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Relationship between quantum speed limit time and memory time in a photonic-band-gap environment

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Non-Markovian effect is found to be able to decrease the quantum speed limit (QSL) time, and hence to enhance the intrinsic speed of quantum evolution. Although a reservoir with larger degree of non-Markovianity may seem like it should cause smaller QSL times, this seemingly intuitive thinking may not always be true. We illustrate this by investigating the QSL time of a qubit that is coupled to a two-band photonic-band-gap (PBG) environment. We show how the QSL time is influenced by the coherent property of the reservoir and the band-gap width. In particular, we find that the decrease of the QSL time is not attributed to the increasing non-Markovianity, while the memory time of the environment can be seen as an essential reflection to the QSL time. So, the QSL time provides a further insight and sharper identification of memory time in a PBG environment. We also discuss a feasible experimental realization of our prediction.

In quantum information and communication theory, a central objective is to know how fast can a quantum system evolve so as to develop the ultra-speed communication channel and quantum computer. The intrinsic minimal evolution time between two states, known as quantum speed limit (QSL) time[1], is a key method in characterizing the maximal rate of quantum evolution. For closed quantum systems, the QSL time is determined by the Mandelstam-Tamm (MT) type bound and the Margolus-Levitin (ML) type bound[2,3].

Over the past few years, a considerable amount of work has been devoted to the extensions of the QSL time for open quantum systems[4–7], since the interaction between quantum systems and environments can not be ignored. Several new QSL time bounds for open quantum systems have been formulated[8]. The analysis of the environmental effects on the QSL time has been recently applied to a number of systems such as spin-boson models[9,10], atoms in photonic crystals[11], and spin chains[12].

Interestingly, it is found that the non-Markovian evolution induced by the memory effect of environment can accelerate the quantum evolution[13–16]. A good example of this is the situation where an atom is coupled to a leaky single mode cavity[13]. For this model, it has been discovered that increasing non-Markovianity will decrease the QSL time, and therefore lead to a faster speed of the intrinsic evolution. Moreover, a monotonic relationship between the non-Markovianity and the QSL time is presented in different settings[17,18]. One question naturally arise: whether can the degree of non-Markovianity directly reflect the length of QSL time in a memory environment.

The purpose of this paper is to examine the relation between non-Markovianity and QSL time in a environment with memory effects. To do so, we will study the QSL time of a single two-level atom that is immersed in a coherent photonic crystal consisting of an upper band, a lower band and a band gap[19–21]. The environmental coherence means that the lower-band and upper-band reservoirs are correlated and can become coherent due to the definite phase difference between the fields from the double-band reservoir. This interaction between atomic system and the PBG reservoir has been widely used to form population trapping[22], spontaneous emission suppression[23–25] and quantum entanglement preservation[26–34].

In this setting, we investigate how a decrease of QSL time can be acquired by manipulating the environmental coherent property and the band-gap width. It is shown that the environmental coherence and the width of the

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band gap, all serve to reduce the QSL time in some cases. For the mechanism of intrinsic quantum speedup, some nontrivial and unexpected results are found. The non-Markovianity can not directly affect the QSL time. In other words, a larger value of the non-Markovianity does not necessarily result in a shorter QSL time. So in order to clear the physical reason, we further focus on the relation between the QSL time and the memory time of the environment. The memory time of the PBG environment plays a decisive role in the memory effect, and can be characterized by the time taken by some information to travel from the system to the environment and back[35]. We find that the QSL time reduction is attributed to the decreasing memory time of environment, which indicates the increase of the return velocity of information. It is not the non-Markovianity, i.e., the total amount of backflow information, but the backflow rate of information can be seen as an essential reflection to the QSL time. Our results suggest that the QSL time can witness the memory time of PBG environments.

In the following, we will first present a model in which a qubit is coupled to a coherent double-band PBG environment. Secondly, we will consider the environmental effects on the QSL time. The relation between the QSL time and the memory time of the PBG environment will also be explored.

Results

The system-environment model. The global system we consider comprises a two-band photonic crystal containing a single two-level atom placed at location $\mathbf{r}_0$. The corresponding Hamiltonian is ($\hbar = 1$)

$$H = H_S + H_E + H_I \tag{1}$$

with the system Hamiltonian

$$H_S = \omega_0 |1\rangle\langle 1| \tag{2}$$

describing a two-level system with the excited state $|1\rangle$, ground state $|0\rangle$ and transition frequency ω_0. The Hamiltonian of the double-band photonic crystal environment,

$$H_E = \sum_u \omega_u a_u^\dagger a_u + \sum_\iota \omega_\iota b_\iota^\dagger b_\iota, \tag{3}$$

represents a environment of harmonic oscillators with field operator $b_\iota^\dagger$ ($a_u^\dagger$) for the lower (upper) band PBG reservoir. The interaction Hamiltonian reads

