "Ce que l'un néglige dans une première approximation comme un fait secondaire et accessoire, un autre le regardera comme le fait principal et dominant" ("Principes," book iv. chap. vii.).

[1] In his important letter on "The Economic Movement in England" in the *Quarterly Journal of Economics* for October 1888.

[2] Cf. Prof. Foxwell *loc. cit.*:—'What the new school protest against is first the unscientific and meagre way in which deduction was used. In their view, though it is worth while to study, and therefore worth while to study accurately, the workings of private interest under a system of competition, yet human nature is not all self-interest. . . ."

popularly suspected of complicity with crime, would do well to take every opportunity of clearing themselves from that imputation, so the mathematical economist is called on to disown emphatically all sympathy with the flagrant abuses to which the injudicious use of abstract reasoning is undoubtedly liable.

To continue the comparison which I was instituting between the mathematical theory of economics and the calculus of probabilities, they have one very unpleasant property in common—a liability to slips. As De Morgan says,[1] "Everybody makes errors in probabilities at times, and big ones." He goes on to mention a mistake committed by both Laplace and Poisson, the ineptitude of which he can only parallel by the reasoning of a little girl whom he had called a "daughter of Eve"; to which she retorted, "Then you must be a daughter of Adam." It is not to be concealed that economic reasoning, even in its severest form, is sometimes equally inconsequent. I should have hesitated to assert that Cournot has made some serious mistakes in mathematics applied to political economy, but that the authority of the eminent mathematician Bertrand[2] may be cited in support of that assertion.

Again, the more abstract theories of value and of probabilities seem to resemble each other in their distance from the beaten curriculum. Each forms, as it were, a little isolated field on the rarely crossed frontier and almost inaccessible watershed between the moral and the physical sciences.

The same character of remoteness belongs perhaps to another province, which is also comparable with ours—the mathematical side of formal logic, the symbolic laws of thought which Boole formulated. There was a certain congruity between Jevons's interest in his logical machine and in what he called the "mechanics of industry." But I venture to regard the latter pursuit as much more liberal and useful than any species of syllogism-grinding.

If you accept these parallels, you will perhaps come to the conclusion that the mathematical theory of political economy is a study much more important than many of the curious refinements which have occupied the ingenuity of scientific men; that as compared with a great part of logic and metaphysics it has an intimate relation to life and practice; that, as a means of discovering truth and an educational discipline, it is on a level with the more theoretical part of statistics; while it falls far short of mixed mathematics in general in respect of that sort of pre-established harmony between the subject-matter and the reasoning which makes mathematical physics the most perfect type of applied science.

But we must remember—and the mention of the theory of probabilities may remind us—that any such judgment is liable to considerable error. We cannot hope to measure the utility of a study with precision, but rather to indicate the estimate on either side of which competent judges would diverge—a central point, which will be found, if I mistake not, equally removed from the position of Gossen, who compares the new science to astronomy, and the attitude of Dr. Ingram towards the researches which he regards as nothing more than "academic playthings, and which involve the very real evil of restoring the metaphysical entities previously discarded."[3]

One more general caution is suggested by another of the technical terms which we have employed. What we are concerned to discover is not so much whether mathematical reasoning is useful, but what is its "final utility" as compared with other means of research. It is likely that a certain amount of mathematical discipline—say as much as Mr. Wicksteed imparts in his excellent "Alphabet of Economic Science"—is a more valuable acquisition to a mind already stored with facts than the addition of a little more historical knowledge.

But, in reverting to the subject of final utility, I am reminded that Presidential addresses, like other things, are subject to this law; and that a discourse on method prolonged beyond the patience of the hearers is apt to become what Jevons called a *discommodity*.

[1] Writing to Rowan Hamilton ("Life of Hamilton," by R. Graves, vol. iii.).

[2] *Journal des Savants*, 1883. I hope to show on some future occasion that M. Bertrand's censures of Cournot and Prof. Walras are far too severe.

[3] See the passage relating to Jevons in the article on "Political Economy" in the "Encyclopædia Britannica," ninth edition.

NOTES.

(*a*) SIMPLE EXCHANGE.—The simplest case of exchange is where there are two large groups of uncombined individuals dealing respectively in two commodities, *e.g.* corn and money. To represent the play of demand and supply, let any abscissa, Ox in Fig. 1, represent a certain price, and let the quantity of commodity demanded at that price be xp. The locus of p may be called the demand-curve. Similarly, xq represents the quantity offered at any price, Ox; and the locus of q is called the supply curve. The price Oa, at which the demand is just equalled by the supply, is determined by the intersection of these curves. This is Cournot's construction. The converse construction, in which the abscissa stands for quantity of commodity, the ordinate for price, is employed by Mr. Wicksteed in his excellent "Alphabet of Economic Science."

The diagrammatic representation which most closely corresponds to Jevons's formulæ is that which the present writer, after Prof. Marshall, and Messrs. Auspitz and Lieben, independently, has adopted. In this construction the two coordinates respectively and symmetrically represent the quantities

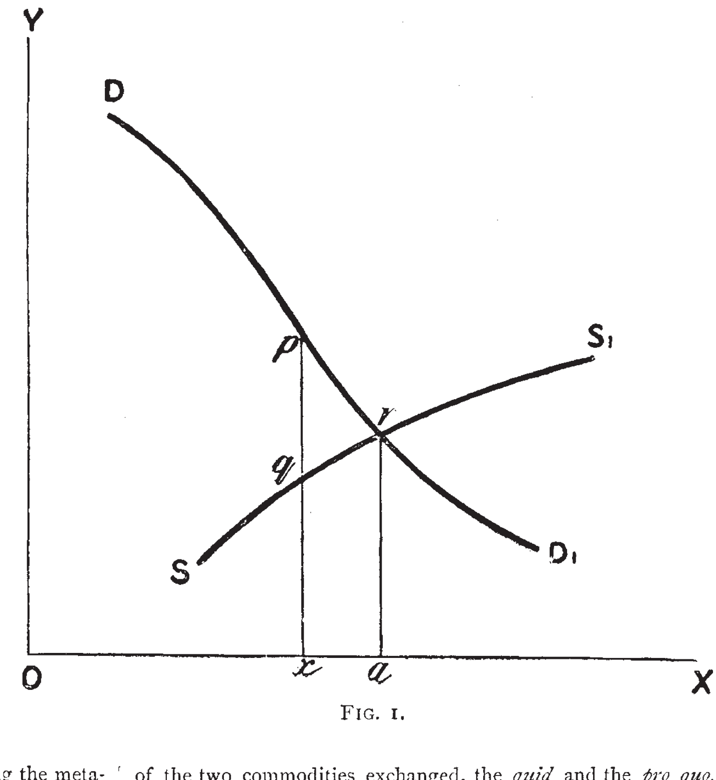

FIG. 1.

of the two commodities exchanged, the *quid* and the *pro quo*. For instance, Fig. 2 may represent the state of supply and demand in the international market between Germany and England. The curve OE denotes that in exchange for any amount of "linen," Oy, England is prepared to supply the quality of "cloth" yp (= Ox); or, in other words, that in exchange for the quantity Ox of cloth England demands xp (= Oy) of linen. The curve OG is similarly related to Germany's supply and demand. The position of equilibrium is determined by the intersection of these curves.

(*b*) VARIATIONS IN SUPPLY.—Suppose, as Mill supposes ("Pol. Econ.," book iii. ch. xviii. § 5), that there has occurred an improvement in the art of producing Germany's export, linen. The altered conditions of supply may be represented by the displaced curve OG′, Fig. 3, indicating that, whereas before the improvement Germany in exchange for any quantity, Ox, of cloth offered only xq, she now offers xq'. The effect of the improvement on the rate of exchange will depend upon the form of the curve OE beyond the point r. If the intersection of the curve OE is at r, vertically above r, we have the case where, as Mill rather awkwardly says, the demand of England for linen increases "in the same proportion with the cheapness." The other cases in which the demand for linen—and accordingly the price,

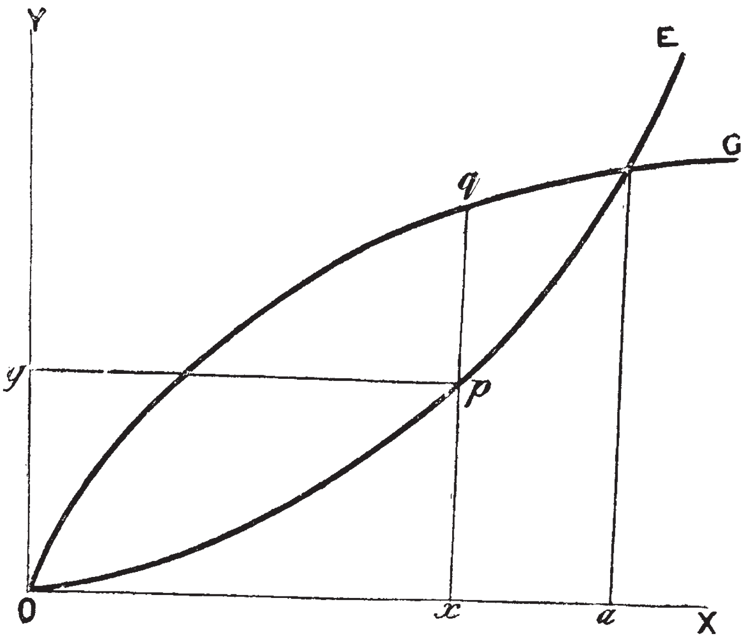

FIG. 2.

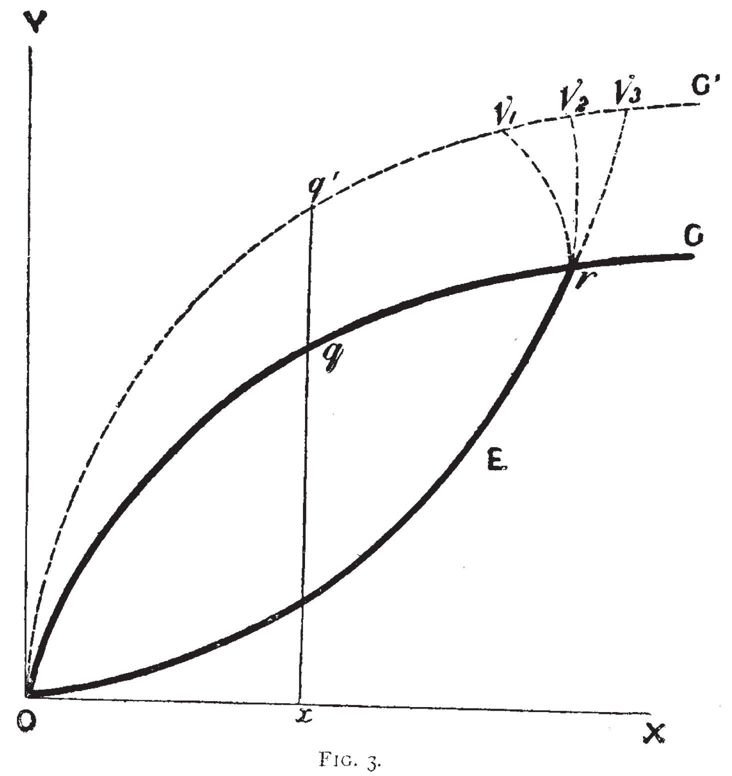

FIG. 3.

so to speak, of cloth in linen—are increased more or less than the cheapness, are represented by the points of intersection r_2, r_3.

The same construction may be used to represent the effect on the rate of exchange produced by a tax on exports or imports. Let OG′ now represent the undisturbed condition of supply, and let OG be what this curve becomes when displaced by a tax on Germany's exports. According to the position of the original intersection, whether at r_1, r_2, or r_3, we have the three cases distinguished by Mill (book v. ch. iv. § 6).

Again, the same construction may be used to facilitate the comprehension of international trade which Prof. Sidgwick has recently proposed. Let the curves OE and OG′ represent the conditions of supply and demand, on the hypothesis that cost of transport is annihilated, that England and Germany are in juxtaposition. Now restore the abstracted sea, and the altered conditions of supply and demand *in a market on the English shore* will be represented by the change of OG′ to OG. According to the form of the curve OE the different effects on the rate of exchange are visible at a glance. (Cf. Sidgwick, "Pol. Econ.," book ii. ch. ii. § 3.)

(*c*) GAIN OF TRADE.—To measure the variations in the advantage acruing from trade by the variations of price—or more generally, rate of exchange—is a confusion which could hardly have occurred to the mathematical economist. The simplest method of illustrating the gain of trade is that proposed by Messrs. Auspitz and Lieben. In Fig. 4, let On be the *locus* of a point t, such that a certain individual in exchange for the quantity Ox of one commodity will just be willing to give the quantity tx of another commodity, will neither gain nor lose by that bargain. Then, if he obtain Ox in return for only rx, he is a gainer by that bargain to the extent of tr. The curve thus defined is called the *utility-curve.*

Now add properly the utility-curves for all the individuals of a community, and we obtain what may be called a collective utility-curve. There is a peculiar propriety in taking one axis, say the ordinate, to stand for money. Let ON, then, in Fig. 5, be the collective utility-curve, in this sense, for the German community with respect to cloth. Let OG represent the demand of Germany for cloth, as before, except that the ordinate now stands for money, not linen. And let OE represent the supply of cloth in exchange for money on the part of England. Then the gain to Germany of the trade with England is represented by the vertical distance tr.

