It would obviously be an advantage if a large excess of dry Grignard reagent could be prepared and then used several times. Unfortunately, it has been found that the butane produced by the action of a second sample of water on a once-used tube of reagent is contaminated by radioactivity from the first sample. This may be due to the fact that the yield is never quite quantitative. Furthermore, the yield on the second occasion is considerably reduced. However, using fresh reagent for each experiment, the method can be conveniently applied to routine tritium assays. The tubes of reagent are prepared in batches of six and baked out together on a manifold prior to use.

Thanks are due to the Calor Gas Company for supplying pure *n*-butane, and to 20th Century Electronics for their very helpful co-operation in designing and selecting counters.

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Solution of the Thomas–Fermi–Dirac Equation

The fundamental equation of the Thomas–Fermi–Dirac statistical atom model[1] is the so-called Thomas–Fermi–Dirac equation

$$\psi'' = x\left[\left(\frac{\psi}{x}\right)^{1/2} + \beta\right]^3, \qquad (1)$$

where ψ'' denotes the second derivative of ψ by x, β represents the constant $\beta = 3^{1/3}/(32^{1/3}\pi^{2/3}Z^{2/3})$ and Z the atomic number. This equation was solved by Umeda[2] for all neutral atoms with the boundary conditions derived by Brillouin[3]:

$$\psi(0) = 1, \quad \psi(x_0) = 0, \quad \psi'(x_0) = 0, \qquad (2)$$

where x_0 corresponds to the border of the atom. The second and the third boundary conditions are equivalent to the minimization of the energy of any single electron in the statistical atom. This requirement and the boundary conditions resulting from it are, however—as has been already mentioned by Jensen[4]—not in agreement with the statistical point of view, according to which the *total* energy of the statistical atom has to be a minimum, which is not equivalent to the minimization of the energy of the single electrons.

From the minimum-principle of the total energy of the neutral statistical atom, the following boundary conditions are obtained:

$$\psi(0) = 1, \ \psi(x_0) = \frac{\beta^2}{16}x_0, \ x_0\psi'(x_0) - \psi(x_0) = 0. \qquad (3)$$

It can be seen easily that the solutions obtained by these boundary conditions differ only at the outer parts of the atom from the solutions of Umeda, and even there only to a small extent. The solution of the Thomas–Fermi–Dirac equation with the boundary conditions (3) may therefore be determined from Umeda's solutions in the following way. The solution is put into the form:

$$\psi(x) = \psi_u(x) + k\varepsilon(x), \qquad (4)$$

where ψ_u denotes the solution of Umeda and $k\varepsilon$ is small compared to ψ_u. By k we denote a numerical factor independent of x. Putting this expression into (1) and expanding the right-hand side in terms of powers of $k\varepsilon$, one obtains for ε, by neglecting the terms of second and higher powers of $k\varepsilon$, the linear differential equation

$$\varepsilon'' = \frac{3}{2}\left[\left(\frac{\psi_u}{x}\right)^{1/4} + \beta\left(\frac{x}{\psi_u}\right)^{1/4}\right]^2\varepsilon. \qquad (5)$$

Singularities apart from $x = 0$ do not appear here, because the solution is of interest only between $x = 0$ and $x = x_0$, and the solution ψ_u disappears only at Umeda's x_0; this is larger than our x_0, which is determined here with the boundary conditions (3).

If one solves this equation with the boundary conditions

$$\varepsilon(0) = 0, \qquad \varepsilon'(0) = 1 \qquad (6)$$

the first of the boundary conditions (3) is fulfilled and also it follows that the constant k determines the initial slope of ψ, for it is:

$$\psi'(0) = \psi'_u(0) + k. \qquad (7)$$

The constants k and x_0 are determined from the second and the third boundary conditions (3). These calculations can be extended also to positive ions and spherical symmetrically compressed atoms.

The calculations are very simple and have been carried out by us for the atoms neon ($Z = 10$), argon ($Z = 18$), krypton ($Z = 36$), xenon ($Z = 54$) and radon ($Z = 86$), and for the single and double positive ions of these atoms. The tabulated solutions as well as the values of k and x_0 will soon be published in the *Acta Physica Hungarica*. By interpolation, solutions for further atoms can be obtained. The deviation of our solutions for argon, krypton and xenon from the exact solutions computed numerically by Jensen, Meyer-Gossler and Rohde[5] is less than 2 per cent.

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Decomposition of Formic Acid by Periodate

Since the introduction of periodic acid as a reagent for glycols[1], it has been widely employed for elucidating the constitution of many organic compounds[2]. In recent years valuable information has been obtained in the study of polysaccharides by oxidation with periodic acid[3]. Under suitable conditions, one molecule of formic acid is liberated from a non-reducing end-group, and two from a reducing end-group[4]. Determination of the formic acid set free may give an idea of the chain-length of a polysaccharide, while the consumption of periodate per repeating unit is useful in locating the position of hydroxyl groups therein. It is therefore necessary to

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