$$\begin{aligned} H_I &= i\sum_u (g_u(\mathbf{r}_0) a_u^+ |0\rangle\langle 1| - H.c.) \\ &\quad + i\sum_\iota (g_\iota(\mathbf{r}_0) b_\iota^+ |0\rangle\langle 1| - H.c.), \end{aligned} \tag{4}$$

where $g_{u(\iota)}(\mathbf{r}_0) = \omega_0 d_0 \left(\frac{1}{2\varepsilon_0 \omega_{u(\iota)} V}\right)^{1/2} \mathbf{u}_d \cdot \mathbf{E}^*_{u(\iota),k}(\mathbf{r}_0)$ is the position-dependent coupling constant of the two-level atom with the upper (lower) band PBG reservoir modes with frequency $\omega_{u(\iota)}$. Here, d_0 refers to the magnitude of the dipolar moment and its direction is represented by $\mathbf{u}_d$. $\mathbf{E}^*_{u,k}(\mathbf{r}_0)$ and $\mathbf{E}^*_{\iota,k}(\mathbf{r}_0)$ represent the eigenmode fields from the two-band PBG reservoir. It has been proved that the eigenmode fields depend on the atomic embedded position and can interfere with each other[36]. Thus, we can write the fields $\mathbf{E}^*_{u,k}(\mathbf{r}_0)$ and $\mathbf{E}^*_{\iota,k}(\mathbf{r}_0)$ as[36]

$$\mathbf{E}^*_{u,k}(\mathrm{r}_0) = \cos\,\theta(\mathbf{r}_0)\mathbf{e}_\mathbf{k}, \tag{5}$$

$$\mathbf{E}^*_{\iota,k}(\mathbf{r}_0) = \sin\,\theta(\mathbf{r}_0)\mathbf{e}_\mathbf{k}, \tag{6}$$

where $\mathbf{e}_\mathbf{k}$ is the unit vector of the electric field. The parameter $\theta(\mathbf{r}_0)$ represents the angle seen by the two-level atom placed at $\mathbf{r}_0$. The model thus describes the coupling of a qubit to a PBG reservoir with coherent property, which results from the $\pi/2$ phase difference between fields $\mathbf{E}^*_{u,k}(\mathbf{r}_0)$ and $\mathbf{E}^*_{\iota,k}(\mathbf{r}_0)$.

We assume that the atom is initially excited, and the two reservoir modes are in the vacuum state $|\tilde{0}_u, \tilde{0}_\iota\rangle$. The state of the total system at time t takes the form

$$\begin{aligned} |\varphi(t)\rangle &= a(t)e^{-i\omega_0 t}|1, \tilde{0}_u, \tilde{0}_\iota\rangle \\ &\quad + \sum_u c_u(\mathbf{r}_0, t)e^{-i\omega_u t}|0, \tilde{1}_u, \tilde{0}_\iota\rangle \\ &\quad + \sum_\iota c_\iota(\mathbf{r}_0, t)e^{-i\omega_\iota t}|0, \tilde{0}_u, \tilde{1}_\iota\rangle, \end{aligned} \tag{7}$$

where $|\tilde{1}_u\rangle$ ($|\tilde{1}_\iota\rangle$) is the radiation state containing one excitation only in the uth (ιth) mode.

The probability amplitudes of the system are governed by the Schrödinger equation, from which we can obtain

$$\begin{aligned} i\dot{a}(t) &= \sum_u g_u(\mathbf{r}_0) c_u(\mathbf{r}_0, t) e^{-i(\omega_u - \omega_0)t} \\ &\quad + \sum_\iota g_\iota(\mathbf{r}_0) c_\iota(\mathbf{r}_0, t) e^{-i(\omega_\iota - \omega_0)t}, \end{aligned} \tag{8}$$

$$i\dot{c}_u(\mathbf{r}_0, t) = g_u(\mathbf{r}_0)a(t)e^{i(\omega_u-\omega_0)t}, \quad (9)$$

$$i\dot{c}_\iota(\mathbf{r}_0, t) = g_\iota(\mathbf{r}_0)a(t)e^{i(\omega_\iota-\omega_0)t}. \quad (10)$$

Eliminating $c_u(\mathbf{r}_0, t)$ and $c_\iota(\mathbf{r}_0,t)$ in Eq. (8), one finds

$$\begin{aligned}\dot{a}(t) \;=\; & -\int_0^t d\tau a(\tau)\{\Gamma_u(t-\tau)\cos^2\theta(\mathbf{r}_0)[\cos^2\theta(\mathbf{r}_0) + e^{i\Delta_c t}\sin^2\theta(\mathbf{r}_0)] \\ & + \Gamma_\iota(t-\tau)\sin^2\theta(\mathbf{r}_0)[\sin^2\theta(\mathbf{r}_0) + e^{-i\Delta_c t}\cos^2\theta(\mathbf{r}_0)]\}\end{aligned} \quad (11)$$