Now let Germany impose a tax on the import of cloth. The effect of the tax will be to displace the supply-curve in the manner indicated by the dotted curve OE′. Let r' be the new point of intersection between the demand- and (displaced) supply-curve. The gain to Germany in the way of trade is now $t'r'$. To which is to be added the tax r'S accruing to Germany. Since t'S may very well be greater than tr, Germany may gain by the imposition of the tax.

What difficulties the reader may feel about this proposition will disappear on reference to Messrs. Auspitz and Lieben's beautiful and original reasoning ("Theorie der Preise." §§ 80–82). In the light of their constructions it will be at once seen what conditions of supply and demand are favourable to the endeavour of one nation to gain by taxing the imports from (or exports to) another. It will be noticed that the particular supposition entertained by Prof. Sidgwick (book iii. ch. v. § 2)—that the quantity consumed of the taxed import is constant—is not essential.

It may be observed that the *utility-curve* is a particular case of the "*indifference-curve*" employed by the present writer ("Mathematical Psychics," p. 21). Also the lines tr and $t'S$ are particular cases of the "preference-curve" (*ibid.* p. 22). If these more general conceptions are employed, the demonstration will not require that we should put the ordinate for money, regarded as a constant measure of utility. The interpretation assigned to the curves OG and OE in our second and third figures may still stand.

(*d*) Economic Equilibrium.—By analogy with well-known physical principles, economic equilibrium may be regarded as determined by the condition that the advantage of all parties concerned, the integrated utility of the whole economic system, should be a *maximum*. This *maximum* is in general subject, or in technical phrase *relative*, to certain conditions; in particular what Jevons called the "law of indifference," that in a market all portions of a commodity shall be exchanged at the same rate. But occasionally this condition is suspended; as often as we take what may be called a socialistic or utilitarian view as distinguished from that incommensurability of pleasures appertaining to different persons, which Jevons in a remarkable passage of his "Theory" (p. 15) has postulated. It will be found that this postulate must be abandoned when we consider the gain of trade, as in our note (*c*), or the theory of combinations, as in note (*l*), and on other occasions.

In general, the first condition of a maximum, that the first term of variation should vanish, gives the Jevonian equations of exchange, the demand-curves of other writers.

The second condition of a maximum, that the second term of variation should be negative, finds its fulfilment in certain well-known propositions which involve the conception of a decreasing rate of increase, viz. the law of diminishing returns, the law, or laws, of diminishing utility and increasing fatigue.

For some propositions it is proper to take account not only of the sign, but also the magnitude, of the second differential of utility. Thus, when Prof. Walker is contending that in case of "any increase of product resulting from the introduction of any new force into industry," the whole increment will fall to be added to the share of the working class, he argues, quite correctly upon his premises, that if the improvement does not "increase the amount of tools and supplies required in production"—since "there is no greater demand for capital in the case supposed— . . . there can be no increase in the rate or amount of interest" (*Quarterly Journal of Economics*, 1887, pp. 283, 284). Analytically we should find that the variation in the rate of interest due to the disturbance of the equilibrium, say $\Delta\iota$, was indefinitely small as compared with the variation in the rate of wages, say $\Delta\omega$, because the decrease in the rate at which the utility of capital increases is indefinitely great. The argument requires that this second differential should be infinite at the position of equilibrium—a postulate which may perhaps give us pause.

(*e*) Complex Exchange is the general case of simplex exchange above analyzed. We have now several, instead of two, categories of dealers and commodities. In both cases equilibrium is determined upon the principle that each individual seeks to maximize his own advantage, subject to the conditions (1) that in a market there is only one price for any article, and (2) that all which is bought is sold, and all which is sold is bought. Let there be m dealers and n articles. And the first article being taken as the measure of value, let the prices of the remaining articles be $p_2, p_3, \ldots p_n$. Let the quantities of commodities bought or sold by any individual, say No. r, be $x_{r1}, x_{r2}, \ldots x_{rn}$; each variable with its sign: *plus*, if bought, *minus*, if sold. Let the

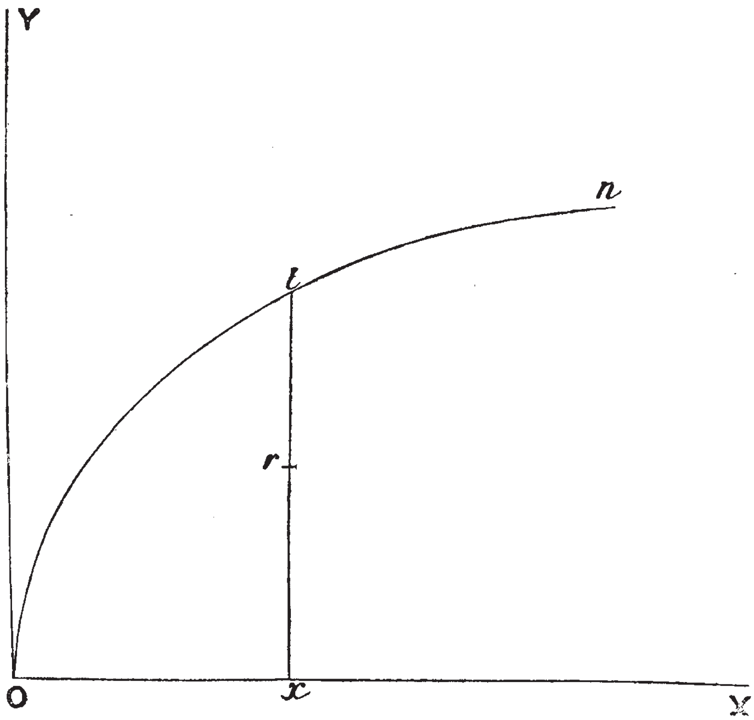

Fig 4.

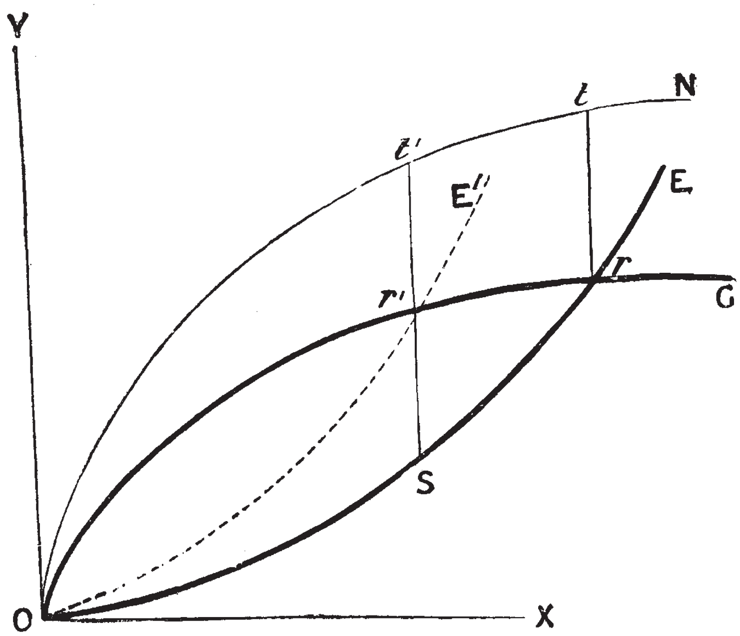

Fig. 5.

advantage of the individual, regarded as a function of his purchases and sales, be $\psi_r(x_{r1}, x_{r2} \ldots x_{rn})$. There is sought the system of values assigned to the variables for which this function is a maximum, subject (a) to the condition which follows from the first assumption above made: $x_{r1} + p_2 x_{r2} +$ &c. $+ p_n x_{rn} = 0$. In order to determine the maximum of ψ_r subject to this condition, we obtain (β) by the calculus of variations $(n - 1)$ equations of the form—

$$\left(\frac{d\psi_r}{dx_{r1}}\right) = \frac{1}{p_2}\left(\frac{d\psi_r}{dx_{r2}}\right) = \ . \ . \ . \ = \frac{1}{p^n}\left(\frac{d\psi_r}{dx_{rn}}\right)$$

(with certain conditions as to the second term of variation). To which is to be added the equation (α). We have thus n equations relating to the rth individual. The same being true of each of the m individuals, we have in all mn equations of the forms (α) and (β). We have also (γ), from the condition that everything which is bought is sold, and conversely, n equations of the following form: $x_{1s} + x_{2s} +$ &c. $+ x_m = 0$.

But of the $(m + n)$ equations of the forms (α) and (γ) only $(m + n - 1)$ are independent. For adding the m equations of the form (α) we have:—

$$\left.\begin{array}{l} \ \ \ (x_{11} + x_{21} + \ . \ . \ . + x_{m1}) \\ + p_2(x_{12} + x_{22} + \ . \ . \ . + x_{m2}) \\ + \ \ \ \ \text{—} \ \ \ \ \text{—} \ \ \ \ \text{—} \ \ \ \ \text{—} \\ + p_n(x_{1n} + x_{2n} + \ . \ . \ . + x_{mn}) \end{array}\right\} = 0.$$

Now, if any $(n - 1)$ of the equations of the form γ, say all but the first, are given, then in the last written equation the coefficients of $p_2 \ . \ . \ . \ . \ p_n$ vanish. Therefore the first equation of the form (γ), viz. $x_{11} + x_{21} +$ &c. $+ x_{m1}$, is also given. We have thus $mn + (n - 1)$ equations to determine $mn + (n - 1)$ quantities, viz. the x variables, which are mn in number, and the $(n - 1)$ p's.

The great lesson to be learnt is this. The equations are *simultaneous*, and their solutions *determinate*. That the factors of economic equilibrium are simultaneously determined is a conception which few of the literary school have received. The reader is referred to Prof. Walras's "Leçon" 12 ("Econ. Pol.," second ed.) for a lengthier exposition, and for a more accurate one to Messrs. Auspitz and Lieben's Appendix IV.

(f) Commercial Competition.—Abstracting that change of occupations which Cairnes ascribed to "industrial" as distinguished from "commercial" competition (comp. Sidgwick's "Pol. Econ.," book ii. ch. i.), let us suppose that the x's of the last note, which primarily denoted commodities ready for immediate consumption, include also agencies of production: (the use of) land, labour, and capital. We may conceive *entrepreneurs* buying these agencies from landlords, labourers, and capitalists, and selling finished products to the public. We have thus the appropriate idea of rent, wages, interest, and (normal) prices determined *simultaneously* (in the mathematical sense).

In a primary view of complex exchange it is proper, with Jevons, to regard each portion of commodity sold, each negative variable, say $- x_{rs}$, as a deduction from an initial store, say ξ_{rs}. But when we consider production, we regard ξ as a function of the outlay of the *entrepreneur*. Supposing that the *entrepreneur* confines himself to the production of a single article, let the gross produce, in money, after replacing capital, be $f_r(c_r, l_r)$, where f_r is a function depending on the individual's skill, energy, opportunities, &c., c_r is the amount of capital borrowed by him, and l_r the number of acres of a certain quality which he rents. The net produce is obtained by deducting from this quantity the payments $c_r\iota - l_r\rho$, where ι is the rate of interest and ρ is the rent per acre. Thus the advantage which the *entrepreneur* seeks to maximize is of the form—

$$\psi_r(x_{r1}, x_{r2} \ . \ . \ . \ [f_r(c_r, l_r) - c_r\iota - l_r\rho] - x_{rr}p_{rr}, \ . \ . \ . \ );$$

whence $\frac{df_r}{dc_r} = \iota$ and $\frac{df_r}{dl_r} = \rho$. The first of these equations expresses a well-known proposition regarding the final utility of capital. The second equation expresses a less familiar condition with respect to the number of acres which will be rented on an ideal supposition of the homogeneity and divisibility of land above the margin of cultivation.

What, then, and where, is the Ricardian theory of rent? Its symbolic statement is $l_r\rho = f(c_r, l_r) - f(c_r, o) = f_r(c_r, l_r) - c_r \times \iota$; where $f(c_r, o)$ is the gross produce of c_r capital laid out by the individual numbered r, on land below the margin obtainable for nothing in as large quantities as desired. It will be found that these equations postulate that the quantity of land above the margin is small as compared with the number of applicants, and that $f(c_r, o)$ is identical with $c_r \times \iota$, which are the common Ricardian assumptions. The validity of these assumptions as a first approximation, the need of correction where greater accuracy is required (truths which some minds seem incapable of holding together), have been admirably pointed out by Mr. Sidgwick ("Pol. Econ.," book ii. ch. vii. § 2). The second approximations made by him may be usefully expressed in the symbols which have been proposed, or rather in those which the student may construct for himself. I do not put forward those which occur to me as the best—if, indeed, there is any absolutely best in the matter of expression. For some purposes it would have been proper to take account of the various qualities of land (as I have elsewhere done—"Brit. Assoc. Rep.," 1886). For other purposes it would be well to put labour hired by the *entrepreneur* as an independent variable. When this or any other variable is omitted, we are to understand that there is implied the best possible arrangements with respect to the variables which are not expressed. The nature of this implication is shown in the following note.

(g) So far we have been taking for granted that the *entrepreneur* does his best, without reference to the motives acting upon him, the pleasures procurable by the sale of his product. Formally it would be proper to take account that the utility-function ψ_r involves the *effort*, say e_r, explicitly, as fatigue diminishes advantage, and implicitly, as exertion increases production. Corresponding to the new variable we have a new equation, the complete differential of ψ_r with reference to e_r, say $\left(\frac{d\psi_r}{de_r}\right) + \left(\frac{d\psi_r}{df_r}\right)\frac{df_r}{de_r} = 0$. It is a nice question how far effort should be regarded as an independent variable; how far the essential principle of piece-work prevails in modern industry.

