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High-Resolution Group Quantization Phase Processing Method in Radio Frequency Measurement Range

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Aiming at the more complex frequency translation, the longer response time and the limited measurement precision in the traditional phase processing, a high-resolution phase processing method by group quantization higher than 100 fs level is proposed in radio frequency measurement range. First, the phase quantization is used as a step value to quantize every phase difference in a group by using the fixed phase relationships between different frequencies signals. The group quantization is formed by the results of the quantized phase difference. In the light of frequency drift mainly caused by phase noise of measurement device, a regular phase shift of the group quantization is produced, which results in the phase coincidence of two comparing signals which obtain high-resolution measurement. Second, in order to achieve the best coincidences pulse, a subtle delay is initiatively used to reduce the width of the coincidences fuzzy area according to the transmission characteristics of the coincidences in the specific medium. Third, a series of feature coincidences pulses of fuzzy area can be captured by logic gate to achieve the best phase coincidences information for the improvement of the measurement precision. The method provides a novel way to precise time and frequency measurement.

Measurement is the basis and premise of scientific research[1–4]. However, the authenticity and precision of the measurement directly determine the success or failure of scientific research[5–9]. The phase processing which has very high resolution is crucial to improve the authenticity and precision of the measurement[10,11] and plays an important role in the field of precise measurement physics such as new atomic frequency standard technology, phase noise measurement and suppression, transmission and comparison of time and frequency, etc[12–16].

The most commonly used phase processing is at present measurement of phase difference that is based on time interval measurement in the international. The more typical methods are here time interval counter method, time-digital conversion method, phase comparison method and dual mixer time difference method, etc. The time interval counter method[17,18] has the advantages of wide measurement range, high linearity and low cost. However, due to the limited filling pulse frequency, it can only achieve measurement result which is much better than one nanosecond(ns) level resolution. The time-digital conversion method[19,20] is easily integrated in a wider dynamic range of measurement, but its resolution is limited by conversion rate and digital signal number. The phase comparison method[21,22] has higher measurement precision, only for the two signals with the same frequency. In the light of serious nonlinear and "dead zone" phenomenon in higher phase comparison frequency, it is usually used in the range of low frequency below 0.1 MHz. The dual mixer time difference method[23,24] is now a high-resolution phase difference measurement method which is much better than one picosecond(ps) level resolution widely used in the international. Due to the limited measurement precision, it is mainly used in measurement of the short-term frequency stability and the phase difference variation.

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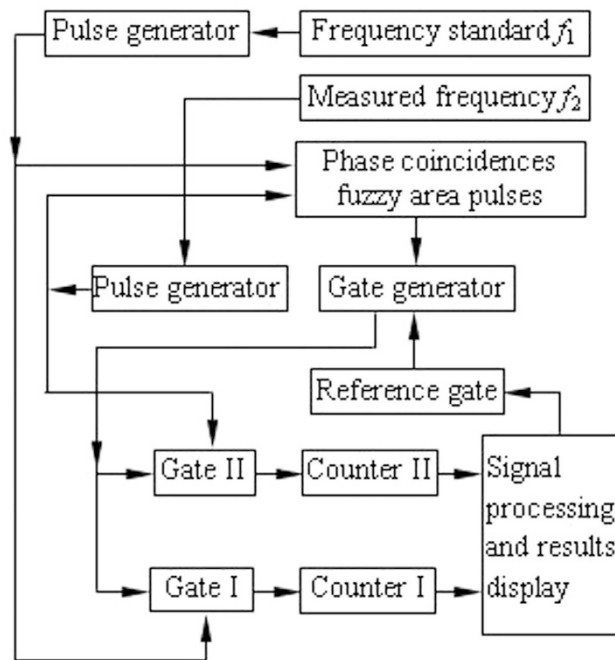
Figure 1. Frequency standard comparison scheme.

In recent years, with the development of aeronautics and astronautics, satellite navigation, precision distance measuring, space location, the modern time service, scientific measurement, radar detection and other high-tech fields, Chinese scholars have also done in-depth research in phase processing and got series of achievement[25]. The dual mixer time difference method is further studied in China. Combing the dual mixer time difference method and the time interval counter method is used in time keeping. NTSC (National Time Service Center of Chinese academy of sciences) has developed a high-precision phase comparator which is better than the dual mixer time difference system by the long-term use of SR620 frequency counter made by Stanford Research System Company for phase comparison of two time signals, which greatly improves measurement resolution of phase processing[26]. A novel phase processing for direct phase comparison between any frequencies signals without frequency normalization is now introduced in China, which is called different frequency phase processing[27]. Aiming at the phase relationships between different frequencies signals, the concepts of the greatest common factor frequency and the equivalent phase comparison frequency are presented[28]. There is no ±1 count error caused by counting in the measurement gate which is set up by the phase coincidences in the traditional phase processing, which is much better than one nanosecond level resolution. Aiming at the variation law of the phase difference between different frequencies signals, a group period phase comparison method[29] with one picoseconds level resolution is proposed, which provides a new way to different frequency phase processing. Aiming at the phase group processing between different frequencies signals, the concept of group phase quantization is established, and the physical characteristics of group phase quantization are analyzed. The measurement resolution which is much higher than one picoseconds level can be achieved in a wide frequency range by the quantitative phase step law of group phase quantization[30]. The concept of phase group synchronization is proposed[31] and the precise frequency linking between different frequencies signals is established, which obtains the measurement precision better than one fs order of magnitude[32].