where $\Gamma_{u(l)}(t-\tau) = \sum_{\mathbf{k}} g_{\mathbf{k}}^2 e^{-i(\omega_{u(l)}-\omega_0)(t-\tau)}$ represents the memory kernels from the upper (lower)-band reservoir. Here, we have applied $g_u(\mathbf{r_0}) \cong g_{\mathbf{k}}\cos\theta(\mathbf{r}_0)$ and $g_l(\mathbf{r_0}) \cong g_{\mathbf{k}}\sin\theta(\mathbf{r}_0)$ with constant $g_{\mathbf{k}} = \omega_0 d_0\left(\frac{1}{2\varepsilon_0\omega_k V_0}\right)^{1/2}\mathbf{u}_d\cdot\mathbf{e}_{\mathbf{k}}$, $c_u(\mathbf{r}_0, t) = c_{\mathbf{k}}(t)\cos\theta(\mathbf{r}_0)$, $c_\iota(\mathbf{r}_0, t) = c_{\mathbf{k}}(t)\sin\theta(\mathbf{r}_0)$, and the band-gap width $\Delta_c = \omega_u - \omega_\iota \cong \omega_{c_1} - \omega_{c_2}$ with the upper (lower) band edge frequency $\omega_{c_1(c_2)}$.

The Laplace transform of $a(t)$ is

$$\begin{aligned}\tilde{a}(s) \;=\; & \widetilde{A}^{(0)}(s)\{1 - \cos^2\theta(\mathbf{r}_0)\sin^2\theta(\mathbf{r}_0)[\widetilde{A}^{(0)}(s + i\Delta_c)\Gamma_\iota(s + i\Delta_c), \\ & + \widetilde{A}^{(0)}(s - i\Delta_c)\Gamma_u(s - i\Delta_c)]\}\end{aligned}$$

with $\widetilde{A}^{(0)}(s) = [s + \Gamma_u(s)\cos^4\theta(\mathbf{r}_0) + \Gamma_\iota(s)\sin^4\theta(\mathbf{r}_0)]^{-1}$. $\Gamma_{u(\iota)}(s)$ represents the Laplace transform of $\Gamma_{u(\iota)}(t-\tau)$ and its calculation is shown in the method section. For the analytical result of $a(t)$ we refer the reader to the Supplementary Material (see the Eq. (S7)).

In the above discussion, we only considered a coherence case, where the PBG reservoir is coherent resulting from the $\pi/2$ phase difference between the lower-band and upper-band fields of PBG (see Equations (5) and (6)). Now, we study a bit of more general case, where the two-band PBG reservoir is incoherent, i.e., the eigenmode fields from the lower-band and upper-band reservoirs are incoherent waves and independent of the atomic embedded position $[\theta(r_0)]$. Thus, the coupling strengths of the atom with the lower-band and upper-band reservoir modes are the same and equal to the constant $g_{\mathbf{k}} = \omega_0 d_0\left(\frac{1}{2\varepsilon_0\omega_k V_0}\right)^{1/2}\mathbf{u}_d\cdot\mathbf{e}_{\mathbf{k}}$. In the non-coherence case, the Laplace transform of the probability amplitude in the state $|1, \tilde{0}_u, \tilde{0}_\iota\rangle$ can be given by $\tilde{a}_{no}(s) = [s + \Gamma_u(s) + \Gamma_\iota(s)]^{-1}$. The calculation process of $\tilde{a}_{no}(s)$ is shown in detail in the Re[37]. The analytical result of the amplitude $a_{no}(t)$ for no coherence case is shown in the Supplementary Material.

Quantum speed limit time. The aim of this section is to investigate the QSL time problem of a two-level system interacting with a PBG environment. The QSL time is defined as the intrinsic minimal time a system evolving between two states. A unified expression for the QSL time in open systems, widely used to evaluate the speed of quantum evolution, can be written as[13]

$$\tau_{QSL} = \max\left\{\frac{1}{E_{op}}, \frac{1}{E_{tr}}, \frac{1}{E_{hs}}\right\}\sin^2[B(\rho_0, \rho_\tau)], \quad (12)$$

where $B(\rho_0, \rho_\tau) = \arccos\sqrt{\langle\varphi(0)|\rho_\tau|\varphi(0)\rangle}$ is the Bures angle between the target state ρ_τ and the initial pure state $\rho_0 = |\varphi(0)\rangle\langle\varphi(0)|$. $E_{op,tr,hs} = \frac{1}{\tau}\int_0^\tau dt\|\dot{\rho}_t\|_{op,tr,hs}$, with $\|\dot{\rho}_t\|_{op,tr,hs}$ denoting the operator norm, trace norm and Hilbert-Schmidt norm of $\dot{\rho}_t$, respectively. Using this QSL time bound, we can evaluate the intrinsic speed of the dynamical evolution by a given driving time τ. When the QSL time τ_{QSL} achieves the actual driving time τ, i.e., $\tau_{QSL} = \tau$, there is no potential capacity for further speedup, or say the speedup evolution can not appear. For intrinsic speedup evolution, it requires $\tau_{QSL} < \tau$, and the shorter the τ_{QSL} the faster the intrinsic speed of evolution (or equivalently, the greater the capacity for potential speedup) will be.