(h) Industrial Competition.—The condition that net advantages should be equal in industries between which there is mobility may thus be contemplated. Let us put the advantage of an individual, say No. r, engaged in the occupation s as a function of his net income, the price of the articles in which his expenditure is made, and the disutility of effort. Say $\phi_{rs}(\mathrm{f}_{rs}(\pi_1, \pi_2 \ . \ . \ . \ . \ e_{rs}), p_1, p_2 \ . \ . \ . \ - e_{rs})$; where ϕ_{rs} is a utility-function, not necessarily the same for the same individual in different occupations, since his indulgences may vary with the nature of his employment; f_{rs}—a symbol not identical with the f of the last but one note—is the individual's net earnings in the business s, involving prices π_1, π_2, &c., of all manner of agents of production, involving also, as stated in note (g), the effort e_{rs}; p_1, p_2, &c., are prices of articles of consumption as a function of which the individual's advantage may be obtained by means of the equations (α) and (β) in note (e)—eliminating the quantities consumed. The last variable in the function ϕ_{rs}, the explicit e_{rs}, has a negative sign prefixed, to indicate that the direct effect of increased fatigue is diminished advantage.

The equation of net advantages imports that the advantage, ϕ_{rr}, of the occupation which the individual chooses is not less than ϕ_{rs}, the advantage of any other occupation open to him. It is important to observe that for all occupations the complete differential with regard to e is zero; in symbols $\left(\frac{d\phi}{d\mathrm{f}}\right)\frac{d\mathrm{f}}{de} + \left(\frac{d\phi}{de}\right) = 0$. But this equation conveys no presumption that the final disutility in different occupations is the same that $\left(\frac{d\phi_{rs}}{de_{rs}}\right) = \left(\frac{d\phi_{rt}}{de_{rt}}\right)$. The equation of final disutility holds only where efforts and sacrifices are capable of being applied in "doses" to any number of occupations. The latter is the only case, I think, contemplated by Jevons in his analysis of cost of production ("Theory," ch. v.). The inquiry, what is meant in general by saying that the cost of production of two articles is equal, must start from right conceptions about final and total utility. But this is not the place to follow up the difficult investigation. I do not attempt here to discuss any matter fully, but only to illustrate the suitability of the subtle language of mathematics to economical discussions.

(i) Prof. Walker's Theory of Business Profits.—Prof. Walker's theory as stated in the *Quarterly Journal of Economics* for April 1887, involves the proposition that the remuneration of the lowest, the least gifted employers, is on a level with that of the labouring class. Concerned as we are here with methods rather than results, it is allowable to posit this premise without expressing an opinion as to its accuracy. It is fortunate not to have to take sides on an issue concerning which the highest authorities are divided, and statistical demonstration is hardly possible.

But, though the expositor of method is not called to dispute the truth of this proposition, he has something to say against the evidence which has been adduced in proof of it. He must enter a protest against the form of the following argument :—

"Let our hypothesis be clearly understood. We assume, first, that there is in a given community a number of employers, more or fewer, who alone are, by law or by custom, permitted to do the business of that community, . . . or else who are so exceptionally gifted and endowed by Nature for performing this industrial function that no one not of that class would aspire thereto, or would be conceded any credit or patronage should he so aspire. Secondly, we assume that neither in point of ability nor opportunity has any one member of this class an advantage as against another, . . . all being, we might say, the exact copies of the type taken, whether that should involve a very high or a comparatively low order of industrial power.

"Now, in the case assumed, what would be true of business profits, the remuneration of the employing class? I answer, that if the members of this class were few, they might conceivably effect a combination among themselves, and . . . fix a standard for their own remuneration. . . . If, however, the community were a large one, and if the business class . . . were numerous, such a combination . . . would be impracticable, . . . the members of the business class would begin to compete with each other. From the moment competition set in it would find no natural stopping-place until it had reduced profits to that minimum which, for the purposes of the present discussion, we call *nil*.

"What, in the case supposed, would be the minimum of profits? I answer: This would depend upon an element not yet introduced into our problem. The ultimate minimum would be the amount of profits necessary to keep alive a sufficient number of the employing class to transact the business of the community. Whether, however, competition would force profits down to this low point would depend on the ability or inability of the employing class to escape into the labouring class. We have supposed that labourers could not become employers; but it does not follow that employers might not become labourers and earn the wages of labourers. . . ." (*Quarterly Journal of Economics*, 1887, p. 270 and context).

This reasoning will puzzle those who have received the abstract theory of supply and demand as formulated by the mathematical section [above, notes (*a*) and (*d*)]. Because the dealers on one side of a market, as the employers in the labour market, compete against each other without combination, it does not follow that the advantage which they obtain from their bargains is *nil*. The minimum to which the play of competition tends is not necessarily small in the sense of a bare subsistence. It is a minimum only in the mathematical sense in which every position of equilibrium is a minimum (of potential energy in physics; in psychics, may we say, of potential utility). See note (*d*).

Representing the *entrepreneur's* demand for work by the curve OG (Fig. 5), where the abscissa measures work done, and the ordinate money payable out of the wages and profit fund, and putting OE for the offer of the workmen, we see that the point r may differ to any extent from the utility-curve ON, which indicates the advantage of a transaction [see note (*d*)]. As far as abstract theory, without specific data, carries us, the competing *entrepreneurs* may make very good bargains. They may be ever so prosperous; they may be, in Burke's fine phrase, "gambolling in an ocean of superfluity."

So far, on the hypothesis that neither in point of ability nor opportunity has any one member of this class an advantage as against another. The heterogeneity of faculty will, of course, introduce a graduation of gain. But in this flight of steps it is not necessary that the lowest should be on a level with the grade of common labour. The scale of profits may be a sort of Jacob's ladder, culminating in a paradise of luxury, and having its lowest rung suspended high above the plain of ordinary wages.

Let us suppose, however, that the writer has tacitly made some assumption as to the numbers of the "numerous" business class relatively to the "large" community (compare the parallel passages in his "Political Economy," Arts.). Still, what does the consideration of business profits as rent do more than the received principle of supply and demand? If the workmen, believing that in the distribution regulated by competition too much has been assigned to brain and too little to muscle, determine to reduce profits by means of a combination, should they stay their hand because they are told that profits (above the lowest grade) are of the nature of rent? The terms "rent" and "margin" may indeed suggest that the extra profits of the abler *entrepreneurs* exactly correspond to their greater ability. It might seem that if, so to speak, we pushed down all the higher faculties to the level of the lowest grade of business power, the diminution of

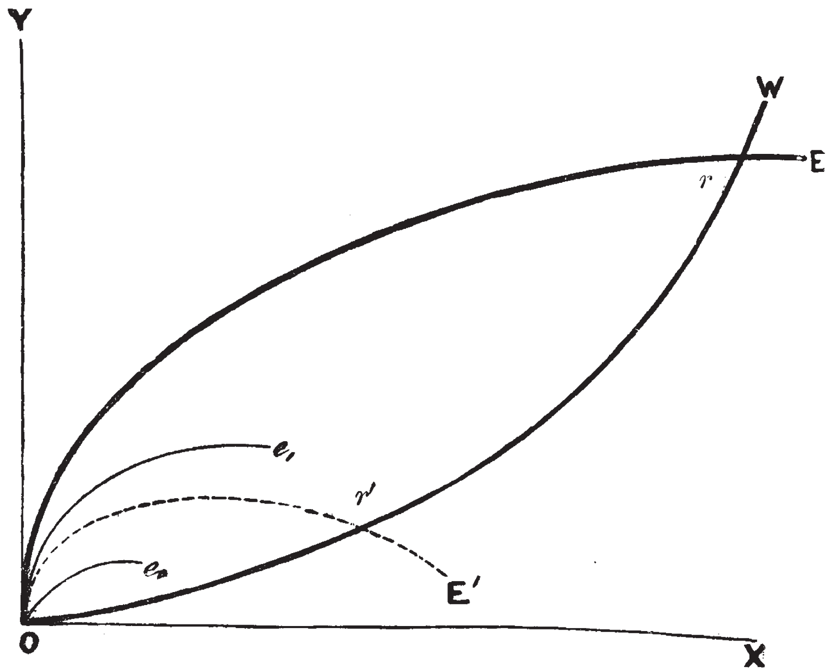

FIG. 6.

the total distributed to the wages and profits fund would exactly correspond to the subtraction from the earnings of the degraded *entrepreneurs*, while everything else remained constant. Conversely, it might be argued that the increment of produce due to the existence of superior ability may justly be assigned as extra profit.

But how little appropriate is this precise conception will at once appear from the annexed diagram. Let OE in Fig. 6 represent the *entrepreneur's* demand, OW the workmen's offer of labour, the abscissa representing work done, and the ordinate wages payable out of the wages and profits funds (abstraction being made of interest and rent for land). Let OE be formed by the superposition of Oe_0, the demand-curve for the lowest collective *entrepreneurs*, and one or more curves, such as Oe_1, appertaining to the *entrepreneurs* of higher ability. Now let us shrink these higher natures to the zero of business ability. The individual demand-curve for each degraded *entrepreneur* will become identical with that from which Oe_0 was formed (by the combination of all the demand curves for the lowest grade). The new demand-curve will therefore be of the form OE′ intersecting with OW at the point r'. (Whether the disturbance will stop there will depend upon the nature of the communication between the departments of employer and workman; whether the mobility is one-sided, like that of fluid allowed by a valve to escape from one vessel to another, but not back again—see the end of the passage cited on p. 505 from the *Quarterly Journal of Economics*—or whether the permeation is perfect.) If Oe_0 is small, if the part played in production by the marginal employers is insignificant, it is probable that the annihilation of the higher grades will result in the destruction of the greater part not only of profits, but also of wages.

Accordingly it appears, in general, inexact to say that the "surplus which is left in the hands of the higher grades of employers . . . is of their own creation" (*Quarterly Journal of Economics*, April 1887, p. 274); if we define their own creation as the difference between the actual produce and that which would have existed if their superior faculties had not been exercised. In that sense (and what other sense is there?) the surplus of the higher grades is likely to be much *less* than their own creation (especially in the case where the marginal employers are relatively few). We seem to have proved too much. But may we not deduce the *quod est demonstrandum*, that actual profits are deserved, from the larger proposition that the *entrepreneurs'* "own creation" is by a certain amount greater than their profits? No; for that larger proposition is blocked by the antinomy that the workmen (or the higher grades of them may by parity claim the greater part of the produce as *their* "own creation"—what would not have existed but for the exertion of their faculties.

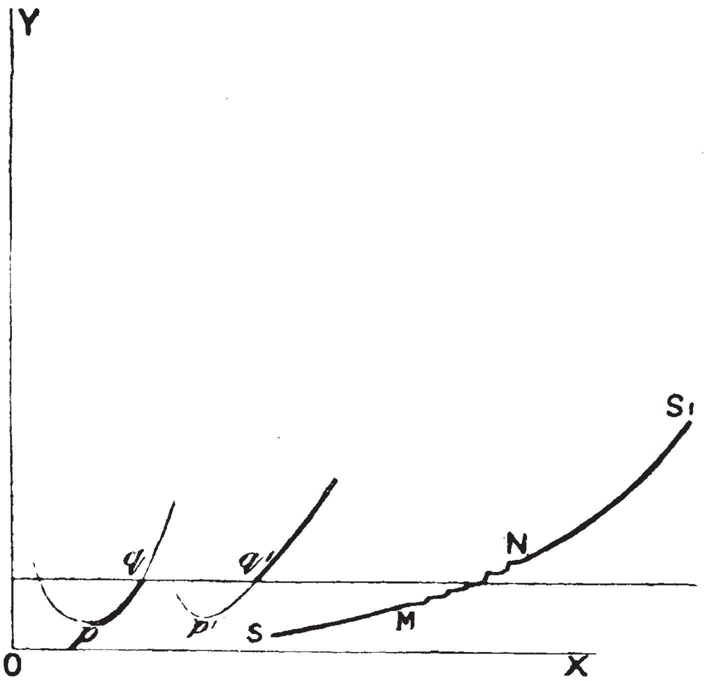

FIG. 7.

In short, we know no more than we knew at first—viz. that the distribuend is produced jointly by the owners of brain and muscle, that the terms of the distribution are determined by supply and demand, and that in this, as in every other market, each more favoured nature enjoys a *rent*, or differential advantage [the nature of which is well illustrated by Messrs. Auspitz and Lieben's construction indicated in our note (*c*)]. That the surplus earning of the superior *entrepreneur* is his own creation is true of the individual, but not of the class; in division, but not in composition.

However, Prof. Walker may have tacitly made some specific assumptions as to the quantities involved (*e.g.* the proportion of produce with which the marginal *entrepreneurs* are concerned); or I may have misinterpreted his statements. Even so, the liability to such misconstruction is a defect in the purely literary method.

It would be easy in the case of less eminent writers to exemplify the part which the mathematical *organon* may play in lopping the excrescences of verbal dialectics. But I must content myself with briefly adverting to one of Prof. Walker's critics, Mr. Sidney Webb. His able paper on the "Rate of Interest and the Laws of Distribution" appears to me to contain several points deserving of attention; with respect to which mathematical conceptions may assist the reader in distinguishing the original from the familiar, and the true from the misleading.