Still, there is some way to go in precise frequency source especially new atomic frequency standard technology in China compared with the international. So a high-resolution phase processing method by group quantization higher than 100 fs level is proposed in a wider frequency measurement range. It is a novel method which is different from the international in phase processing. In terms of group quantization phase processing, there are not limit of frequency normalization in the traditional phase processing, and also demand of nominal frequency in the different frequency phase comparison. It can implement high-resolution phase measurement, comparison and control of any frequency signals in a wide radio frequency range without frequency synthesis and conversion.

Results and Discussion

In the following experiment, the reference signal and the measured signal are from the same frequency source. The reference signal f_1 is produced by 5071A Cs atomic clock. The measured signal f_2 is from precise frequency synthesizer by the reference signal caused by 5071A Cs atomic clock.

Frequency standard comparison with simple phase relationships. The experimental scheme of the frequency standard comparison is shown in Fig. 1. Here, the frequency of the reference signal f_1 is 10 MHz, and the frequency of the measured signal f_2 is 8 MHz.

First, the reference signal f_1 and the measured signal f_2 are converted to the same frequency pulse with a suitable pulse width by pulse generator, as shown in Fig. 2.
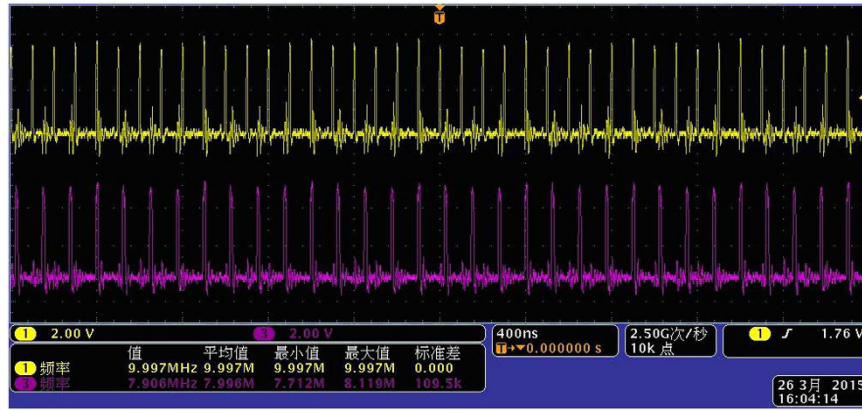
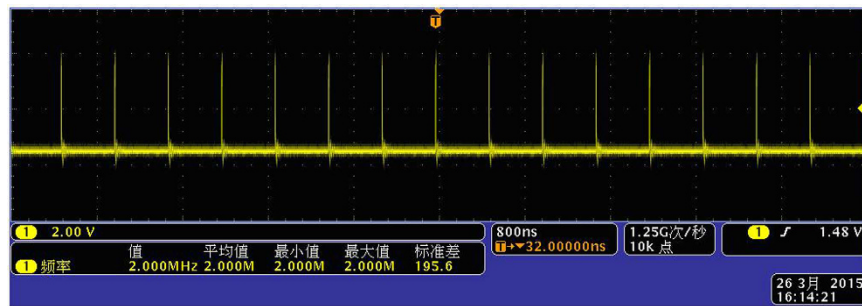**Figure 2. The same frequency pulse of two comparison signals.**

Figure 3. Phase coincidences between f_1 and f_2.

Second, the same frequency pulse of f_1 and f_2 is sent to the phase coincidence detector in the same time. The result of the phase coincidence leads to the phase coincidences fuzzy area, as shown in Fig. 3.

The quantized phase resolution $T_{\max c}$ denotes here measurement resolution, and is given by $T_{\max c} = (1/10, 1/8) = 25$ ns. However, the detection resolution of the phase coincidence circuit is usually 2 ns. Obviously, every phase difference obtained by phase comparison between f_1 and f_2 can be identified by the detector. The fuzzy area is in fact a series of the phase coincidences pulse. These phase coincidences pulses are strict phase synchronization by the $T_{\min c}$. That is to say,

$$T_{\min c} = m_1 m_2 T_{\max c} = 0.5 \ \mu s \tag{1}$$

where $m_1 = 4$, $m_2 = 5$ and $T_{\max c} = 25$ ns. The phase comparison result f_{out} is a pulse signal with 2 MHz frequency.

Third, the open signal and stop signal are produced by gate generator. The reference signal and the measured signal are counted as follows.

$$N_1 T_1 = N_2 T_2 \tag{2}$$

$$f_2 = f_1 N_2 / N_1 \tag{3}$$

where T_1 and T_2 are their periods, N_1 is counter value of the reference signal f_1 from counter I, N_2 is counter value of the measured signal f_2 from counter II.

The obtained measurement data is processed by the signal processing circuit. The experimental result shows that the frequency stability in Fig. 1 can be reached E-13/s order of magnitude.

Frequency measurement with the complex phase relationships. The experimental scheme of frequency measurement is shown in Fig. 4.