For convenience, we assume that the atom starts in the excited state $|\varphi(0)\rangle = |1\rangle$, that is $\rho_0 = |1\rangle\langle 1|$. The density matrix of the atom for arbitrary time t can be evaluated as[38] $\rho_t = |a(t)|^2|1\rangle\langle 1| + (1 - |a(t)|^2)|0\rangle\langle 0|$. In the light of Eq. (12), the QSL time of the above model can be given by

$$\tau_{QSL} = \frac{1 - P_\tau}{\frac{1}{\tau}\int_0^\tau dt|\partial_t P_t|} \quad (13)$$

with $P_t = |a(t)|^2$ denoting the atomic excited state population. In the following we will pay special attention to the problem of the effect of environment coherence and width of the band gap Δ_c on the QSL time.

In Fig. 1a, we investigate QSL time τ_{QSL} versus parameter ω_0. A reasonable comparison between the solutions within and without the environment coherence is clearly shown. Here, we choose the driving time τ sufficiently large. This guarantees that the system reaches its steady state. We find that, when ω_0 is in the region of the photonic band gap, τ_{QSL} of coherence case (dashed line) is smaller than that of the non-coherence case (solid line), which means that the environment coherence can reduce the τ_{QSL}, i.e., increase the intrinsic speed of

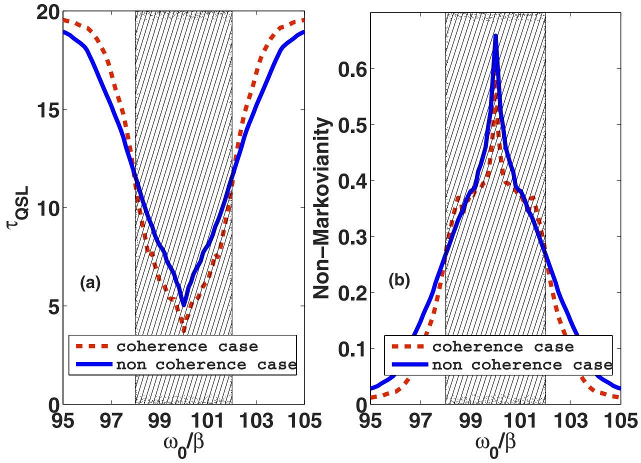

Figure 1. (**a**) The QSL time τ_{QSL} and (**b**) non-Markovianity $\mathcal{N}$ (in unit of $1/\beta$) as a function of ω_0/β, for the coherence case with $\theta(\mathbf{r}_0) = \pi/4$ (dashed line) and the non-coherence case (solid line), respectively. Here we set the driving time $\tau = 20$ (in unit of $1/\beta$). The shadow region refers to the bang gap of the tow-band photonic crystal.

quantum evolution. While, when ω_0 goes far outside the band gap, the QSL time of coherence increases quickly and becomes larger than that of non-coherence case.

As noted in refs 13 and 10, the non-Markovianity $\mathcal{N}$ within the driving time can lead to smaller QSL times. In order to further study the relation between the QSL time and the non-Markovianity, we also plot the non-Markovianity for coherence and non-coherence cases. Non-Markovian dynamics, being linked to memory effects of the environment, implies that the lost information flows from environment back to the system[39]. The non-Markovianity $\mathcal{N}$ can be defined as the total amount of backflow information, $\mathcal{N} = \max_{\rho_{1,2}(0)} \int_{\sigma>0} dt\sigma(t,\ \rho_{1,2}(0))$, where $\sigma(t,\ \rho_{1,2}(0)) = \frac{d}{dt}D(\rho_1(t),\ \rho_2(t))$ denotes the changing rate of the trace distance. The trace distance is defined as $D(\rho_1(t),\ \rho_2(t)) = \frac{1}{2}tr|\rho_1(t) - \rho_2(t)|$ with $|o| = \sqrt{o^+ o}$. It should be note that the dynamical process is non-Markovian if there exists a pair of initial states such that $\sigma(t,\rho_{1,2}(0)) > 0$, and the maximum is taken over all pairs of initial states. As noted in ref. 40, the pair of optimal states is proved to be the states $\rho_1(0) = |0\rangle\langle 0|$ and $\rho_2(0) = |1\rangle\langle 1|$. This allows us to derive the rate of change of the trace distance in the form $\sigma(t,\rho_{1,2}(0)) = \partial_t P_t$.

The corresponding non-Markovianity $\mathcal{N}$ is shown in Fig. 1b. The presence of environmental coherence produces two interesting features. First, except for the regions near the band edges, the non-Markovianity for the non-coherence environment is always greater than the one for the coherence environment. It is implies that environmental coherence can suppress the non-Markovianity. Second, by contrasting Fig. 1a and b, we find that decreasing non-Markovianity will decrease the τ_{QSL} when ω_0 is near the middle of the band gap, while, decreasing non-Markovianity will increase the QSL time outside the band gap region. Therefore, it is not always true that increasing non-Markovianity can lead to a smaller QSL time.