(1) Mr. Webb restates the theory formulated by Jevons, that capital is ideally distributed according to the law of "equal returns to the last increments" ("Rate of Interest and Laws of Distribution," by S. Webb, pp. 10, 11, 21 of paper reprinted from *Quarterly Journal of Economics*, January 1888).

In symbols [see above, note (*f*), p. 504] let the net earning of any individual be $f_r(c_r) - \iota c_r$; where f_r is a function differing for different individuals according to their faculties and opportunities, c_r is the amount of borrowed capital employed by the individual; ι is the rate of interest; land and labour are not expressed. In equilibrium,

$$\frac{df_r(c_r)}{dc_r} = \iota = \frac{df_s(c_s)}{dc_s} = \frac{df_t(c_t)}{dc_t} = \ldots$$

(2) Again, Mr. Webb discerns that the "law of diminishing returns" is applicable to capital as well as to land (*ibid.*, pp. 9, 20, &c.). This is probably a new truth to the literary economist, who will have some difficulty in reconciling it with the *law of increasing returns* received into the text-books. To the mathematician it is evident that, in order to maximize the net earnings $f(c) - \iota c$, not only must the first differential of this expression vanish, but also the second differential, $\frac{d_2 f}{dc^2}$, must be negative, which is the *law of diminishing returns*. It is quite consistent with the supposition that for certain values of the variable, not admissible as a solution of the problem, $\frac{df}{dc}$ should be positive, agreeably to the *law of increasing returns*.

Diagrammatically, let us represent the conditions under which capital is applied by a certain individual, according as the scale of production is large or small, by the curves pq and $p'q'$ in Fig. 7; where the abscissa denotes quantity of capital, and the ordinate the increment of (gross) produce due to an increment of capital. There cannot be equilibrium, unless the increment denoted by the ordinate is just balanced by the sacrifice thereby incurred—in the case which I have supposed, the payment of interest. There cannot, then, on this supposition, be stable equilibrium, unless the curve is ascending. The descending (dotted) branches correspond to the law of increasing returns. (The explanation of other features in the figure is given in the next note.)

(3) Mr. Webb dwells much on "the special industrial advantages not due to superiority of site or skill" which are enjoyed by some individuals. The use of an expression for the product like our f_r may serve at least to keep in mind the existence of such specialities. It also brings into view a difficulty which has not been sufficiently noticed by those who use *rent* in its metaphorical or secondary sense.

Suppose the extra produce is a function involving several variables (or parameters) like land, ability, opportunity. Say, $f_r = F(\lambda, \alpha, \omega \ldots)$, where F is a form common to the community, and λ, α, ω denote the quality of land, ability, and opportunity peculiar to the individual. If the extra produce is $F(\lambda, \alpha, \omega) - F(o, o, o)$; is $F(\lambda, o, o) - F(o, o, o)$ the "economic rent" of land; $F(o\ \alpha\ o) - F(o, o, o)$ the rent of ability; $F(o\ o\ \omega) - F(o\ o\ o)$ the extra produce due to opportunity? (*ibid.*, pp. 16, 17). If so, the three parts do not make up the whole.

(4) Anyway, to call the third extra produce *interest* is very unhappy. Its affinities are evidently with rent (cf. Sidgwick, "Pol. Econ.," book ii. ch. vii. § 4).

(5) I should not have complained about the use of a term, but that it is connected with Mr. Webb's main contention against Prof. Walker, to which I am unable to attach significance:

that "this, not the 'rent of ability,' is the real keystone of the arch" (*ib.*, p. 17). From the point of view here taken (above, p. 497) this search for the "keystone" among the factors of distribution is nearly as hopeless as the speculation of the ancients about the real *up* or *down*.

(*j*) DETERMINATENESS OF ECONOMIC EQUILIBRIUM.—To investigate the possibility of there being more than one rate of exchange at which the supply is just carried off by the demand, let us consider the most favourable or, at least, the most familiar case in which there may be two scales of production, and therefore two series of terms on which the producer is willing to deal —two supply-curves. Let the two branches in Fig. 7 represent such a double supply-curve, the ordinate denoting price, and the abscissa the quantity of product which is offered at that price by a certain individual. Now, the essential idea or leading property of an individual supply-curve is that it represents the quantity which the individual will prefer to offer at any given price. Hence the *descending* part of the branches, dotted in the figure, cannot form a genuine supply-curve. For at any point on that part of the *locus* it is evidently the interest of the individual to increase his production, price being constant. Stable equilibrium, therefore, can exist only on the ascending, the unbroken branches. The thick curve lines in the figure indicate the *locus* of *greatest possible*, as distinguished from *maximum*, utility. Suppose that at and above the price, corresponding to the point q, it is the interest of the producer to adopt the larger scale of production. Up to that price his industrial dispositions will be represented by the inner curve; on reaching that point he will jump from q to q' and ascend along the outer curve. The *locus* of greatest possible utility may be called the genuine or effective supply-curve. A similarly shaped supply-curve may exist for other producers. Suppose, now, all these individual effective supply-curves superposed, and we have the effective supply-curve for the community, SS^1, which continually trends outwards. This character is not annulled by the existence of steps in the tract MN, corresponding to the prices at which the leap of each individual occurs. *A fortiori* the demand-curve continually trends outwards like that in Fig. 1 (cp. Auspitz and Lieben). It should seem, therefore, that theoretically on the supposition of enlightened self-interest there is only one rate of exchange at which supply is just equal to demand. No doubt there is something to be said on the other side. Suppose the jump from q to q', or from q' to q, involves a breach of habit, the inconvenience which the "economic man" will not neglect. A little attention will show that in this case the tract MN′ of the collective-curve might break up in two separate branches. Moving from M upwards we should not be on the same *locus* as from N downwards.

$Oy_r rx$ should be a *maximum* ("Recherches," Art. 25), or rather the *greatest possible*. The solution is not likely to be indeterminate, except in the particular case where the demand-curve is an equilateral hyperbola (contrast Sidgwick, "Pol. Econ.," book ii. chap. ii. § 4).

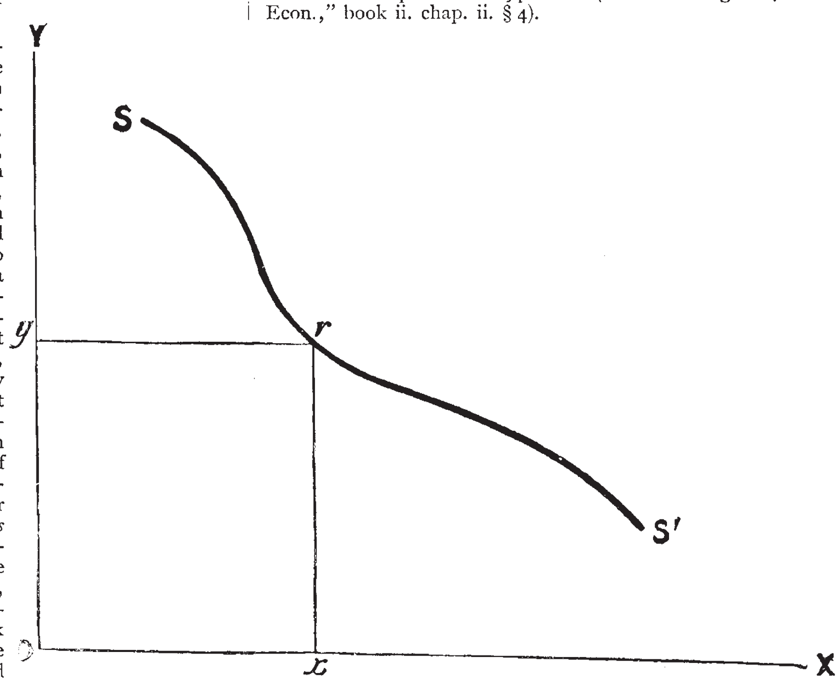

FIG. 8.

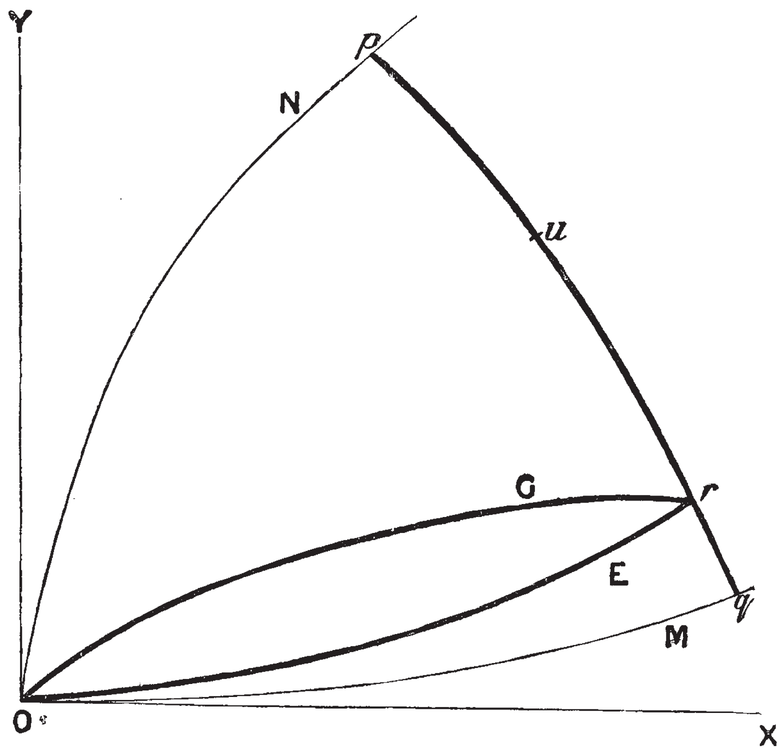

FIG. 9.

(*k*) ONE-SIDED MONOPOLY.—In Fig. 8 let the curve SS′ represent the demand of the public for a monopolized article, the abscissa denoting price, the ordinate quantity. Then, as Cournot shows, it is the interest of the monopolist that the rectangle

(*l*) TWO-SIDED MONOPOLY.—In Fig. 9 let Op and Oq represent the curves of constant satisfaction, or indifference-curves [above, note (*d*); "Theorie der Preise," Appendix II.; "Mathematical Psychics," p. 21] drawn through O for two individuals or combinations respectively. Then the *locus* of bargains which it is not the interest of both parties to disturb is the *contract-curve*, *pq* ("Math. Psych.," *loc. cit.*). At what point, then, on this curve will the transaction settle down? If we assume that the conditions of a market are retained, the required point is at the intersection of the supply- and demand-curves, which is on the contract-curve. That is the solution of Messrs. Auspitz and Lieben ("Theorie," p. 381). It corresponds to the principle laid down by Prof. Marshall for the action of arbitrators (referred to above in note). But Prof. Menger, who has a numerical scheme equivalent to a rudimentary contract-curve ("Grundsätze," pp. 176–78), and Prof. Böhm-Bawerk, referring to the "*Spielraum*" afforded by the indeterminates of bargain, recommend to "split the difference." Instead of "equal," "*equitable*" division has been proposed by the present writer—namely, that adjustment which produces the maximum of utility to all concerned; not subject to the conditions of a market, but irrespectively thereof [equations (β) and (γ), without equation (α) in note (*e*) above], the utilitarian arrangement, which also is represented by a point in the contract-curve, say *u* in Fig. 9. Such might seem to be the ideally most desirable arrangement; but very likely the practically best, the πρακτὸν ἀγαθόν, is in the neighbourhood indicated by Prof. Marshall and Messrs. Auspitz and Lieben.