Here the frequency of the reference signal f_1 is 10 MHz, and the frequency of measured signal f_2 is 10.23 MHz.

First, the reference signal f_1 and the measured signal f_2 are converted to the same frequency pulse with a suitable pulse width by pulse generator, as shown in Fig. 5.

Second, the same frequency pulse of f_1 and f_2 is sent to the phase coincidence circuit in the same time. The result of the phase coincidence generates the phase coincidences fuzzy area, as shown in Fig. 6.

The quantized phase resolution is $T_{\max c} = (1/10, 1/10.23) \approx 10$ ps. Every phase difference less than the detection resolution 2 ns in phase comparison result f_{out} can not be obviously identified by the phase coincidence detection circuit. The fuzzy area occurs and is made of a series of the phase coincidences pulse. These fuzzy areas are very stable and strict synchronization by the group period T_{gp}. That is to say,
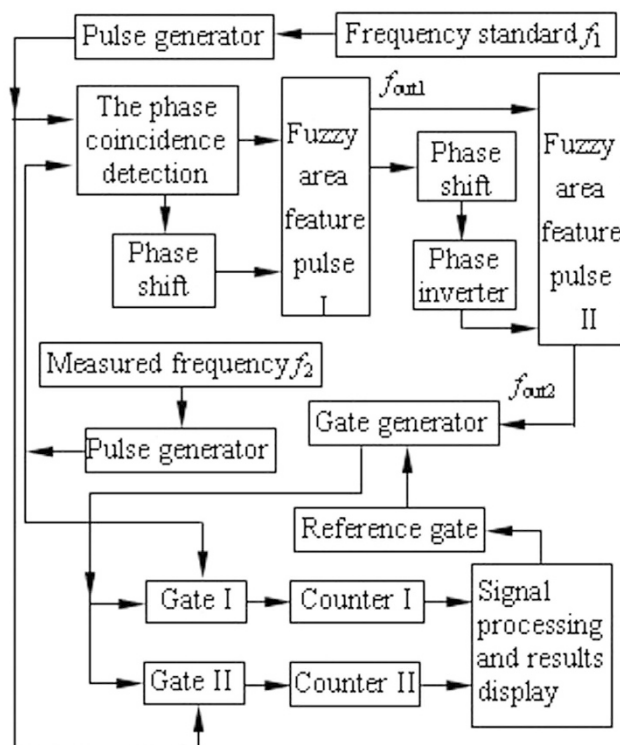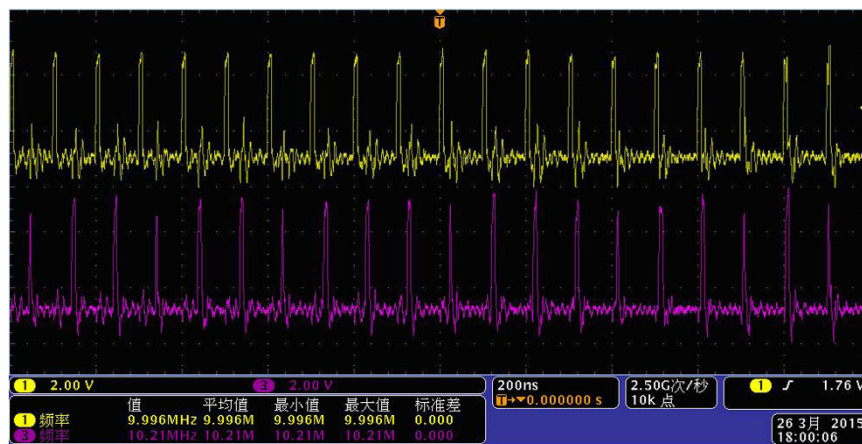
Figure 4. Frequency measurement experiment scheme.

Figure 5. The same frequency pulse of f_1 and f_2.

$$T_{gp} = 1/\Delta f \approx 4.4 \ \mu s \tag{4}$$

where $\Delta f = 0.23$ MHz. If the phase coincidences are used as the open signal and stop signal of measurement gate in this time, a larger measurement error is produced, which leads to low measurement precision and authenticity. That is to say, it is very difficult to achieve frequency stability of E-13/s order of magnitude in the time. The phase coincidences fuzzy area is required to be further processed for high measurement precision and authenticity.

Third, the phase coincidences fuzzy area is precisely shifted in phase, and there is an exclusive OR processing with the original fuzzy region in order to obtain the phase coincidences pulse of fuzzy region edge. The processing result f_{out1} is shown in Fig. 7.

In order to obtain single fuzzy area edge pulse, the fuzzy area is further processed by precise phase shift, non logic, and logic, etc. The further processing result f_{out2} is shown in Fig. 8.