Next, we will consider how the QSL time is affected by the width of the band gap Δ_c. Both the τ_{QSL} and the non-Markovianity for various of Δ_c are shown in Fig. 2a and b. We observe that the QSL time decreases with the increase of Δ_c. If averaging the non-Markovianity over the presented region of ω_0, one can see that the averaged value of non-Markovianity also decreases with the increasing of Δ_c. Therefore, the reason of the τ_{QSL} reduction in this case is not due to the enhancement of non-Markovianity.

In concluding this section, we would like to emphasize that, the environmental coherence and the width of the band gap are all play an important role in accelerating the intrinsic speed of the atomic evolution. More importantly, the decrease of the τ_{QSL} is not directly due to the non-Markovianity, i.e., the total amount of backflow information. What is the mechanism of the intrinsic speedup in a memory environment? What can directly affect the τ_{QSL} in a reservoir with memory effects? To address the above questions, we first describe the memory effect of the model by using the memory time in the next section.

Relationship between the QSL time and the memory time. The memory effect of a reservoir connects tightly with the reservoir correlation time, i.e., memory time. It has been shown that the dynamics of an open system is Markovian when the memory time is very short and non-Markovian when the memory time is long[41]. We first describe the memory time of this model.

For a two-level atom with transition frequency near resonant with the band edge of a photonic crystal, a emitted photon will penetrate a localization length[42] and back. Such a feedback mechanism can result in information backflow, hence non-Markovianity. Also, the time taken by a photon to perform a round trip to the atom should reasonably behave as a memory time, which is a key parameter to the occurrence of memory effect.

Based on this, we take the memory time as $T = 2l/v$, where l is the localization length and v is the photon group velocity. For our two-band PBG reservoir, we will obtain two localization lengths l_l and l_u coming from the lower-band reservoir and the upper-band reservoir. The analytical results of l_u and l_l are shown in the Supplementary Material. The dispersion relation of a two-band PBG environment reads

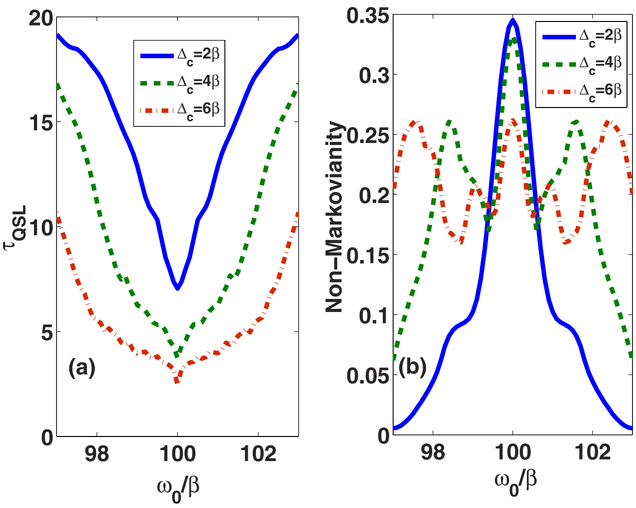

Figure 2. (**a**) The QSL time τ_{QSL} and (**b**) non-Markovianity $\mathcal{N}$ (in unit of $1/\beta$) as a function of ω_0/β for different values of the width of the band gap Δ_c with $\tau = 20$ (in unit of $1/\beta$) and $\theta(\mathbf{r}_0) = \pi/4$.

$$\omega = \begin{cases} \omega_{c_1} + A_1(k - k_0)^2 & \omega \geq \omega_{c_1}, \\ \omega_{c_2} - A_2(k - k_0)^2 & \omega \leq \omega_{c_2}, \end{cases} \tag{14}$$

where $A_m = \omega_{c_m}/k_0^2$ ($m = 1, 2$), and k_0 is a constant characteristic of the dielectric material. Thus, the photon group velocity can be given by $v = d\omega/dk$. To simplify our calculations, we approximate the group velocity as $|v_u| = |v_l| = \chi$, where χ is chosen sufficiently small. That is, we assume that the magnitudes of group velocities coming from the upper and lower band reservoirs are the same and equal to χ. These approximations are valid because we will focus on processes where the photonic band gap is narrow ($\omega_{c_1} \simeq \omega_{c_2}$) and the atomic frequencies are close to the photonic band edge.