(*m*) THE AUSTRIAN SCHOOL.—Prof. Menger and his followers have expressed the leading propositions of the economic calculus—the law of diminishing utility, the law of demand and supply, and so forth—by means of particular numerical examples, supplemented with copious verbal explanation. Their success is such as to confirm the opinion that the mathematical method is neither quite indispensable nor wholly useless, *nec nihil nec omnia*, like most scientific appliances. Conceding that in the main they impart a saving knowledge of the true theory of value, it may still be maintained that they occasionally emphasize the accidents of a particular example as if they formed the essence of the general rule; that their explanations are excessively lengthy; and yet their meaning sometimes is obscure. For instance, Prof. Böhm-Bawerk may seem to attach undue importance to his conception of the *Grenzpaar*. He illustrates the play of demand and supply by supposing a market in which on the one hand there are a number of dealers each with a horse to sell, and on the other hand a number of would-be buyers (*Konrad's Jahrbuch*, 1886; "Kapital," p. 211; cp. Mr. James Bonar's excellent article in the *Quarterly Journal of Economics*, 1888). The latter are arranged in the order of their strength: first, the one who is prepared to give most for a horse, the highest price which the second can afford is less, and so on. Parallel to this

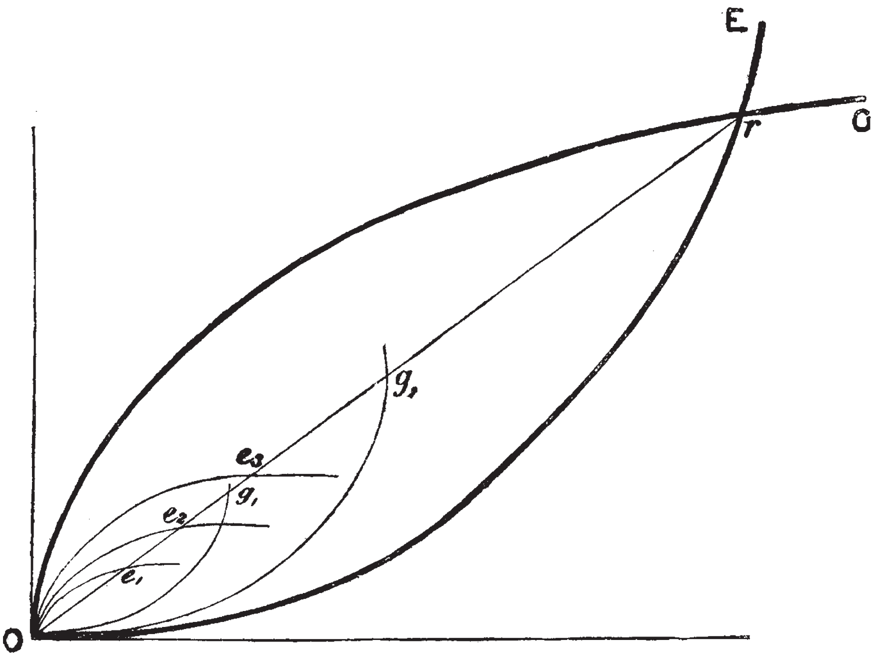

FIG. 10.

arrangement is that of the would-be sellers: first, he who can sell cheapest, and so on. Upon this hypothesis it might happen that the fifth would-be buyer is willing to give a little more than the lowest figure which the fifth would-be seller will take; while the sixth on the side of the buyers is not willing to give quite as much as the sixth horse-dealer stands out for. In this case five horses only will be sold; and the couple who are the last between whom a bargain is possible—buyer No. 5 and seller No. 5—enjoy a mighty distinction as the *Grenzpaar*; an honour which is to some extent shared with No. 6, the first couple between whom a bargain is impossible.

Now this attention to a particular couple is not always appropriate. How if the weakest actual buyer should prove to be, not buyer No. 5 but buyer No. 1, as to a *second* horse? Prof. Böhm-Bawerk, indeed, has thought of this case, and called attention to it in a note to his later redaction ("Kapital," p. 218). So far—although the whole simplicity of the scheme is destroyed when we permit second and third horses to the different buyers and sellers—the conception of a "limiting couple" may still be retained. It will be found, however, that this idea is not appropriate to the general case of a divisible commodity, which a single individual on one side of the market may buy from or sell to a large number on the other side. That general case is much more clearly represented by a diagram like Fig. 10, where the inner thin-lined curves represent the dispositions of the individual dealer, the outer thick curves the collective supply and demand (cp. Auspitz and Lieben, "Theorie der Preise"). No doubt Prof. Böhm-Bawerk's conception is appropriate to a particular case, that in which the *kleinste marktübliche Mengeneinheit*, in the phrase of Messrs. Auspitz and Lieben (*ibid.*, p. 123), is considerable. But it is better, with those eminent theorists, to begin with the general or, at least, the simple case.

As an instance of the excessive circumlocution in which the purely literary method is liable to be entangled, we may notice the doctrine of objective and subjective value, which occupies many pages in the works which we have cited. I have been unable to find more in the distinction than what is visible on a glance at the appropriate diagram. The individual's subjective estimate of worth is expressed by his particular demand- or supply-curve, Oe_1, Oe_2, &c., Og_1, Og_2, &c., in Fig. 10. The proper combination of those individual curves gives the collective demand- and supply-curves, of which the intersection represents the "objective" value.

Moreover, verbal circumlocutions are so little adapted to express mathematical conceptions that we are sometimes left in uncertainty as to our author's meaning. When Prof. Böhm-Bawerk remarks that there is something special in the labour-market, in that the buyer will vary his arrangements according to the price of the article, the rate of interest ("Kapital," p. 407), does he specify the property which Messrs. Auspitz and Lieben have stated as general—that the utility function [our ψ, note (e) above] is discontinuous, being different for large and small values of the variable under consideration?

These deficiencies are more conspicuous in other writings of the Austrian school. A glance at Fig. 10, an intuition of the corresponding algebraic formulæ, will show that the notion of an *average* imported into the doctrine of value by Dr. Emil Sax ("Staatswirthschaft") is not quite appropriate. As an instance in which great abridgment would be effected by mathematical expression, we might notice the last three chapters of Dr. Zuckerkandl's "Theorie der Preise." Again the difficulty of conveying technical propositions without the proper phraseology may be illustrated by Prof. Wieser's "Der natürliche Werth," when he speaks of value and final utility having place in a Communistic or Socialistic State (p. 26, note, and *passim*). May his meaning thus be formulated? In an economical *régime* distribution and exchange are regulated by the condition that the final utility of all concerned should be zero, the total utility a minimum, *subject to the law that there should be only one rate of exchange in a market*. In a communistic or utilitarian *régime* the limitation which the last italicized clause expresses is removed. In terms employed in our note (e) the economical adjustment is determined by the equations (α), (β), and (γ); the utilitarian adjustment is determined by (β) and (γ) only—in short, the distinction between the points r and u in our Fig. 9 referring to note (e).

In offering these too brief criticisms I regret that my limits impose a curtness which is hardly consistent with courtesy.

SECTION G.

MECHANICAL SCIENCE.

Opening Address by William Anderson, M.Inst.C.E., President of the Section.

I have had considerable difficulty in selecting a subject which should form the main feature of my address. This meeting being held in Newcastle, it seemed almost imperative that I should dwell upon two industries which may be said to have had their genesis here: that I should direct your attention to the extraordinary development of the system of transmitting power by hydraulic agency, and the use of the same agency for lifting enormous weights and exerting mighty pressures; and that I should not neglect to notice a manufacture of specially national importance—that of heavy artillery, and of ships of war sent forth fully equipped and ready to take their places in our first line of defence.

The desire which I felt of treating of these subjects was heightened by the opportunity which it would have afforded of paying a tribute of respect and admiration to the distinguished citizen of this town, who, by his genius and perseverance, created the Elswick Works, raised the character of British engineering, and rendered his country services so eminent that Her Majesty has seen fit to recognize them by bestowing honours higher than any which an engineer has hitherto been able to achieve.

But I felt that the themes mentioned, important as they are, have been frequently treated of by able men, and that I would perhaps render more service to Mechanical Science if I drew your attention to a subject which appears to me to be bearing with daily augmenting force on the practical manipulation of the materials used in construction. I allude to the molecular structure of matter. This branch of science has, up to the present time, been left very much in the hands of the chemist and the physicist, and I dare say that many engineers may think that it is by no means desirable to change the arrangement; but I hope to show that the progress of engineering, the more exact methods of dealing with the properties of materials, the increased demands on their powers of endurance, render it imperatively necessary that mechanics should interest themselves more deeply in their internal structures and the true meaning of the laws by which their properties are defined.

Five years ago, at Montreal, in his address to the Mathematical Section, Sir William Thomson took for his subject the ultimate constitution of matter, and discussed, in a most suggestive manner, the very structure of the ultimate atoms or molecules. He passed in review the theories extant on the subject, and pointed out the progress which had been made in recent years by the labours of Clausius, of Clerk Maxwell, of Tait, and of others, among whom his own name, I may add, stands in unrivalled prominence.

I will not presume to enter into the field of scientific thought and speculation traversed by Sir William Thomson, because I am only too conscious that both my mathematical knowledge and my acquaintance with the natural sciences is too limited to entitle the views which I may have formed to any respect; I propose to draw attention only to some general considerations, and to point out to what extent they practically interest the members of this Section.

In a lecture delivered at the Royal Institution last May, Prof. Mendeleeff attempted to show that there existed an analogy between the constitution of the stellar universe and that of matter as we know it on the surface of the earth, and that from the motions of the heavenly bodies down to the minutest interatomic movements in chemical reactions the third law of Newton held good, and that the application of that law afforded a means of explaining those chemical substitutions and isomerisms which are so characteristic, especially of organic chemistry.

Examined from a sufficient distance, the planetary system would appear as a concrete whole, endowed with invisible internal motions, travelling to a distant goal. Taken in detail, each member of the system may be involved in movements connected with its satellites, and again each planet and satellite is instinct with motions which, there is good reason to believe, extend to the ultimate atoms, and may even exist, as Sir W. Thomson has suggested, in the atoms themselves. The total result is complete equilibrium, and, in many cases, a seeming absence of all motion, which is, in reality, the consequence of dynamic equilibrium, and not the repose of immobility or inertness.

The movements of the members of the stellar universe are, many of them, visible to the eye, and their existence needs no demonstration; but the extension of the generalization just mentioned to substances lying, to all appearances, inert on the earth's surface is not so apparent. In the case of gases, indeed, it is almost self-evident that they are composed of particles, so minute as to be invisible, in a condition of great individual freedom. The rapid penetration of odours to great distances, the ready absorption of vapour and of other gases, and the phenomena connected with diffusion, compression, and expansion seem to demonstrate this. One gas will rapidly penetrate another and blend evenly with it, even if the specific gravities be very different. The particles of gases are, as compared with their own diameters, separated widely from each other; there is plenty of room for additional particles; hence any gas which would, by virtue of its molecular motion, soon diffuse itself uniformly through a vacuum will also diffuse itself through one or more other gases, and once so diffused, it will never separate again. A notable example of this is the permanence of the constitution of the atmosphere, which is a mere mixture of gases. The oxygen and the nitrogen, as determined by the examination of samples collected all over the world, maintain sensibly the same relative proportions, and even the carbonic acid, though liable to slight local accumulations, preserves, on the whole, a constant ratio; and yet the densities of these gases differ very greatly.

Liquids, though to a much less degree than gases, are also composed of particles separated to a considerable relative distance from each other, and capable of unlimited motion where no opposing force, such as gravity, interferes: for under such circumstances their energy of motion is not sufficient to overcome the downward attraction of the earth; hence they are constrained to maintain a level surface.

The occlusion of gases without sensible comparative increase of volume shows that the component particles are widely separated. Water, for example, at the freezing-point occludes above one and three-quarter times its own volume of carbonic oxide, and about 480 times its volume of hydrochloric acid, with an increase of volume, in the latter case, of only one-third; and sulphuric acid absorbs as much as 600 times its bulk of methylic ether. The quantity of gas occluded increases directly as the pressure, which seems to indicate that the particles of the occluded gas are as free in their movements among the particles of the liquid as they would be in an otherwise empty containing vessel.

Liquids, therefore, are porous bodies whose constituent particles have great freedom of motion. It is no wonder, consequently, that two dissimilar liquids, placed in contact with each other, should interpenetrate one another completely, if time enough be allowed; and this time, as might be expected, is considerably greater than that required for the blending of gases, because of the vastly greater mobility of the particles of the latter. The diffusion of liquids takes place not only when they are in actual contact, but even when they are separated by partitions of a porous nature, such as plaster of Paris, unglazed earthenware, vegetable or animal membranes, and colloidal substances, all of which may be perfectly water-tight in the ordinary sense of the term, but yet powerless to prevent the particles of liquids making their way through simultaneously in both directions.

The rate of diffusion increases with the temperature; but an increase of temperature, we know, is synonymous with increased molecular motion of the body, and with increased activity of this kind we would naturally look for more rapid interchanges of the moving atoms. Such phenomena are only conceivable on the supposition that active molecular motion is going on in an apparently still and inert mass.

When we come to solid substances the same phenomena appear.

The volumes of solids do not differ greatly from the volumes of the liquids from which they are congealed, and the solid volumes are generally greater. The volume of ice, for example, is one tenth greater than that of the water from which it separates. Solid cast-iron just floats on liquid iron, and most metals behave in the same way; consequently, if the liquids be porous the solids formed from them must be so also; hence, as might be expected, solids also occlude gases in a remarkable manner. Platinum will take up five and a half times its own volume of hydrogen, palladium nearly 700 times, copper 60 per cent., gold 29 per cent., silver 21 per cent. of hydrogen and 75 per cent. of oxygen, iron from eight to twelve and a half times its volume of a gaseous mixture chiefly composed of carbonic oxide.

Not only are gases occluded, but they are also transpired under favourable conditions of temperature and pressure, and even liquids can make their way through. Red-hot iron tubes will permit the passage of gases through their substance with great readiness, common coal-gas under high pressure transpires through the steel of the containing vessel, and it is well known that mercury will penetrate tin and other metals with great rapidity, completely altering their structure, their properties, and even their chemical compositions.

The evidence of the mobility of the atoms or molecules of solid bodies is overwhelming. Substances when reduced to powder, may, even at ordinary temperatures, be restored to the homogeneous solid condition by pressure only. Thus, Prof. W. Spring, some ten years ago, produced from the powdered nitrate of potassium and sodium, under a pressure of thirteen tons to the square inch, homogeneous transparent masses of slightly greater specific gravity than the original crystals, but not otherwise to be distinguished from them. More than that, from a mixture of copper filings and sulphur he produced, under a pressure of thirty-four tons per square inch, perfectly homogeneous cuprous sulphide, Cu_2S, the atoms of the two elements having been brought, by pressure, into so intimate a relation to each other that they were able to arrange themselves into molecules of definite proportion; and, still more remarkable, the carefully dried powders of potash, saltpetre, and acetate of soda were, by pressure, caused to exchange their metallic bases and form nitrate of soda and acetate of potash.