The single fuzzy area edge pulse is here used as the open signal and stop signal of measurement gate. The reference signal and the measured signal are counted in gate time. The obtained counter value is processed by the signal processor, and the frequency stability data of the measured signal can be easily obtained in this experiment.
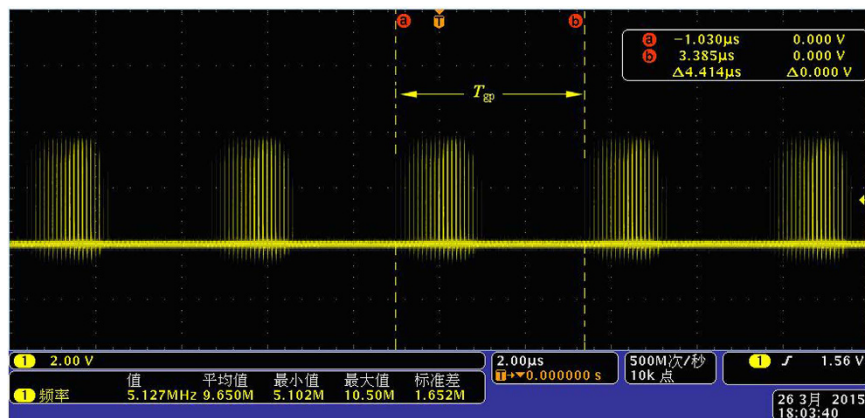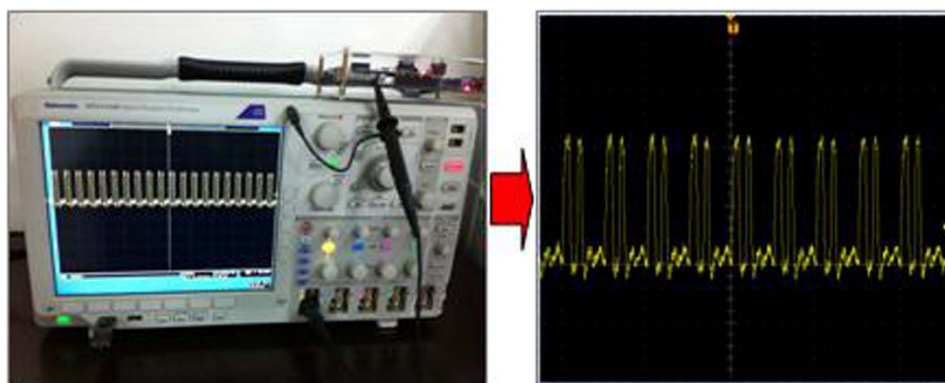
Figure 6. Fuzzy area of the phase coincidences.

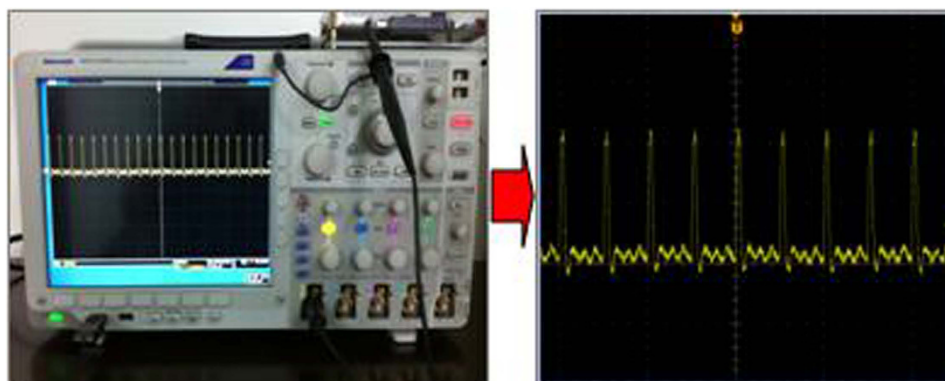**Figure 7. Double edge pulses of the coincidences fuzzy area.**

Figure 8. Single edge pulse of the coincidences fuzzy area.

The experimental result demonstrates that the frequency stability in Fig. 4 can be also reached E-13/s order of magnitude.

The above experiments show that the principle of the group quantization and its synchronization is scientific and reasonable, the quantization phase resolution is the measurement resolution, and high measurement resolution does not necessarily achieve the high measurement precision due to the limited resolution of detection circuit. The key is whether the measurement resolution is stable. The stability of the measurement resolution depends on the stability of frequency relationships between the reference signal and the measured signal. However, the stability of the fuzzy area depends on the stability of the detection resolution of the detector. If the measurement resolution and the detection resolution are stable in a measurement system, the system hardware error caused by the inconsistence and mismatching of detection device can be reduced or eliminated, which greatly improves the authenticity and precision of the measurement to ensure the success of the scientific research.
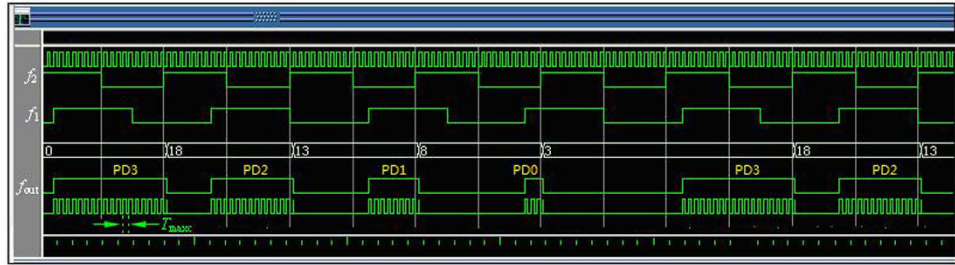
Figure 9. Quantization of phase differences.

