A New Type of Focusing X-Ray Monochromator

FOLLOWING up a suggestion which arose during a conversation with Dr. T. Ll. Richards recently, I have been experimenting with a new type of focusing X-ray monochromator. The material used is single texture copper strip, as described by Cook and Richards[1,2]. This material has a very high degree of preferred orientation, with a <100> direction lying in the rolling direction, and a {100} plane in the strip surface. The orientation is suitable for curved monochromators set to reflect 200, and since the copper can be bent and ground much more easily than the crystals usually used for this purpose, and is also much more robust, its advantages are obvious. The reflected beam is quite intense and can easily be traced with a fluorescent screen.

The development of this type of monochromator is continuing, and fuller details will be published later.

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[1] Cook and Richards, *J. Inst. Metals*, **66**, 1 (1940).
[2] Cook and Richards, *J. Inst. Metals*, **67**, 203 (1941).

Two Relativistic Models of Dirac's Electron

THE view has often been expressed that some at least of the difficulties encountered in the quantum-mechanical theory of the electron arise from the inadequateness of the underlying classical model. Perhaps some progress might be achieved by substituting a spin particle obeying the laws of relativistic dynamics for the spinless particle used as starting-point by Dirac. At least two different relativistic models of an electron (or, more generally, of any elementary particle with spin, as it seems probable that it is only through the process of quantization that the individuality of different sorts of particles is brought in) may be thus constructed, corresponding to the *first* and the *second cases* dealt with in earlier communications[1]. I shall only comment here on the *second* model.

This consists of a unipole-dipole singularity moving with the velocity of light according to the laws of relativistic dynamics. The essential point, however, is that it is not the singularity itself, but rather its mean position—or the small circle on which it moves—which has to be considered as representing the electron. The following is a list of some analogies and some promising differences between my *second model* of an electron and its quantum-mechanical counterpart. (*a*) **G** has ceased to be parallel to **v** just as in Dirac's theory, where the linear momentum and the velocity have different operators. (*b*) The scalar velocity of the singularity is always c, whereas the scalar velocity of the circle as a whole may acquire all values between 0 and c. (*c*) In both theories the moments of momentum are not constants of motion by themselves but they may be easily supplemented in such a way as to acquire that property; the additional terms involve the vector of angular momentum or its operator which are thus introduced in a very similar manner. It might be objected that the above items are expressly brought in to stress the analogy, but in any event the following come out quite automatically, according to the rules of the theory of relativity and without any new arbitrary assumptions. (*d*) If we put M_σ equal to the mass of an electron and s_σ equal to $\hbar/2$, the frequency of revolution around the circle will be just equal to the frequency of Schrödinger's *Zitterbewegung*; and (*e*) the proper diameter of the circle $2r_\sigma = \hbar/M_\sigma c$ becomes equal to the maximum accuracy with which the position of a particle of mass $M\sigma$ can be ascertained. The amplitude of the *Zitterbewegung* has the same order of magnitude. (*f*) Two isotropic tensors, namely, w^a, for which $w_a w^a = 0$, and $s^{\alpha\beta}$ for which $s_{\alpha\beta}s^{\alpha\beta} = 0$ play a dominant part in our theory; it is well known that isotropic four-vectors and isotropic four-dimensional bivectors are closely connected to spinors.

(*g*) The motion of the spin singularity in an electromagnetic field is far more complicated than in a field-free space, but as a first approximation, if the intensity of the field is small enough, we may go on speaking of motion in a circle even in an electromagnetic field, the circle being subject to a small acceleration as a whole and to slight deformations. Obviously the inequality to be fulfilled by the intensity of the field may be obtained by expressing the fact that in the rest system of the circle the displacement of the centre of the circle during the time of one revolution should be vanishingly small in comparison with the proper radius of the circle. The resulting inequality (in the case of an electrostatic field of intensity E)

$$\varepsilon E\hbar \leqslant M_\sigma{}^2 c^3,$$

is just the same as the condition of no jumps from positive to negative energy states in Dirac's theory.