The same movements and changes have taken place, and are still going on, in Nature's laboratory. During the countless ages with which geology deals, and under the enormous pressures of superincumbent masses, stratified sedimentary rocks become crystallized and assume the appearance of rocks of igneous origin, and not only so, but rocks of whatever origin, crushed and ground to pieces by irresistible geological disturbances, reconstruct themselves into new forms by virtue of the still more irresistible and constant action of molecular forces and movements. Those who had the privilege of hearing Prof. A. Geikie's brilliant lecture at the Royal Institution last session will remember the striking series of microscopic slides which he exhibited, and by the aid of which he illustrated the changes of structure to which I have alluded.

At high temperatures the same effects are more easily produced on account of the greater energy of motion of the atoms or molecules. In the process of the manufacture of steel by cementation, or in case-hardening, the mere contact of iron with solid substances rich in carbon is sufficient to permit the latter to work its way into the heart of the former, while in the formation of malleable cast-iron the carbon makes its way out of the castings with equal facility; it is a complete case of diffusion of solid substances through each other, but, on account of the inferior and restricted mobility of the particles at ordinary temperatures, a higher degree of heat and longer time are needed than with liquids or gases.

Again, when, by the agency of heat, molecular motion is raised to a pitch at which incipient fluidity is obtained, the particles of two pieces brought into contact will interpenetrate or diffuse into each other, the two pieces will unite into a homogeneous whole, and we can thus grasp the full meaning of the operation known as "welding." By the ordinary coarse methods but few substances unite in this way, because the nature of the operation prevents, or at any rate hinders, the actual contact of the two substances; but when molecular motion is excited to the proper degree by a current of electricity, the faces to be joined can be brought into actual contact, the presence of foreign substances can be excluded, and many metals not hitherto considered weldable, such as tool steel, copper, and aluminium are readily welded, as many of us witnessed at the hands of Prof. Ayrton in the highly instructive lecture on electricity delivered last year at our Bath meeting. Again, a mere superficial union of different metals takes place readily under the influence of high temperature and moderate pressure, as we see in the operations of tinning, soldering, and brazing. The surfaces of the metals must be made as clean as possible; the solder, which melts at a lower temperature than the metal to be soldered or brazed, is applied, and at a comparatively moderate temperature and under very slight pressure the particles interpenetrate each other; the two metals unite and form an alloy, by the intervention of which the two surfaces are joined. This effect is very well illustrated by the action which takes place at the surface of contact of two dissimilar liquids. If brine, for example, be placed in the lower part of a glass tube, and ordinary water, coloured in some way, be carefully poured on the top, a sharp plane of demarcation will appear, but in a short time the plane of separation will become blurred, and will ultimately disappear, a local blending of the two waters will take place, and will thus present a case of fluid-welding.

It seems plain, therefore, that apparently inert solid masses are also built up of moving particles in dynamic equilibrium, for without such an assumption it would be hard to explain the phenomena to which I have alluded. But in addition to this evidence we can adduce the effects of other forms of energy, which we recognize under the names of radiant heat, light, and electricity. These we know to be forms of motion which can be communicated and converted from one to the other, from the invisible to the visible. The movement which we term radiant heat, acting through the instrumentality of the luminiferous ether which is believed, on the strongest grounds, to pervade all space and all matter, is competent to augment the quantity of movement in the particles of substances, and generally to cause an enlargement of volume. Conversely, when the particles, by contact or by radiation, part with their heat, either to surrounding objects or to space, the quantity of motion is reduced, the body contracts, and this contraction goes on down to temperatures far below those at which we have to work in practice, and consequently at all ordinary temperatures there must be abundant room for molecular motion.

Again, energy in the form of light operates changes in the surface of bodies, causing colours to fade, and giving to photography the marvellous power which it possesses. Light decomposes the carbonic acid of the atmosphere in the chlorophyll of green leaves, and determines chemical combinations, such as chlorine with hydrogen to form hydrochloric acid, or carbonic oxide with chlorine to form chlorocarbonic acid. It is inconceivable that these effects could be produced unless the undulations of light were competent to modify the molecular motions already existing in the solid liquid and gaseous bodies affected.

Electricity exerts a similar influence. Generated by the molecular movements caused by chemical activity, whether directly, as in the primary battery, or indirectly, as in the dynamo, it is competent to increase the molecular movements in bodies so as to produce the effects of heat directly applied; it is capable of setting up motions of such intensity as to produce chemical changes and decompositions, to say nothing of the whole series of phenomena connected with magnetism, with induction, or the action through space and through non-conducting bodies, which, as in the case of radiant heat and light, seems to imply the existence of an interatomic ether. Conversely, changes of molecular equilibrium, brought about by the action of external forces, produce corresponding changes in electrical currents: witness the effects of heat, for example, on conductivity and the wondrous revelations of molecular change obtained by the aid of Prof. Hughes' induction balance. The behaviour of explosives illustrates also, and in a striking manner, the effects of disturbing molecular equilibrium. An explosive is a substance which contains in itself, in a solid or liquid form, all the elements necessary to produce a chemical change by which it is converted into the gaseous state. The application of heat, of pressure, or of impact, causes, as in Prof Spring's experiments, chemical union to take place, first at the spot where the equilibrium is disturbed by the application of external force, and afterwards, with great rapidity, throughout the mass, the disturbance being propagated either by the air surrounding the particles or by the luminiferous ether with all the rapidity of light; the chemical reaction is accelerated by the pressure which may arise, for example, if the explosive be confined in the chamber of a gun or in the bore-hole of a blast. High explosives, as they are termed, are comparatively inert to ordinary ignition; but when the molecular equilibrium is suddenly disarranged throughout the mass by the detonation of a percussion fuse, combination takes place instantly throughout, and violent explosion follows. In a similar manner some gases, such as acetylene, cyanogen, and others, can be decomposed by detonation and reduced to their solid constituents. Prof. Thorpe has devised a very beautiful lecture experiment, in which carbon disulphide is caused to fall asunder into carbon and sulphur by the detonation of fulminate of mercury fired by an electric spark. In these cases a reverse action takes place, but illustrates equally well the conversion of one form of energy into others, and the consequent disturbance of molecular equilibrium in the substances affected. It seems to me clear, therefore, that the time has come when the conception of dynamic equilibrium in the ultimate particles of matter in all its forms must take the place of the structural system of inert particles. I cannot conceive how the phenomena which I have enumerated can be explained on the supposition that matter is built up of motionless particles—how, for example, a stack of red and yellow bricks could ever change the order of arrangement without being completely pulled asunder and built up again, in which case an intermediate state of chaos would exist; but I can easily comprehend how a dense crowd of people may appear as a compact mass, streaming, it may be, in a definite direction, and yet how each member of that mass is endowed with limited motion, by virtue of which he may push his way through without disturbing the general appearance; how the junction of two crowds would form one whole, though, perchance, altered in character; and how even Prof. Spring's experiments may be explained by the supposition that bystanders on the edge of a crowd would be forced, by external pressure, to form part of it and partake of its general movements.

It is a suggestive fact that the product of the atomic weight of certain groups of substances and their specific heats is a constant quantity which, for the greater number of the elements, does not differ much from 6·5. This implies that the quantity of heat necessary to raise the temperature of the atoms of any one of the groups to any given extent is the same; hence these atoms will be endowed with the same amount of energy at any given temperature, and therefore would be competent to replace each other without disturbing the general dynamic equilibrium.

When it is conceded that molecular motion pervades matter in all its forms, and that the solid passes, often insensibly, into the fluid, or even direct into the gaseous, it follows, almost of necessity, that there must be a border-land, the limits of which are determined by temperature and pressure, in which substances are constantly changing from one state to another. This is observable in fusion, but to a more marked degree in evaporation, where the particles are being incessantly launched into space as gas and return as constantly to the liquid state. Henri Ste. Claire Deville has investigated similar phenomena in chemical reactions; he has found that at certain temperatures and pressures substances fall asunder and combine much in the way in which evaporation takes place, and has given the name of "dissociation" to this property of matter. Prof. Mendeleeff and others have extended the great French chemist's observations, and have formulated the general law that substances are capable of dissociation at all temperatures, not only in the case of chemical unions, but also in that of solutions.

If steel be looked upon as a solution of carbon and iron, then the hardening of steel is explained by the theory that dissociation has taken place at the temperature at which it is suddenly cooled, the sudden cooling fixing the molecular motion at such an amplitude or phase that it gives a characteristic structure, one of the properties of which is extreme hardness. In tempering, the gradual communication of heat causes dissociation again to take place, the molecular equilibrium is modified by the increased energy imparted to the particles, and when suddenly cooled at any point there remains again a distinct substance, composed of iron and carbon, partly in various degrees of solution and partly free, and again possessing special mechanical qualities. That steel, and probably other alloys, differ in the nature of their composition according to the way in which they are worked, both with respect to heat and mechanical pressure, has been abundantly proved by many eminent metallurgists, and especially by Sir Frederick Abel, in the extended researches which he has recently carried out, on the hardening of steel, for the Institution of Mechanical Engineers, and it would appear as a natural sequence that the properties of steel would be greatly affected by the manner in which its temperature was changed, as we indeed find that it is when these changes are produced by baths of melted metals, by oil, or by water at different temperatures. The action which takes place may be illustrated by what would happen supposing that a complicated dance, such as the lancers, were suddenly stopped in various phases of the figures. The component parts would always remain the same, but the relative distribution of the partners would vary continually, and analysis would show that at one time each gentleman was associated with a particular lady; at another, that two ladies were attached to a single gentleman, while a number of gentlemen had no partners at all; and yet again, at another, that the movements which were once rectilinear have become circular. In each case the groups would assume a totally distinct appearance.

In support of these views it may be stated that, as far as I know, no pure element is capable of being hardened or tempered, the reason being that no chemical change can take place when there is only one substance; the effect of heat or pressure, however suddenly applied, produces merely a change of form which does not appear to carry with it any corresponding alteration of mechanical properties.

It may be urged, however, that it is unlikely that alloys or solutions could be affected in a manner so marked merely by small changes at comparatively low temperatures; but I would observe that "great and little" are relative terms, and we have abundant evidence of the immense effects produced by what would be called "little" causes. Sir Frederick Bramwell, in his address last year, drew attention to the importance of the "next to nothing." It is not so very long ago that anyone would have been considered a dreamer for propounding a theory that the presence of the fraction of a per cent. of carbon, phosphorus, or sulphur would totally alter the character of iron; it is known that the addition of one two-thousandth part of aluminium to molten iron makes the pasty mass as fluid as water; that the presence of the smallest impurity in copper has a disastrous effect on its powers of conducting electricity; and that the addition of one-thousandth part of antimony converts the "best selected" copper into the worst conceivable. I need hardly allude to the great part played in Nature by microscopic organisms, and how much of the beauty of our seas and rivers is derived from substances so minute that nothing but the electric beam of Prof. Tyndall is capable of revealing their presence.

There is one more circumstance connected with my subject to which I must draw your attention, because, though its application to the mechanical properties of substances is very recent, it promises to be of great importance. I allude to the periodic law of Dr. Mendeleeff. According to that law, the elements, arranged in the order of their atomic weights, exhibit an evident periodicity of properties, and, as Prof. Carnelley has observed, the properties of the compounds of the elements are a periodic function of the atomic weights of their constituent elements. Acting on these views, Prof. Roberts-Austen has recently devoted much time and labour to testing their exactness with reference to the mechanical properties of metals. The investigation is surrounded by extraordinary difficulties, because one of the essential features of the inquiry is that the metals operated on should be absolutely pure. For chemical researches a few grains of a substance are all that is needed, and the requisite purity can be obtained at a moderate cost of time and labour; but when mechanical properties have to be determined considerable masses are needed, and the funds necessary for obtaining these are beyond the reach of most private individuals. I cannot help suggesting that wealthy institutions, such as many of those connected with our profession, could not employ their resources more wisely than in giving the means of following up the researches which Prof. Roberts-Austen has inaugurated.

In view of the difficulty of obtaining metals of sufficient purity, he selected gold as his base, because that metal can be more readily brought to a state of purity than any other, and is not liable to oxidation. In a communication to the Royal Society, made last year, he shows that the metals alloyed with gold which diminish its tenacity and extensibility have high atomic volumes, while those which increase these properties have either the same atomic volumes as gold or have lower ones. The inquiry has only just been commenced, but it appears to me to promise results which, to the engineer, will prove as important and as fruitful of progress as the great generalization of Mendeleeff has been to chemists. A law which can not only indicate the existence of unknown elements but which can also define their properties before they are discovered, if capable of application to metallurgy, must surely yield most valuable results, and will make the compounding of alloys a scientific process instead of the lawless and haphazard operation which it is now.

The practical importance of the views I have enunciated are, I think, sufficiently obvious. Everyone will admit that an external force cannot be applied to a system in motion without affecting that motion; consequently matter, in whatever state, cannot be touched without changes taking place, which will be more or less permanent. The application of heat will cause a change of volume, and, at last, a change of condition; the application of external stresses will also produce a change of volume; and it is natural to infer that there must be some relation between the two, and, accordingly, Prof. Carnelley has drawn attention to the fact that the most tenacious metals have high melting-points, though here again there is a great want of exactness, partly on account of the difficulty of measuring high temperatures, and partly by reason of the scarcity of pure materials.