Hence, the memory time is reduced to $T_{u(l)} = 2l_{u(l)}/\chi$, which depends entirely on the localization length $l_{u(l)}$. If $l_u \geq l_l$, i.e., $T_u \geq T_l$, we choose the long time T_u as the memory time of the two-band PBG reservoir, and vice versa. Clearly, the memory time is

$$T = \begin{cases} 2l_u/\chi & l_u \geq l_l, \\ 2l_l/\chi & l_l \geq l_u. \end{cases} \tag{15}$$

As an illustration, the memory time is plotted as a function of ω_0/β in Fig. 3. The comparison between the solutions within and without the environment coherence is shown in Fig. 3a. It is remarkable to find that the memory time exhibits the same behaviors as the QSL time in Fig. 1a. That is, both τ_{QSL} and T of coherence (non-coherence) cases are shorter (longer) than that of non-coherence (coherence) cases for ω_0 is in (out) the band gap region. By contrasting Figs 3b and 2a, the results also confirm that decreasing memory time will decrease the QSL time. We thus conjecture that the memory time could be seen as an essential reflection to the QSL time of the dynamical evolution. The decrease of memory time can make the information return more quickly to the system, which helps to accelerate the intrinsic speed of evolution and therefore lead to a smaller QSL time. On the other hand, from Figs 3b and 2b, we can find that a shorter memory time can cause a smaller non-Markovianity in PBG reservoirs. That is to say, it is not the total amount of backflow information but the backflow rate of information that directly affects the QSL time.

Discussion

In summary, we have studied a qubit that is coupled to a coherent PBG reservoir. We have investigated how the coherent property and the width of the band gap affect the QSL time of the qubit. We find that the width of the band gap serves to reduce the QSL time. However, the environment coherence can play dual effects. We have also explored the mechanism of the QSL time reduction in our model. It is revealed that the memory time of the reservoir can be seen as an essential reflection to the QSL time.

The ideal physical system to test the phenomenon we illustrated in this work is InAs quantum dots coupled to a planar GaAs photonic crystal[43]. In experiment, to control the embedded position of the quantum dot in order to observe how the QSL time is influenced by the coherent property of the PBG environment, we can use the method of electrohydrodynamic jet printing[44]. For the relevant experimental parameters, the transition frequency and dipole moment of InAs quantum dots are observed to be $\omega_0 \sim 1.3 PHz$ and $d_0 \sim 3.3 \times 10^{-18} CM$[45,46], respectively, which in turn gives $\beta \sim 160\, MHz$. We can tune the ω_0 of quantum dots by the Stark shift with typical shifts $\Delta \sim 1\, GHz$. Therefore, $\Delta/\beta \sim 6$, and the conditions for intrinsic speedup can be accomplished. Our work may be of theoretical and experimental interests in controlling the QSL time in memory environments.

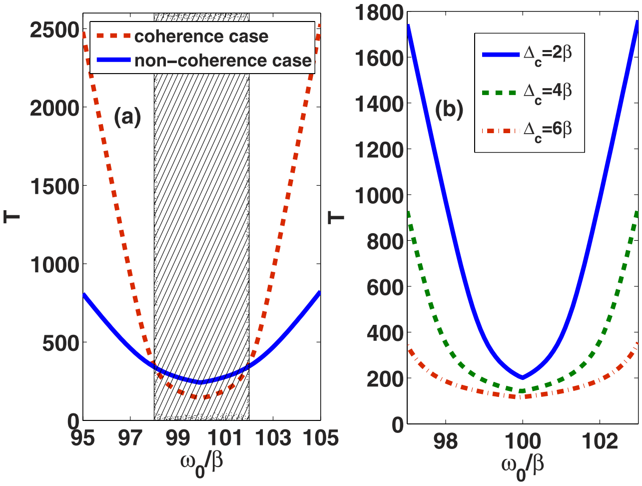

Figure 3. The memory time T as a function of ω_0/β for (**a**) the coherence case with $\theta(\mathbf{r}_0) = \pi/4$ (dashed line) and the non-coherence case (solid line), and (**b**) for various values of band-gap width Δ_c. The shadow region refers to the bang gap of the tow-band photonic crystal. Here we set $\chi = 0.1$ (in the unit of $1/\beta$).

Method

The calculations of $\Gamma_u(s)$ and $\Gamma_l(s)$. Using the above dispersion relation (Re. (14)), the Laplace transforms of the memory kernels $\Gamma_u(t-\tau)$ and $\Gamma_l(t-\tau)$ can be obtained analytically as

$$\Gamma_u(s) = \beta_1^{3/2} \int_{-\infty}^{\infty} \frac{\rho_1(\omega)}{s + i(\omega - \omega_0)} d\omega = \frac{\beta_1^{3/2}}{i\sqrt{-is - \delta_1}}, \tag{16}$$

$$\Gamma_\iota(s) = \beta_2^{3/2} \int_{-\infty}^{\infty} \frac{\rho_2(\omega)}{s + i(\omega - \omega_0)} d\omega = \frac{i\beta_2^{3/2}}{\sqrt{is + \delta_2}}, \tag{17}$$

where

$$\rho_1(\omega) = \frac{1}{\pi} \frac{\Theta\,(\omega - \omega_{c_1})}{\sqrt{\omega - \omega_{c_1}}} \quad \omega \geq \omega_{c_1}, \tag{18}$$

$$\rho_2(\omega) = \frac{1}{\pi} \frac{\Theta\,(\omega_{c_2} - \omega)}{\sqrt{\omega_{c_2} - \omega}} \quad \omega \leq \omega_{c_2}, \tag{19}$$

with the Heaviside step function Θ. $\beta_{1(2)}^{3/2} = [(\omega_0 d_0)^2/6\pi\varepsilon_0] k_0^3/\omega_{c1(2)}^{3/2}$ [37]. $\delta_{1(2)} = \omega_0 - \omega_{c1(2)}$. To keep it simple in this work, we assume $\beta_1^{3/2} = \beta_2^{3/2} = \beta^{3/2}$.