(*h*) *Variability of mass and pair production.* Strictly speaking, all that has been said hitherto lacks real foundation so long as we have not proved that $M\sigma$ is a constant of motion, and that it may hence be put equal to the mass of an electron. Less restrictively, the question may be put as follows. At the outset, let the electron move in a field-free space; it is then represented by a circle of constant proper radius r_σ, constant mass M_σ and constant spin s_σ. Now, let the electron enter an electromagnetic field and, after remaining there for a while, get out of it into a field-free space again; there it becomes once more a regular circle (the rest-system of which is in general different from what it was before passage through the field). But do M_σ, and hence r_σ and s_σ, return to their previous values? Strictly speaking, the answer is in the negative, but it may be proved that M_σ is approximately constant when the field fulfils a certain condition, which for the field of a plane monochromatic wave of frequency ν reduces to the inequality

$$h\nu \leqslant 2M_\sigma c^2.$$

Thus we see that the condition of constancy of mass in the classical theory is equivalent to the condition of non-production of pairs in quantum-mechanics. In other words, it may be expected that it is only through the process of quantization that the mass of the electron becomes constant—so long as it is not created or annihilated by pair production.

It is not at all clear how the quantization of our relativistic model has to be performed. In any event, it will have to be done in a very different manner from the present one, for Schrödinger's *Zitterbewegung* is a consequence of the superposition of states of positive and negative energies: to-day we imagine that a particle is either in a state of positive or in a state of negative energy, whereas our circular model of an electron, being a model of the *Zitterbewegung*, must correspond simultaneously to states of positive and of negative energies.

The full details of the theories referred to in this and earlier communications will be published in five papers in the first post-war issue of *Acta Physica Polonica*.

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[1] *Nature*, **157**, 766, 809 (1946).

Deflexion of Light in the Gravitational Field without using Einstein Geometry

IN a communication in *Nature*[1], Cheng has shown that the advance of perihelion can be easily obtained from a classical Lagrangian with contraction of length and time in the gravitational field but without the use of the Einstein geometry. Applying this same result to a beam of light passing through the sun's surface with $h = r^2\dot\theta\beta^2 \doteq r_0 c$, it is found that the deflexion angle would be 2·61″.

The equation of path is

$$\frac{d^2\mu}{d\theta^2} + \mu = \frac{GM}{h^2} + 3\lambda\mu^2.$$

The approximate solution to the first order of GM/c^2 is

$$\mu = \frac{1}{r_0}\cos\theta + \frac{GM}{h^2}\left(5 + \cos(\pi + 2\theta)\right).$$

Therefore, the angle of deflexion ε is obtained from the equation

$$\mu = \left\{\cos\left(\frac{\pi}{2} + \frac{t}{2}\right) + \frac{GMr_0}{h^2}\left(5 - \cos(\pi + \varepsilon)\right)\right\}\frac{1}{r_0} = 0$$

$$\varepsilon = \frac{6MG}{r_0 c^2} = 2{\cdot}61''.$$

This result seems in better agreement with corrected astronomical data than Einstein's original value, which is only two thirds of the above value.

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[1] Cheng, *Nature*, **155**, 574 (1945).

Statistical Thermodynamics of Mixtures

FROM some work employing the Bethe technique, I deduced[1] a formula for the number of configurations of mixtures of p-mer and single molecules on a lattice. Afterwards, a kinetic derivation of this formula, together with its generalization to the case of a mixture of simple or branched chain molecules of any number of species, was given by Guggenheim[2] using the principle of detailed balancing. The formula in question can be written in the form:

$$\log g(N_i) = \tfrac{1}{2}z \log (\sum_i q_i N_i)\,! - \sum_i \log N_i\,! - (\tfrac{1}{2}z - 1)\log (\sum_i r_i N_i)\,! + \sum_i N_i \log \rho_i \quad . \quad . \quad (1)$$

where $q_i z$ is the number of sites which are closest neighbours of a molecule of species i which contains r_i submolecules (a slight modification of the definition covers the case of flexible open-chain molecules which can bend back on themselves), and the other symbols have their customary meanings. The value of q_i is given by

$$z(r_i - q_i) = 2(r_i - 1) \quad (\text{all } i) \quad . \quad . \quad (2)$$

In my work only simple chain molecules were considered. Guggenheim's derivation is equally applicable to molecules with branched chains as to simple-chain molecules, but "the exclusion of molecules having closed rings is an essential condition" for Guggenheim's argument. It would therefore be expected that the formulæ obtained by