Jordan proposes[2] the following expression :

$$\sqrt{\nu}\, b(\nu) = \int_{-\infty}^{+\infty} \gamma^{+}(\alpha)\gamma(\nu+\alpha)\, d\alpha \qquad (1)$$

But it may be shown that the operators $b(\nu)$ so defined commute with one another as well as with the conjugate operators. In fact, applying the Fourier transformation from $\gamma(\alpha)$ to $\Psi(x)$, it is easily seen that (1) is equivalent to

$$\sqrt{\nu}\, b(\nu) = \int_{-\infty}^{+\infty} e^{-i\nu x}\, \Psi^{+}(x)\, \Psi(x)\, dx \qquad (2)$$

The operator on the right-hand side has in configuration space of n particles (neutrinos) the meaning : multiplication by $\sum_{k=1}^{n} e^{-i\nu x_k}$ and thus commutes with the conjugate one. The converse conclusion reached by Jordan is obviously due to some mistake in his calculations, most probably to the fact that he implicitly uses in his arguments indefinite expressions of the form $\infty - \infty$.

Since the relation (1) is the mathematical basis of Jordan's theory, the disproof of this relation entails the failure of the whole theory, at least in its present form.

From general arguments developed above, it is to be expected, however, that no consistent neutrino theory of light based on a relation between F and Ψ can be constructed.

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[1] *Z. Phys.*, **93**, 464 (1935).
[2] P. Jordan, "Anschauliche Quantentheorie" (Berlin : Springer, 1936), p. 269.

Minimal Lines and Geodesics within Matter: a Fundamental Difficulty of Einstein's Theory

ACCORDING to Einstein's relativity theory *the minimal lines*, $ds^2 = g_{\iota\kappa}dx_\iota dx_\kappa = 0$, represent, in any circumstances (that is, in space-time upon which any metrical tensor $g_{\iota\kappa}$ satisfying the field-equations has been impressed), *light-lines*, and thus the laws of propagation of light. Now, *in vacuo*, the representation is of course correct, giving—apart from minor refinements—uniform, isotropic propagation with the velocity c, as pre-arranged. But inside matter, considered as a continuous medium characterized by the material tensor $T_{\iota\kappa}$, the minimal lines manifestly *cannot represent light propagation*, even to a rough approximation. For in such a medium, supposed isotropic, the light velocity is, essentially, c/μ (where μ is the refractive index, say, for light of a fixed frequency), whereas $T_{\iota\kappa}$, determining the $g_{\iota\kappa}$, contains no trace of μ, in fact, no properly optical feature of the medium in question. Thus, even without detailed mathematical deduction, one can see that, *within matter*, minimal lines are *not* light-lines ; that is, $ds = 0$ does not represent light propagation.

To illustrate my point, let us consider the case of Schwarzschild's incompressible liquid sphere in equilibrium, that is, $T_1^1 = T_2^2 = T_3^3 = -p/c^2$, $T_4^4 = \rho$ (density) = constant, for which the complete solution is available. Take, for example, the case of purely radial propagation. Then, rigorously,

$$ds^2 = \frac{1}{4}\left(3\cos\frac{a}{R} - \cos\frac{r}{R}\right)^2 c^2dt^2 - dr^2,$$

where r is radial length in 'natural measure', $r = a$ the boundary of the sphere, and $R = c\sqrt{3/8\pi k\rho}$ the curvature radius of the manifold within the sphere (k = gravitational constant). Thus the minimal lines give for the velocity of propagation :

$$v = \frac{c}{2}\left(3\cos\frac{a}{R} - \cos\frac{r}{R}\right).$$

This velocity, then, is a curious function of the distance r from the centre of the sphere, but manifestly wrong. For of the properties of the medium, say, water, it contains only the mass-density ρ, through R. If $\rho = 0$, we have $R = \infty$ and the expression reduces to $v = c$, as it should. But this (absence of matter) *is the only case* when things are correct.

The net result is that within any material medium, as water or glass, Einstein's minimal lines express nothing of physical interest.

In much the same way, the *geodesics*, $\delta\int ds = 0$, which are claimed to represent the motion of free particles in any field $g_{\iota\kappa}$, have this property *in vacuo*, but not within a material medium. This may again be illustrated by Schwarzschild's solution, ds, for a liquid sphere as medium. The corresponding geodesic equations are readily written down, and they yield the following result.

If the Schwarzschild sphere is comparatively small (a/R a small fraction), a 'particle' placed in it at rest, at a distance r from the centre, is subjected, according to Einstein's theory, to an initial acceleration

$$E = -\frac{4\pi k\rho}{3} r,$$

towards the centre, no matter what the density (ρ_p) of the particle itself, whereas the (approximately) correct, Newtonian acceleration is, of course,

$$N = \frac{4\pi}{3} k\rho r \left(\frac{\rho}{\rho_p} - 1\right).$$

The Einsteinian acceleration E, as claimed by the geodesic, happens to agree with the Newtonian acceleration N when $\rho_p = \infty$, but in no other case. Of course, the correct acceleration would come out if we had supplemented the geodesic equations by the resultant of the pressures of the liquid upon the surface of the particle (Archimedes). But we are concerned here with the pure *geodesics* as characteristic lines of the four-dimensional manifold.

In fine, in the geodesic scheme, no account whatever is taken of the 'density' of the immersed test particle. The failure of the geodesics as representatives of motion within a material medium is quite analogous to that of the minimal lines, where no account is taken of the features of the immersed light, so to speak (namely, of its frequency entering into μ or into the dielectric constant of the medium). In fine, similarly to the minimal lines, the geodesics within matter express nothing of physical interest.

The whole part of Einstein's theory which claims to deal with the phenomena within material media would, then, have to be thoroughly rebuilt, which seems scarcely possible without entering into the granular structure of matter—a formidable task.

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