References

1. Anandan, J. & Aharonov, Y. Geometry of quantum evolution. *Phys. Rev. Lett.* **65,** 1697 (1990).
2. Margolus, N. & Levitin, L. B. The maximum speed of dynamical evolution. *Phys. D* **120,** 188 (1998).
3. Mandelstam, L. & Tamm, I. The uncertainty relation between energy and time in non-relativistic quantum mechanics. *J. Phys. (USSR)* **9,** 249 (1945).
4. del Campo, A., Egusquiza, I. L., Plenio, M. B. & Huelga, S. F. Quantum speed limits in open system dynamics. *Phys. Rev. Lett.* **110,** 050403 (2013).
5. Taddei, M. M., Escher, B. M., Davidovich, L. & de Matos Filho, R. L. Quantum speed limit for physical processes. *Phys. Rev. Lett.* **110,** 050402 (2013).
6. Marvian, I. & Lidar, D. A. Quantum speed limits for leakage and decoherence. *Phys. Rev. Lett.* **115,** 210402 (2015).
7. Sun, Z., Liu, J., Ma, J. & Wang, X. Quantum speed limits in open systems: Non-Markovian dynamics without rotating-wave approximation. *Sci. Rep.* **5,** 8444 (2015).
8. Wu, S. X., Zhang, Y., Yu, C. S. & Song, H. S. The initial-state dependence of quantum speed limit. *J. Phys. A* **48,** 045301 (2015).
9. Dehdashti, S., Bagheri Harouni, M., Mirza, B. & Chen, H. Decoherence speed limit in the spin-deformed boson model. *Phys. Rev. A* **91,** 022116 (2015).
10. Zhang, Y.-J., Han, W., Xia, Y.-J., Cao, J. P. & Fan, H. Quantum speed limit for arbitrary initial states. *Sci. Rep.* **4,** 4890 (2014).
11. Xu, Z.-Y., Luo, S., Yang, W.-L., Liu, C. & Zhu, S. Quantum speedup in a memory environment. *Phys. Rev. A* **89,** 012307 (2014).
12. Wei, Y.-B., Zou, J., Wang, Z.-M. & Shao, B. Quantum speed limit and a signal of quantum criticality. *Sci. Rep.* **6,** 19308 (2016).
13. Deffner, S. & Lutz, E. Quantum speed limit for non-Markovian dynamics. *Phys. Rev. Lett.* **111,** 010402 (2013).
14. Cimmarusti, A. D. *et al.* Environment-assisted speed-up of the field evolution in cavity quantum electrodynamics. *Phys. Rev. Lett.* **114,** 233602 (2015).
15. Meng, X., Wu, C. & Guo, H. Minimal evolution time and quantum speed limit of non-Markovian open systems. *Sci. Rep.* **5,** 16357 (2015).
16. Liu, C., Xu, Z. Y. & Zhu, S. Quantum-speed-limit time for multiqubit open systems. *Phys. Rev. A* **91,** 022102 (2015).
17. Zhang, Y.-J. *et al.* Classical-driving-assisted quantum speed-up. *Phys. Rev. A* **91,** 032112 (2015).