Conclusion

According to the analysis of the high-resolution phase processing in the international, and by combining the inherent phase or frequency relations between different frequency comparison signals, a high-resolution group quantization phase processing method in radio frequency range is presented. The method depends on the quantization of phase difference to achieve the higher measurement resolution, and relies on the stability of the fuzzy area to obtain the higher measurement precision. It has a significant meaning in precise measurement. Compared to high-resolution dual mixer time difference method which is widely used in the international, there are the advantages of simple circuit structure, low development cost, small phase noise, quick response time , high authenticity and stability.

Methods

As a key step in phase measurement, the resolution conflicts with the stability of detection equipment. Resolution which directly leads to measurement error is considered to be the decisive factor affecting the measurement precision, especially in the condition of super-high resolution quantization measurement. Thus, high-resolution phase processing method is an important way for improving measurement precision in time and frequency measurement. For the same frequency signals, there is a high measurement resolution in the traditional phase processing. However, for the different frequency signals, some frequencies synthesis and transformation, such as mixing and multiple frequency, etc., are first used for frequency normalization of comparison signals. There are some additional errors caused by frequency conversion circuit with the inherent noise characteristics, which partly affects the improvement of measurement precision. Even if the switch phase comparison with the higher phase comparison precision is used in phase comparison, it is also confined to further improve the measurement precision due to the "dead zone" phenomenon and "nonlinearity" problem of phase comparison. Hence, the research of novel measurement principle and the exploration of frequency relationships of comparison signals and its variation law which is used in precise measurement fields for the improvement of measurement precision and resolution have a realistic significance. Frequency signal as an object of phase comparison and processing is one of periodic motion phenomenon in nature, and it follows the law of periodic motion in phase comparison. Hence, the essence of the law can be described by some new concepts such as the greatest common factor period, group quantization, phase quantization, group quantization synchronization and group quantization period, etc.

Suppose f_1 and f_2 are two stable frequencies of phase comparison signals with the initial phase difference from the same frequency source, where T_1 and T_2 denote their periods.

Let $T_1 = m_1 T_{maxc}$ and $T_2 = m_2 T_{maxc}$, where m_1 and m_2 are two positive integers without common factor and $m_1 > m_2$. The T_{maxc} is here called **the greatest common factor period** between f_1 and f_2 frequencies signals, and can be calculated by equation (5),

$$T_{maxc} = (T_1, T_2). \tag{5}$$

Every phase difference in phase comparison results f_{out} is quantized by T_{maxc}, as shown in Fig. 9.

With the time, there is a series of phase coincidences caused by the stability of electromagnetic wave signal transmission in the specific medium and the differences of two signals periods T_1 and T_2 in phase comparison. The obtained ideal coincidences are a minimum of phase comparison results f_{out}. The coincidences with a stable transmission can parallel shift with the time. When the shifting time is just multiples of the least common multiple period T_{minc}, the coincidences occur again. The least common multiple period can be calculated by equation (6),

$$T_{minc} = m_1 m_2 T_{maxc}. \tag{6}$$

As shown in Fig. 10. So the continuity and periodicity of the coincidences such as A, B, C, D, E, etc. for the interval of the least common multiple period is the key characteristics of different frequency phase comparison.

The least common multiple period directly reflects the mutual relationships of phase between two frequencies signals. The relationships can be obtained by equation (7),

$$T_{minc} = m_1 T_2 = m_2 T_1. \tag{7}$$

Hence it is not difficult to conclude that the phase comparison for the interval of one signal period (T_1 or T_2) as reference phase can not be implemented in the different frequency phase processing. This is the main reason
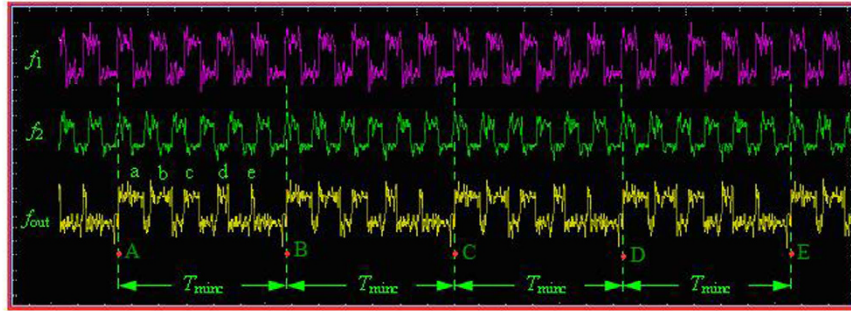
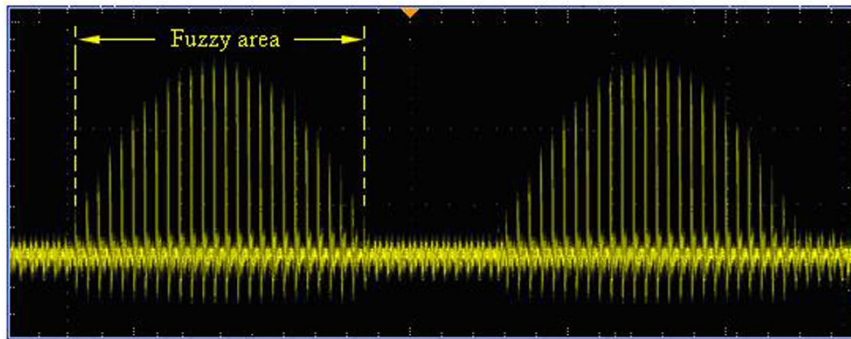**Figure 10. Continuity of different frequency phase comparison.**

Figure 11. The phase coincidences and fuzzy area.

why the same frequency and frequency normalization are used for different frequency signals in the traditional phase comparison.

