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Bose–Einstein Statistics and Helium Films

FOLLOWING the proposal of London[1] that the peculiar transition in liquid helium may be due to the accumulation of a significant fraction of the total number of particles in the ground-state of a Bose–Einstein gaseous assembly, many attempts have been made to understand the behaviour of this liquid in terms of the quantum statistics of a Bose gas. Most of these investigations have been confined to the study of helium in bulk. A few years ago, Osborne[2] applied Bose–Einstein statistics to helium films and showed that for films thicker than about 10^{-7} cm., the transition temperature should be the same as that for bulk liquid, whereas films thinner than this are effectively two-dimensional and should have transition temperatures much lower than the bulk value. The monotonic lowering of the 'onset' temperature of superfluidity with decreasing film thickness, observed in the study of unsaturated helium films[3] (which are essentially three-dimensional), thus remained unexplained[4].

Since it is to be expected that the properties of a statistical assembly would be modified when at least one of its dimensions becomes comparable with the mean de Broglie wave-length of the particles, we believe that a proper analysis based upon the Bose–Einstein statistics should be able to explain the observed phenomenon. In the present communication it is shown that such a study is possible provided the eigenfunctions in the bounded continuum are enumerated more exactly than is ordinarily done. We have therefore calculated the total number of particles in an ideal Bose–Einstein gaseous assembly by the equation:

$$N = \sum_i \frac{g_i}{\exp(\alpha + \varepsilon_i/kT) - 1} \tag{1}$$

and while replacing the summation over states by an integral we use the expression[5]:

$$g(\varepsilon)\,d\varepsilon = 2\pi V\left(\frac{2m}{h^2}\right)^{3/2} \varepsilon^{1/2}\left[1 + \frac{S}{8V}\left(\frac{h^2}{2m}\right)^{1/2} \frac{1}{\varepsilon^{1/2}}\right]d\varepsilon \tag{2}$$

We have taken periodic boundary conditions, and S/V is the ratio of the surface area of the enclosure to its volume.

Now, for the degenerate case ($\alpha = 0$) the integral over the correction term in (2) diverges. This is because we have replaced the summation by integration. On physical grounds we should expect that, to a crude approximation, the final result should not be much different from the one calculated from (2) if we replace $\varepsilon^{1/2}$ in the correction term by its average value:

$$\overline{\varepsilon^{1/2}} = (kT)^{1/2}\,[\Gamma 2\zeta(2)]/[\Gamma 3/2\zeta(3/2)]$$

whence one gets at the transition temperature T_0:

$$N = V\left(\frac{2\pi mkT_0}{h^2}\right)^{3/2} \zeta(3/2)\left[1 + \frac{0{\cdot}176\,S}{V}\left(\frac{h^2}{2mkT_0}\right)^{1/2}\right] \tag{3}$$

For a cuboid the thickness t of which is much smaller than the other two sides, we have $S/V \simeq 2/t$. Expressing t in terms of the mean de Broglie wave-length, $h\,.\,(2mkT_0)^{-1/2}$, and T_0 as a fraction r of T_0^* (the transition temperature for an infinitely large assembly, $S/V \to 0$), we may write:

$$1 = r^{3/2}[1 + 0{\cdot}352/t] \tag{4}$$

Equation (4) gives the variation of the transition temperature with the film thickness, as shown in Fig. 1. This is in good qualitative agreement with the experimental results of Long and Meyer[3].

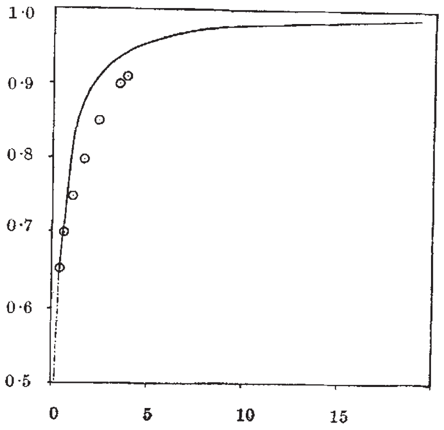

Fig. 1. The onset temperature of superfluidity T_0 (expressed as a fraction r of the transition temperature for bulk) as a function of film thickness t (in units of the mean de Broglie wave-length of the particles). ———, Theoretical curve; ⊙, experimental points of Long and Meyer

It may be noted here that the effect of the decreasing film thickness becomes important when it approaches about ten times the mean de Broglie wave-length. Further, it may be mentioned that the correction term in (4) becomes equal to unity for $t = 0{\cdot}352$, which implies a film of thickness 3·9 A. This limiting thickness is very close to the atomic distance in helium, which means that the three-dimensional approach, modified as above in order to include the effect of the surface term, is valid for films of thickness down to one atomic distance, below which the problem becomes two-dimensional and ceases to remain of practical interest.

A more detailed investigation along these lines, taking into account the interactions in the assembly, is in progress and will be reported elsewhere.

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