18. Liu, H. B., Yang, W. L., An, J. H. & Xu, Z. Y. Mechanism for quantum speedup in open quantum systems. *Phys. Rev. A* **93,** 020105 (2016).
19. John, S. Electromagnetic absorption in a disordered medium near a photon mobility edge. *Phys. Rev. Lett.* **53,** 2169 (1984).
20. John, S. & Wang, J. Quantum electrodynamics near a photonic band gap: photo bound states and dressed atoms. *Phys. Rev. Lett.* **64** 2418(1990).
21. John, S. & Wang, J. Quantum optics of localized light in a photonic band gap. *Phys. Rev. B* **43** 12772 (1991).
22. de Vega, I., Alonso, D. & Gaspard, P. Two-level system immersed in a photonic band-gap material: A non-Markovian stochastic Schrödinger-equation approach. *Phys. Rev. A* **71,** 023812 (2005).
23. Bay, S., Lambropoulos, P. & Molmer, K. Atom-atom interaction in strongly modified reservoirs. *Phys. Rev. A* **55,** 1485 (1997).
24. Lecamp, G., Lalanne, P. & Hugonin, J. P. Very Large Spontaneous-Emission β Factors in Photonic-Crystal Waveguides. *Phys. Rev. Lett.* **99,** 023902 (2007).
25. Wu, J.-N., Huang, C.-H., Cheng, S.-C. & Hsieh, W.-F. Spontaneous emission from a two-level atom in anisotropic one-band photonic crystals: A fractional calculus approach. *Phys. Rev. A* **81,** 023827 (2010).
26. Bellomo, B., Lo Franco, R., Maniscalco, S. & Compagno, G. Entanglement trapping in structured environments. *Phys. Rev. A* **78,** 060302(R) (2008).
27. Wu, Y. N., Wang, J. & Zhang, H. Z. Threshold for formation of atom-photon bound states in a coherent photonic band-gap reservoir. *Opt. Commun*. **366,** 431 (2016).
28. Bellomo, B., Lo Franco, R., Maniscalco, S. & Compagno, G. Two-qubit entanglement dynamics for two different non-Markovian environments. *Phys. Scr. T* **140,** 014014 (2010).
29. Bellomo, B., Lo Franco, R. & Compagno, G. Long-time protection of nonlocal entanglement. *Adv. Sci. Lett.* **2,** 459 (2009).
30. Lo Franco, R., Bellomo, B., Maniscalco, S. & Compagno, G. Dynamics of quantum correlations in two-qubit systems within non-Markovian environments. *Int. J. Mod. Phys. B* **27,** 1345053 (2013).
31. Man, Z.-X., Xia, Y.-J. & Lo Franco, R. Cavity-based architecture to preserve quantum coherence and entanglement. *Sci. Rep*. **5,** 13843 (2015).
32. Zhang, Y.-J., Han, W., Fan, H. & Xia, Y.-J. Enhancing entanglement trapping by weak measurement and quantum measurement reversal. *Ann. Phys.* **354,** 203 (2015).
33. Li, H., Xie, S., Xu, J. & Yang, Y. Effects of engineering initial states and quantum interference near the edge of a photonic bandgap on the entanglement. *J. Opt. Soc. Am. B* **32,** 1050 (2015).
34. Aolita, L., de Melo, F. & Davidovich, L. Open-system dynamics of entanglement: a key issues review. *Rep. Prog. Phys.* **78,** 042001 (2015).
35. Tufarelli, T., Kim, M. S. & Ciccarello, F. Non-Markovianity of a quantum emitter in front of a mirror. *Phys. Rev. A* **90,** 012113 (2014).
36. Cheng, S.-C., Wu, J.-N., Yang, T.-J. & Hsieh, W.-F. Effect of atomic position on the spontaneous emission of a three-level atom in a coherent photonic-band-gap reservoir. *Phys. Rev. A* **79,** 013801 (2009).
37. Yang, Y., Fleischhauer, M. & Zhu, S.-Y. Spontaneous emission from a two-level atom in two-band anisotropic photonic crystals. *Phys. Rev. A* **68,** 043805 (2003).
38. Breuer, H.-P. & Petruccione, F. *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, UK, 2002).
39. Breuer, H.-P., Laine, E.-M. & Piilo, J. Measure for the degree of non-Markovian behavior of quantum processes in open systems. *Phys. Rev. Lett.* **103,** 210401 (2009).
40. Wißmann, S., Karlsson, A., Laine, E.-M., Piilo, J. & Breuer, H.-P. Optimal state pairs for non-Markovian quantum dynamics. *Phys. Rev. A* **86,** 062108 (2012).
41. Laine, E.-M., Piilo, J. & Breuer, H.-P. Measure for the non-Markovianity of quantum processes. *Phys. Rev. A* **81,** 062115 (2010).
42. John, S. & Quang, T. Photon-hopping conduction and collectively induced transparency in a photonic band gap. *Phys. Rev. A* **52,** 4083 (1995).
43. Yoshie, T. *et al.* Vacuum Rabi splitting with a single quantum dot in a photonic crystal nanocavity. *Nature* **432,** 200 (2004).
44. See, G.-G. *et al.* Polarized quantum dot emission in electrohydrodynamic jet printed photonic crystal. *Appl. Rhys. Lett.* **107,** 051101 (2015).
45. Alén, B., Bickel, F., Karraib, K., Warburton, R.-J. & Petroff, P.-M. Stark-shift modulation absorption spectroscopy of single quantum dots. *Appl. Phys. Lett.* **83,** 2235 (2003).
46. Yang, Y. & Zhu, S.-Y. Spontaneous-emission enhancement and population oscillation in photonic crystals via quantum interference. *Phys. Rev. A* **61,** 043809 (2000).

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Author Contributions
J. Wang, Y. N. Wu and M. L. Mo contributed the idea. J. Wang performed the calculations, and prepared the figures. J. Wang wrote the main manuscript, H. Z. Zheng checked the calculations and made an improvement of the manuscript. All authors contributed to discussion and reviewed the manuscript.

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