It is demonstrated by experiments that there is a strict phase coincidence for the interval of the least common multiple period. However, **the obtained ideal coincidence is not a single pulse, but a fuzzy area including a series of the coincidences pulses,** as shown in Fig. 11.

The decisive factor of the actual measurement precision is the stability of the fuzzy area. Thus, reducing coincidence pulses number and width of fuzzy area to improve measurement precision is an important aspect that we should work hard continuously.

The frequency f_1 is used as a reference signal in Fig. 10. For every positive pulse of f_1, the phase comparison result is the first positive pulse phase difference of the measured frequency f_2 such as a, b, c, etc. Then a phase difference group here called "group" is formed in the neighboring phase coincidences. So there are the following three important characteristics for all the phase differences in a group.

First, it is not monotonous by sequence of the phase differences. Second, it is not continuous for the neighboring phase difference. Third, it is repeatable for the interval of the least common multiple period.

For every positive pulse of f_2 in Fig. 9, phase comparison results are $\Delta p_1, \Delta p_2, \ldots, \Delta p_{m2}$, where m_2 is a number of T_1 in a group, Δp_{m2} is phase difference of the phase coincidence. The phase comparison results can be calculated by equation (8),

$$
\begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \Delta p_3 \\ \vdots \\ \Delta p_{m2} \end{bmatrix} = \begin{bmatrix} n_1 T_2 - T_1 \\ n_2 T_2 - 2T_1 \\ n_3 T_2 - 3T_1 \\ \vdots \\ m_1 T_2 - m_2 T_1 \end{bmatrix} = \begin{bmatrix} n_1 m_2 T_{\max c} - m_1 T_{\max c} \\ n_2 m_2 T_{\max c} - 2m_1 T_{\max c} \\ n_3 m_2 T_{\max c} - 3m_1 T_{\max c} \\ \vdots \\ m_1 m_2 T_{\max c} - m_2 m_1 T_{\max c} \end{bmatrix}
$$

$$
= T_{\max c} \begin{bmatrix} n_1 m_2 - m_1 \\ n_2 m_2 - 2m_1 \\ n_3 m_2 - 3m_1 \\ \vdots \\ m_1 m_2 - m_2 m_1 \end{bmatrix} = T_{\max c} \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ \vdots \\ K_{m2} \end{bmatrix}
\tag{8}
$$

where $n_1, n_2, \ldots, m_1$ are periods number of f_2, $K_1, K_2, K_3 \ldots, K_{m_2}$ are positive integers, $m_1 T_2 = m T_1 = T_{\min c}$, and $\Delta p_1, \Delta p_2, \ldots, \Delta p_{m2}$ are the integral multiples of $T_{\max c}$. All the phase differences in a group are quantized by $T_{\max c}$. The
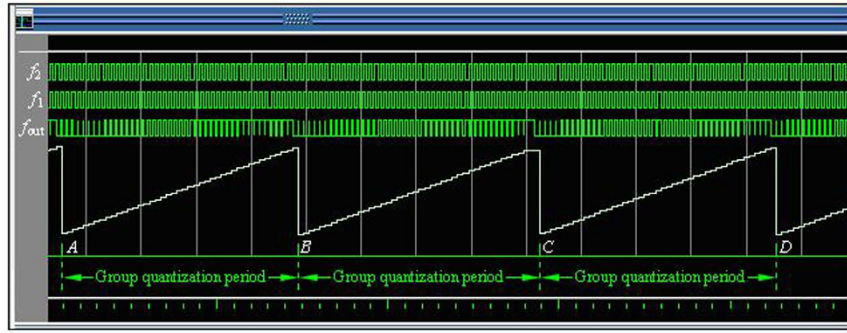
Figure 12. Step law of phase quantization.

Figure 13. The phase quantization frequency.

T_{maxc} called **phase quantization** is the least phase difference in all phase differences, and it is used as a basic quantization unit of the phase differences. The detection circuit can distinguish T_{maxc}, while its resolution is only better than T_{maxc}. The T_{maxc} is usually used as a reference standard of detection circuit resolution in the different frequency phase processing. Hence, the T_{maxc} is also called the **quantized phase resolution**. It is found that the quantization step of T_{maxc} is linear by rearranging the phase differences from small to large in a group, as shown in Fig. 12, where A, B, C, D are the coincidences.