Again, long-continued stresses, or stresses frequently applied, may be expected to produce permanent changes of form, and so we arrive at what is termed the fatigue of substances. Stretched beyond their elastic limits, which limits I do not suppose to exist except when stresses are applied quickly, substances are permanently deformed, and the same effects follow the long application of heat. The constant recurrence of stresses, even of those within the elastic limit, causes changes in the arrangement of the molecules of substances which slowly alter the properties of the latter, and in this way pieces of machinery, which theoretically were abundantly strong for the work they had to perform, have failed after a more or less extended period of use. The effect is intensified if the stresses are applied suddenly, if they reach nearly to the elastic limit, and if they are imposed in two or more directions at once, for then the molecular disturbance becomes very intense, the internal equilibrium is upset, and a tendency to rupture follows. Such cases occur in artillery, in armour-plates, in the parts of machinery subject to impact; and, as might be expected, the destructive effects do not always appear at once, but often after long periods of time.

When considerable masses of metal have to be manipulated by forging or by pressure in a heated condition, the subsequent cooling of the mass imposes restrictions on the free movement of some, if not all, of the particles; internal stresses are developed which slowly assert themselves, and often cause unexpected failures. In the manufacture of dies for coining purposes, of chilled rollers, of shot and shell hardened in an unequal manner, spontaneous fractures take place without any apparent cause, and often after long delay, the reason being that the constrained molecular motion of the inner particles gradually extends the motion of the outer ones until a solution of continuity is caused.

Similar stresses occur in such masses as crank shafts, screw shafts, gun hoops, &c. The late General Kalakoutsky, some seventeen years ago, commenced a systematic investigation into the internal stresses in the tubes and hoops of guns and in armour-piercing shells. The method he pursued was to cut disks or rings about half an inch thick off the hoops and shells, to divide the metal of each disk into from four to six rings, to fix by means of silver plugs, on which very finely marked cross-lines were drawn, from four to eight points on the surface of each ring, and then to measure, with extreme exactness, the changes in diameter produced in every ring by the successive cutting out of the rings. Knowing by direct tests the mechanical properties of his material, he was able, from the changes in the diameters, to calculate what the tangential stresses in every part of each disk were, and to draw inferences as to their fitness for the work they were intended to perform. The same method of investigation has been pursued by Captain Noble of the Elswick Works, and by Lieutenant Crozier of the United States Artillery, with the practical result that probably much more attention will be paid in future to the principles on which the annealing and hardening of steel is carried on. A gun hoop or tube, to be in the best condition to resist a bursting stress, should have its inner surface in a state of compression, and its outer in a state of tension, and the hoops should be shrunk on to the tubes or on to each other with but very little pressure. General Kalakoutsky proposed, in order to set up beneficial internal stresses, that tubes which were being annealed should be cooled from the inside by a jet of steam, of air, of water, or of oil; and he advocated the practice of testing the effects of each new method of manufacture or of treatment by the careful measurements of slices of the finished material instead of working at random, as is still very much the practice. It is evident, also, that a sample of steel cut out of a gun hoop or crank shaft, and tested, can afford no indication of the available tenacity of the same sample *in situ*. When released from the constraint of its surroundings, the particles must, of necessity, change their condition, for the disturbing forces have been removed; and the probability is that, if the steel be good, the test will prove satisfactory, especially if some time be allowed to elapse between cutting out the sample and testing it, and a false security will be engendered such as has often led to disastrous results.

The influence of time on steel seems to be well established; the highest qualities of tool steel are kept in stock for a considerable period; and it seems certain that bayonets, swords, and guns are liable to changes which may account for some of the unsatisfactory results which have manifested themselves at tests repeated after a considerable interval of time. As all these things have been hardened and tempered, there must necessarily have been considerable constraint put upon the freedom of motion of the particles, this constraint has gradually been overcome, but at the expense of the particular quality of the steel which it was originally intended to secure.

I have now laid before you the views respecting the constitution of matter which I think are gaining ground, which explain many phenomena with which we are familiar, and which will serve as guides in our treatment of metals, and especially of alloys; but I must admit that the subject is still by no means clear, that a great deal more definition is wanted, and that we are still awaiting the advent of the man who shall do for molecular physics what Newton did for astronomy in explaining the structure of the universe.

One of the most remarkable features of the last thirty years is the introduction of petroleum, and the wonderful development to which the trade in it has attained.

Under the generic name of petroleum are embraced a vast variety of combinations of carbon and hydrogen, each of which is distinguished by some special property. At ordinary temperatures and pressures some are gaseous, some are liquid, and some solid, and most are capable of being modified by suitable treatment under various temperatures and pressures. The employment of petroleum in the arts is still extending rapidly. Used originally for illuminating purposes, it is now employed as fuel for heating furnaces and steam-boilers; as a working agent in heat-engines; valuable medicinal properties have been discovered; and as a lubricant it stands unrivalled.

As an illuminant, even in this country, it is, to a large extent, superseding every other in private houses, and even in public lamps, because it gives a cheaper and more brilliant light than ordinary gas, and leaves the consumer free from the tyranny of great and privileged companies.

As fuel it is especially convenient, cleanly, and economical. Stored in tanks of suitable construction, it is sprayed into the furnace without labour and without creating dust and dirt; it is especially convenient in locomotive and marine work on account of the rapidity, ease, and cleanliness with which it can be run into the tender or into the oil-bunkers of a ship. As a working agent in heat-engines it is employed in two ways. First as vapour, generated from the liquid petroleum contained in a boiler, very much in the same way as the vapour of water is used in an engine with surface condenser, the fuel for producing the vapour being also petroleum. Very signal success has been obtained by Mr. Yarrow and others in this mode of using mineral oil, especially for marine purposes and for engines of small power; there seems to be no doubt that by using a highly volatile spirit in the boiler a given amount of fuel will produce double the power obtainable by other means, and at the same time the machinery will be lighter and will occupy less space than if steam were the agent used. The other method is to inject a very fine spray of hot oil, associated with the proper quantity of air, into the cylinder of an ordinary gas-engine, and ignite it there by means of an electric spark or other suitable means. Attempts to use oil in this way date back many years, but it was not till 1888 that Messrs. Priestman Brothers exhibited at the Nottingham Show of the Royal Agricultural Society an engine which worked successfully with oil, the flashing-point of which was higher than 75° F., and was therefore within the category of safe oils. The engine exhibited was very like an ordinary Otto gas-engine, and worked in exactly the same cycle. A pump at the side of the engine forced air into a small receiver at a few pounds' pressure to the square inch. The compressed air, acting by means of a small injector, carried with it the oil in the form of fine spray, which issued into a jacketed chamber heated by the exhaust, in which the oil was vaporized. The mingled air and oil was thus raised to a temperature of about 300°, and was then drawn, with more air, into the cylinder, where, after being compressed by the return stroke of the piston, it was exploded by an electric spark, and at the end of the cycle the products of combustion were discharged into the air after encircling the spray chamber and parting with most of their heat to the injected oil. The results of careful experiments made by Sir William Thomson and by myself on different occasions were, that 1·73 pound of petroleum was consumed per brake-horse power per hour; but the combustion was by no means perfect, for a sheet of paper held over the exhaust-pipe was soon thickly spattered with spots of oil.

At the Windsor Show of the Royal Agricultural Society this year, Messrs. Priestman again exhibited improved forms of their engine; the consumption of oil fell to 1·25 pound per brake-horse power per hour, and a sheet of paper held over the exhaust remained perfectly clean. They also showed a portable engine of very compact construction, and quite adapted to agricultural use; the ordinary water cart, which has, in any case, to attend a portable steam-engine, being adapted to supply the water necessary to keep the working cylinder of the engine cool.

It is hardly necessary to state that the use of petroleum for furnace purposes of all kinds is increasing very rapidly, and the demand has naturally reacted on the supply in promoting improved means of transport; and Newcastle, again, has led the van in this matter, for Sir William Armstrong, Mitchell, and Co., Messrs. Palmer, and others have sent out a fleet of steamers constructed to carry the oil in bulk with perfect safety, both as regards the stowage of a cargo so eminently shifting, and with respect to risk from fire and from explosion.

The enormous consumption of petroleum and of natural gases frequently raises the question as to the probability of the proximate exhaustion of the supply; and, without doubt, many fear to adopt the use of oil, from a feeling that if such use once becomes general the demand will exceed the production, the price will rise indefinitely, and old methods of illumination, and old forms of fuel, will have to be reverted to From this point of view it is most interesting to inquire what are the probabilities of a continuous supply; and such an investigation leads at once to the question, "What is the origin of petroleum?" In the year 1877, Prof. Mendeleeff undertook to answer this question; and as his theory appears to be very little known, and has never been fully set forth in the English language, I trust you will forgive me for laying a matter so interesting before you. Dr. Mendeleeff commences his essay by the statement that most persons assume, without any special reason—excepting, perhaps, its chemical composition—that naphtha, like coal, has a vegetable origin. He combats this hypothesis, and points out, in the first place, that naphtha must have been formed in the depths of the earth. It could not have been produced on the surface, because it would have evaporated; nor over a sea bottom, because it would have floated up and been dissipated by the same means. In the next place, he shows that naphtha must have been formed beneath the very site on which it is found; that it cannot have come from a distance, like so many other geological deposits, and for the reasons given above—namely, that it could not be water-borne, and could not have flowed along the surface, while in the superficial sands in which it is generally found no one has ever discovered the presence of organized matter in sufficiently large masses to have served as a source for the enormous quantity of oil and gas yielded in some districts; and hence it is most probable that naphtha has risen from much greater depths under the influence of its own gaseous pressure, or floated up upon the surface of water, with which it is so frequently associated.

The oil-bearing strata in Europe belong chiefly to the Tertiary or later geological epochs; so that it is conceivable that in these strata, or in those immediately below them, carboniferous deposits may exist and may be the sources of the oil; but in America and in Canada the oil-bearing sands are found in the Devonian and Silurian formations, which are either destitute of organic remains, or contain them in insignificant quantities. Yet if the immense masses of hydrocarbons have been produced by chemical changes in carboniferous beds, equally large masses of solid carboniferous remains must still exist; but of this there is absolutely no evidence, while cases occur in Pennsylvania where oil is obtained from the Devonian rocks underlying compact clay beds, on which rest coal-bearing strata. Had the oil been derived from the coal, it certainly would not have made its way downwards; much less would it have penetrated an impermeable stratum of clay. The conclusion arrived at is, that it is impossible to ascribe the formation of naphtha to chemical changes produced by heat and pressure in ancient organized remains.

One of the first indices to the solution of the question lies in the situation of the oil-bearing regions. They always occur in the neighbourhood of, and run parallel to, mountain ranges,—as, for example, in Pennsylvania, along the Alleghanies; in Russia, along the Caucasus. The crests of the ranges, formed originally of horizontal strata which had been forced up by internal pressure, must have been cracked and dislocated, the fissures widening outwards, while similar cracks must have been formed at the bases of the ranges; but the fissures would widen downwards, and would form channels and cavities into which naphtha, formed in the depths to which the fissures descended, would rise and manifest itself, especially in localities where the surface had been sufficiently lowered by denudation or otherwise.

It is in the lowest depths of these fissures that we must seek the laboratories in which the oil is formed; and once produced, it must inevitably rise to the surface, whether forced up by its own pent-up gases or vapours, or floated up by associated water. In some instances the oil penetrating or soaking through the surface layers loses its more volatile constituents by evaporation, and, in consequence, deposits of pitch, of carboniferous shales, and asphalte take place; in other cases, the oil, impregnating sands at a lower level, is often found under great pressure, and associated with forms of itself in a permanently gaseous state. This oil may be distributed widely according to the nature of the formations or the disturbances to which they have been subjected; but the presence of petroleum is not in any way connected with the geological age of the oil-bearing strata: it is simply the result of physical condition and of surface structure.

According to the views of Laplace, the planetary system has been formed from incandescent matter torn from the solar equatorial regions. In the first instance this matter formed a ring analogous to those which we now see surrounding Saturn, and consisted of all kinds of substances at a high temperature, and from this mass a sphere of vapours, of larger diameter than the earth now has, was gradually separated. The various vapours and gases which, diffused through each other, formed at first an atmosphere round an imaginary centre, gradually assumed the form of a liquid globe, and exerted pressures incomparably higher than those which we experience now at the base of our present atmosphere. According to Dalton's laws, gases, when diffused through each other, behave as if they were separate; hence the lighter gases would preponderate in the outer regions of the vaporous globe, while the heavier ones would accumulate to a larger extent at the central portion, and at the same time the gases circulating from the centre to the circumference would expand, perform work, would cool in consequence, and at some period would assume the liquid or even the solid state, just as we find the vapour of water diffused through our present atmosphere does now. That which is true of changes of physical condition, Henri Ste. Claire Deville, in his brilliant theory of dissociation, has shown to be equally true with respect to chemical changes; and the cooling of the vapours forming the earth while in its gaseous condition was necessarily accompanied by chemical combinations, which took place chiefly on the outer surface, where oxides of the metals were formed; and as these are generally less volatile than the metals themselves, they were precipitated on to what there then was of liquid or solid of the earth, in the form of metallic rain or snow, and were again probably decomposed, in part at least, to their vaporous condition. The necessary consequence of this action is that the inner regions of the earth must consist of substances the vapours of which have high specific densities and high molecular weights—that is to say, composed of elements having high atomic weights—and that the heavier elementary substances would collect nearer the centre, while the lighter ones would be found nearer the surface. Our knowledge of the earth's crust extends but to an insignificant distance; yet, as far as we do know it, we find that the arrangement above indicated prevails. Hydrogen, carbon, nitrogen, oxygen, sodium, magnesium, aluminium, silicon, phosphorus, sulphur, chlorine, potassium, calcium—substances whose atomic weights range from 1 to 40—became condensed, entered into every conceivable combination with each other, and produced substances the specific gravity of which averages about $2\frac{1}{2}$, never exceeds 4, and are found near the immediate surface of the globe.