From the aspect of the quantized phase difference, all phase differences in a group can be calculated by equation (9),

$$P_{gq} = (K_1 T_{\max c} + K_2 T_{\max c} + \cdots + K_{m_2} T_{\max c})$$
$$= T_{\max c} \sum_{i-1}^{m_2} K_i, \ i = 1, 2, \cdots, m_2, \tag{9}$$

where P_{gq} called **group quantization.** The P_{gq} is repeatable for the interval of the least common multiple period, so the least common multiple period is also called **group quantization period** T_{gqp} in different frequency phase processing. That is to say, $T_{gap} = T_{minc}$. Eventually it is the nominal frequency of two comparison signals that determines the group quantization, which is not affected by phase noise or frequency drift. The quantized rate of the phase differences in a group depends on the f_{pq} which can be calculated by equation (10),

$$f_{pq} = \frac{1}{T_{\max c}}. \tag{10}$$

The f_{pq} is here called **phase quantization frequency**. It is shown by equation (4) that any phase difference Δp in a group is the integer multiple of the T_{maxc}, so the quantized situation of the Δp is shown in Fig. 13.

The T_{maxc} is the least phase difference in phase comparison results, so the concept of the f_{pq} is based on the analysis of phase by equation (10). However, the change situation of power caused by the T_{maxc} can be observed, only while the variation of phase which is not a physical quantity is converted into the change of voltage by phase comparison circuit. So the frequency spectrum of the f_{pq} cannot be observed in frequency analyzer.

There is a strict phase synchronization of group quantization for the interval of group in phase comparison, which generates the phase coincidences in the least common multiple period. The phase coincidences fuzzy area can be obtained by different frequency phase coincidence detection circuit. It is concluded by the concept of group quantization that different frequency phase comparison is essentially the periodic accumulation of the T_{maxc} for the interval of group with the time. The monotonous situation of the accumulation is determined by the frequency relationships of two comparison signals. While the T_{maxc} changes to the P_{gq} or the opposite, there is a strict phase coincidence between two frequencies signals. The T_{maxc} is the least unit of phase difference which is used as the step value for the monotonous variation of any phase difference in a group. The step variation of the T_{maxc} is repeatable, periodic and synchronous for the interval of the least common multiple period. It is a fact that
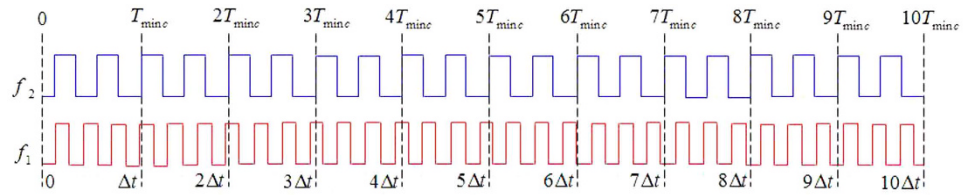
Figure 14. Group quantization phase and its volatility.

the initial value of the T_{maxc} is linearly converted into group quantization P_{gq} or the opposite, which is the step law of the quantized phase, as shown in Fig. 12 above.

It can be seen by the analysis above that the greatest phase difference in a group is the T_1, and $T_1 = m_1 T_{maxc}$. Hence, the time that the T_1 is quantized by the T_{maxc} is just T_{minc}. That is to say, while a single phase difference accumulated by phase quantization T_{maxc} in a group is increased to the T_1, there is a strict phase coincidence between two comparison signals.

Similarly, the formation of fuzzy area width of the phase coincidences caused by step of the phase quantization is similar to the formation of group quantization caused by the accumulation of the T_{maxc}. The fuzzy area is made of a number of very narrow coincidence pulses less than the detection circuit resolution in the form of Gauss distribution. That is to say, the very narrow coincidence pulses less than the detection resolution are also constantly accumulated in the process of the formation of group quantization. Their distribution is only random in the form of fuzzy area. The number of them is continuously increased until the formation of group quantization. Experimental results show that it is a variation law of periodic movement from quantitative changes to qualitative changes in precise measurement fields. The results of the qualitative changes with certain physical phenomenon which is periodically repeated finally return to a new starting point. If the width of fuzzy area is reduced by the periodic variation law of the quantized phase, the precision of measured quantity can be improved two, three or more orders of magnitude. The super-high resolution better than ps even 100 fs level is also obtained by further reducing the value of the T_{maxc}.

The influence of the errors caused by system hardware is mainly reflected in the volatility of group quantization phase Δt. By establishing the model of the Δt, the frequency of the measured signal can be easily obtained, as shown in Fig. 14.

Suppose f'_1 and f'_2 are two frequencies of phase comparison signals with a relative frequency difference Δf caused by system hardware or phase noise, and $f'_1 = f_1 + \Delta f, f'_2 = f_2$. The f_1 and f_2 are nominal frequencies of f'_1 and f'_2, and their periods are $T_1 = m_1 T_{maxc}$ and $T_2 = m_2 T_{maxc}$, respectively, where m_1 and m_2 are two positive integers without common factor, and $m_1 < m_2$. So the periods of two comparison signals are as follows.