But the mean specific gravity of the earth, as determined by Maskelyne, Cavendish, and others, certainly exceeds 5, and consequently the inner portion of our globe must be composed of substances heavier than those existing on the surface, and such substances are only to be found among the elements with high atomic weights. The question arises, What elements of this character are we likely to find in the depths of the earth? In the first place, since gases diffuse through each other, a certain proportion of the elements of high atomic weight will also be found on the surface of the earth. Secondly, the elements forming the bulk of the earth must be found in the atmosphere of the sun—if, indeed, the earth once formed part of its atmosphere; and of all the elements, iron, with a specific gravity exceeding 7, and with an atomic weight of 56, corresponds best with these requirements, for it is found in abundance on the surface of the earth; and the spectroscope has revealed the very marked presence of iron in the sun, where it must be partly in the fluid and partly in the gaseous state; and consequently iron in large masses must exist in the earth; so that the mean specific gravity of our planet may well be 5, the value which has been determined by independent means.

It is not easy, however, to define in what condition the mass of iron which must exist in the heart of the earth is likely to be. Iron is capable of forming a vast number of combinations, depending upon the relative proportion of the various elements present. Thus, in the blast-furnace, oxygen, carbon, nitrogen, calcium, silicon, and iron are associated, and produce, under the action of heat, besides various gases, a carburet of iron and slag, the latter containing chiefly silicon, calcium, and oxygen—that is to say, substances similar to those which form the bulk of the surface of the earth. But these same elements, if there be an excess of oxygen, will not yield any carburet of iron; and the same result will follow if there be a deficiency of silicon and calcium, because of the large proportion of oxygen which they appropriate. In the same way, during the cooling of the earth, if oxygen, carbon, and iron were associated, and if the carbon were in excess of the oxygen, the greater part of the carbon would escape in the gaseous state, while the remaining part would unite with the iron. It is certain that, in the heart of the earth, there must have been a deficiency of oxygen, because of its low specific gravity; and the argument is supported by the fact that free oxygen and its compounds with the lighter elements abound on the surface. Further, it must be presumed that much of the iron existing at great depths must be covered over and protected from oxygen by a coating of slag; so that, taking all these considerations into account, it is reasonable to conclude that deep down in the earth there exist large masses of iron in part at least in the metallic state or combined with carbon.

The above views receive considerable confirmation from the composition of meteoric matter, for it also forms a portion of the solar system, and originated, like the earth, from out of the solar atmosphere. Meteorites are most probably fragments of planets, and a large proportion of them include iron in their composition, often as carbides, in the same form as ordinary cast-iron—that is to say, a part of the carbon is free and a part is in chemical union with the iron. It has been shown, besides, that all basalts contain iron, and basalts are nothing more than lavas forced by volcanic eruptions from the heart of the earth to its surface. The same causes may have led to the existence of combinations of carbon with other metals.

The process of the formation of petroleum seems to be the following:—It is generally admitted that the crust of the earth is very thin in comparison with the diameter of the latter, and that this crust incloses soft or fluid substances, among which the carbides of iron and of other metals find a place. When, in consequence of cooling or some other cause, a fissure takes place through which a mountain range is protruded, the crust of the earth is bent, fissures are formed; or, at any rate, the continuity of the rocky layers is disturbed, and they are rendered more or less porous, so that surface waters are able to make their way deep into the bowels of the earth, and to reach occasionally the heated deposits of metallic carbides, which may exist either in a separated condition or blended with other matter. Under such circumstances it is easy to see what must take place. Iron, or whatever other metal may be present, forms an oxide with the oxygen of the water; hydrogen is either set free, or combines with the carbon which was associated with the metal, and becomes a volatile substance—that is, naphtha. The water which had penetrated down to the incandescent mass is changed into steam, a portion of which finds its way through the porous substances with which the fissures are filled, and carries with it the vapours of the newly formed hydrocarbons, and this mixture of vapours is condensed wholly or in part as soon as it reaches the cooler strata. The chemical composition of the hydrocarbons produced will depend upon the conditions of temperature and pressure under which they are formed. It is obvious that these may vary between very wide limits, and hence it is that mineral oils, mineral pitch, ozokerit, and similar products, differ so greatly from each other in the relative proportions of hydrogen and carbon. I may mention that artificial petroleum has been frequently prepared by a process analogous to that described above.

Such is the theory of the distinguished philosopher, who has framed it, not alone upon his wide chemical knowledge, but also upon the practical experience derived from visiting officially the principal oil-producing districts of Europe and America, from discussing the subject with able men deeply interested in the oil industry, and of collecting all the available literature on the subject. It is needless to remark that Dr. Mendeleeff's views are not shared by every competent authority; nevertheless the remarkable permanence of oil-wells, the apparently inexhaustible evolution of hydrocarbon gases in certain regions, almost force one to believe that the hydrocarbon products must be forming as fast as they are consumed, that there is little danger of the demand ever exceeding the supply, and that there is every prospect of oil being found in almost every portion of the surface of the earth, especially in the vicinity of great geological disturbances. Improved methods of boring wells will enable greater depths to be reached; and it should be remembered that, apart from the cost of sinking a deep well, there is no extra expense in working at great depths, because the oil generally rises to the surface or near it. The extraordinary pressures, amounting to 300 pounds per square inch, which have been measured in some wells seem to me to yield conclusive evidence of the impermeability of the strata from under which the oil has been forced up, and tend to confirm the view that it must have been formed in regions far below any which could have contained organic remains.

The weights and measures in use in this country are a source of considerable trouble and confusion. Besides the imperial measures, which are complicated enough, a great number of local units are in use, so that unwary strangers are not unfrequently deceived, or, at any rate, if they hope to escape from mistakes, have to apply themselves to the study of local customs. In the scientific world, again, the metric system is now almost exclusively used, and the same may be said of engineers and manufacturers who have to do with foreign countries in which French measures are in vogue. The same difficulty surrounds the measurement of the power of motors. The unit of power is, indeed, from the nature of the case, common to the whole world—it is unit of weight multiplied by unit of height—and with us the foot-pound, or 33,000 times the foot-pound, is generally accepted; but the difficulty lies in determining how the measure is to be applied. Thus, in the case of a water-motor, should the power be calculated by the energy latent in the falling water, or in the actual work given off by the motor? In heat-engines we have to deal with many variables. There is the initial pressure of the working agent, the terminal pressure, the length of stroke, the number of revolutions per minute, the indicated power in the cylinder, the effective power given off, and the adequacy of the means of supplying the working agent. In the early days of steam, when pressures were pretty uniform, and speed bore a certain relation to the stroke, the diameter of a cylinder was a tolerably close index to the power of the engine, and such simple rules as "10 circular inches to the horse-power," which prevailed among agricultural engineers, were tolerably intelligible. But in these days, when pressures, speeds, and rates of expansion vary so greatly, the size of the cylinder, or cylinders, is no longer a guide, and I imagine that most manufacturers have ceased to class their engines by their nominal horse-power. The problem is pretty simple in the case of pumping-engines, for there the nominal power may be taken, as it is in Holland, to be the actual work performed upon the water, and perhaps a similar rule might apply to motors driving dynamos, but for most other purposes no simple law is possible. In my own practice I have, for many years, been in the habit of classing engines by their indicated horse-power per one revolution for every probable initial pressure, below the maximum one for which the engines were designed, and for various rates of expansion. To facilitate the calculations I use curves which give the theoretical horse-power, on the supposition that steam expands according to Boyle's law, for 10,000 cubic inches of steam measured at the moment of exhaust, which is, in fact, the volume of the cylinder in single-cylinder engines, and the volume of the last cylinder in compounds. These curves are calculated for initial pressures rising by 25 pounds, and, in non-condensing engines, for the extreme range of expansion possible, and to fourteen expansions in condensing engines. The true indicated horse-power ranges from 80 per cent. to 85 per cent. of the theoretical, as above stated, the precise percentage depending upon the construction of the engine. As large engines are now almost always compound, the size of the cylinders is no guide to the lay mind; hence, in answering inquiries, it is necessary by some means to get at the actual horse-power expected and to settle the initial pressure, for on this point there is still much timidity among steam-users, so that the engine-builder has to adapt himself in this and other particulars to the needs or prejudices of his customer.

In marine engines, again, the difficulty is still greater, because the only measure of the effective power of the engines is the speed of the ship under given conditions of immersion. But the resistance of ships is a complicated matter, not perfectly ascertained yet, so that the speed attained in any new combination of engines and hull is by no means a certainty; hence some recognized measure of the power of a marine engine, depending only on the measurement of the cylinders and boilers, becomes very desirable.

So strongly has the want of a standard horse-power been felt by shipbuilders and marine engine makers, that the Council of the North-East Coast Institution of Engineers and Shipbuilders appointed a Committee to investigate the subject, and to devise, if possible, a set of rules which would be generally acceptable. The Committee made its report in the spring of 1888. They took as their basis the indicated horse-power, under certain normal conditions, and propose to call this the normal indicated horse-power (N.I.H.P.). The normal conditions are, briefly, the following:—

(1) That the steam, of whatever boiler pressure, is expanded to the same terminal pressure.

(2) That the expansion is effected by all engines with the same degree of efficiency.

(3) That the piston speeds of engines of different lengths of stroke are proportional to the cube root of their respective strokes, and, further, that the actual loaded trial-trip value of piston-speed may be taken as 144 times the cube root of the stroke in inches ($144\sqrt[3]{S''}$).

(4) That in cases in which the engines and boilers bear to each other such proportions as to prevent condition (1) from being fulfilled without thereby violating condition (3) the coal consumption per I.H.P. will not be affected, but will be constant for the same boiler-pressure.

(5) That the boilers are constructed in accordance with what will be generally recognized as the average practice of the present day in respect of the allowance of steam room in relation to power, the diameter, area, and pitch of tubes, the relation of grate to heating surface, and the area of uptakes and funnel; that average natural chimney draught is used, or, if forced draught be employed, that it does not exceed the natural draught; that the horse-power is proportional to the heating surface (H), and to the cube root of the pressure ($\sqrt[3]{P}$); and, further, that the actual loaded trial-trip horse-power may be taken as equal to one-sixteenth of the heating surface multiplied by the cube root of the pressure $\frac{(H\sqrt[3]{P})}{16}$.

(6) That the efficiency of the engine mechanism is constant, and that the propeller is such as to secure that the engines will utilize the boiler power referred to in condition (5) in the manner prescribed by conditions (3) and (4).

Subject to these conditions the normal indicated horse-power is found by multiplying the square of the diameter of the low-pressure cylinder in inches by the cube root of the stroke in inches, adding to the product three times the heating surface of the boiler in square feet, multiplying the sum by the cube root of the pressure, and dividing the product by 100.

$$\text{N.I.H.P.} = \frac{(D^2\sqrt[3]{S} + 3H)\sqrt[3]{P}}{100}.$$

It is evident from this formula, and from the conditions, that account is taken of all the variables, and that the boiler is regarded as an integral part of the engine. The report gives several useful formulæ deduced from the above. Whether the expressions given are the most convenient possible for general marine practice or not, I am not competent to say; but it seems to me that a step has been taken in the right direction in the attempt which has been made to measure marine engines by some rational standard. The members of the Committee were all thoroughly practical as well as scientific men; they determined their constants by reference to a large number of successful cases; and I sincerely hope that the question will be pursued by the marine engine builders on the west coast, and by the constructors of land engines. As engineer to the Royal Agricultural Society, I have frequently had to define the power of engines entered for competition for the Society's prizes, and I have experienced the greatest difficulty in laying down rules for the guidance of intending competitors, being fearful, on the one hand, of restricting originality, and, on the other, of admitting engines of greatly varying powers.

I have expressed an opinion that the numerous engineering Societies which exist at this day have it in their power to promote the advancement of mechanical science in a notable manner by appointing Research Committees, or by aiding individual investigations from their abundant means. The North-East Coast Institution of Engineers and Shipbuilders has done good service in their endeavours to establish a practical measure of the power of marine engines, while the Institution of Mechanical Engineers has, for the last ten years, been steadily promoting researches of an eminently practical nature. Their expenditure has reached the handsome sum of £1700, and their Proceedings have been enriched with Reports on the hardening, tempering, and annealing of steel, on the form of riveted joints, on friction at high velocities, on marine-engine trials, and on the value of the steam jacket. The names of those who are acting on these Committees are a guarantee that the investigations conducted by them will rank among the classical works of the profession, and will abundantly justify the liberal expenditure which has been incurred.

It is impossible to conclude the address which I have had the honour of delivering, without an allusion to the most important structure which engineering skill and enterprise has ever attempted. The Forth Bridge is rapidly approaching completion, and on Saturday, September 14, Mr. Baker is to deliver a lecture, in which he will, no doubt, tell us when the great work is likely to be completed. I think that the members of this Section belong sufficiently to the "working classes" to have a claim to admission to the lecture, and to hear from the lips of the creator of the bridge the story of its inception, of its progress, and his hopes as to its completion.