$$T'_1 = \frac{1}{f'_1} = \frac{1}{f_1 + \Delta f} = m_1 T_{\max c} + \Delta T' \tag{11}$$

$$T'_2 = \frac{1}{f'_2} = m_2 T_{\max c}. \tag{12}$$

From Fig. 14, the Δt is a phase drift quantity generated by the Δf for the interval of T_{minc}. Hence the mathematical model of the Δt can be easily established by equation (7), equation (10), equation (11) and equation (12), which is followed by

$$\begin{aligned}
\Delta t = m_1 T'_2 - m_2 T'_1 &= \frac{m_1}{f'_2} - \frac{m_2}{f'_1} = \frac{m_1 f'_1 - m_2 f'_2}{f'_1 f'_2} \\
&= \frac{m_1 f_1 + m_1 \Delta f - m_2 f_2}{f'_1 f'_2} \left| \begin{aligned} T_1 &= 1/f_1 = m_1 T_{\max c} \\ T_2 &= 1/f_2 = m_2 T_{\max c} \\ f_{pq} &= 1/T_{\max c} \end{aligned} \right. \\
&= \frac{f_{pq} + m_1 \Delta f - f_{pq}}{f'_1 f'_2} \\
&= \frac{m_1 \Delta f}{f'_1 f'_2} \left| T_{gp} = \frac{1}{\Delta f} = \frac{m_1}{T_{gp} f'_1 f'_2}. \right.
\end{aligned} \tag{13}$$

Here, the T_{gp} is group period[29] between the frequencies signals f'_1 and f'_2, it reflects the state of phase coincidence between two comparison signals in frequency standard comparison and can be calculated by reverse cycle of Lissajous figure obtained by measurement on the oscilloscope.

From the equation (11), the $\Delta T'$ is the phase jitter of frequency signals f_1, it reflects a pollution level of the f_1 caused by phase noise, and can be precisely calculated by the following equation (14)

$$\Delta T' = \frac{1}{f_1 + \Delta f} - m_1 T_{\max c} \bigg| T_1 = \frac{1}{f_1} = \frac{T_1}{1 + T_1 \Delta f} - m_1 T_{\max c}$$

$$= \frac{T_1 - m_1 T_{\max c} - T_1 \Delta f m_1 T_{\max c}}{1 + T_1 \Delta f} \bigg| T_1 = m_1 T_{\max c}$$

$$= -\frac{(m_1 T_{\max c})^2 \, \Delta f}{1 + m_1 T_{\max c} \Delta f} \bigg| T_{gp} = \frac{1}{\Delta f} = -\frac{(m_1 T_{\max c})^2}{T_{gp} + m_1 T_{\max c}}.$$

$$(14)$$

The phase quantization error is also an important factor that affects the accuracy of measurement. It affects the measurement resolution $T_{\max c}$ of detection system and the minimum phase difference in a $T_{\min c}$, that determines the size of phase quantization error. The smaller the $T_{\max c}$, the higher the resolution of measurement and the smaller the phase quantization error.

In the practical application circuit based on group quantization phase processing, the precision of measurement is mainly determined by the measurement resolution and its stability, and the reliability of the system hardware. The resolution of measurement can be improved by possibly reducing the periods of two comparison signals and increasing the $T_{\min c}$. The stability of measurement resolution is mainly determined by the stability of frequencies relationships between two comparison signals that can be easily achieved by precise frequency linking method based phase group synchronization[32]. The variation law of the group phase differences can be therefore revealed by the relationships between the group quantization phase Δt and the measurement resolution $T_{\max c}$.

Similar to equation (8), for every positive pulse of f_2 in Fig. 14, all phase differences in the Nth $T_{\min c}$ are $\Delta p'_1$, $\Delta p'_2$, ..., $\Delta p'_{m1}$, where m_1 is a number of T_2 in a group, $\Delta p'_{m1}$ is phase difference of the phase coincidence. The phase comparison results can be calculated by equation (15)

$$
\begin{bmatrix} \Delta p'_1 \\ \Delta p'_2 \\ \Delta p'_3 \\ \vdots \\ \Delta p'_{m2} \end{bmatrix} =
\begin{bmatrix} n_1 T'_1 - T'_2 + (N-1)\Delta t \\ n_2 T'_1 - 2T'_2 + (N-1)\Delta t \\ n_3 T'_1 - 3T'_2 + (N-1)\Delta t \\ \vdots \\ m_2 T'_1 - m_1 T_2 + (N-1)\Delta t \end{bmatrix}
$$

$$
=
\begin{bmatrix} n_1 (m_1 T_{\max c} + \Delta T') - m_2 T_{\max c} + (N-1)\Delta t \\ n_2 (m_1 T_{\max c} + \Delta T') - 2m_2 T_{\max c} + (N-1)\Delta t \\ n_3 (m_1 T_{\max c} + \Delta T') - 3m_2 T_{\max c} + (N-1)\Delta t \\ \vdots \\ m_2 (m_1 T_{\max c} + \Delta T') - m_1 m_2 T_{\max c} + (N-1)\Delta t \end{bmatrix}
$$

$$
=
\begin{bmatrix} n_1 m_1 - m_2 \\ n_2 m_1 - 2m_2 \\ n_3 m_1 - 3m_2 \\ \vdots \\ m_2 m_1 - m_1 m_2 \end{bmatrix} T_{\max c} +
\begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ m_2 \end{bmatrix} \Delta T' + (N-1)\Delta t
$$

$$
=
\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \vdots \\ Y_{m2} \end{bmatrix} T_{\max c} +
\begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ m_2 \end{bmatrix} \Delta T' + (N-1)\Delta t
$$

$$(15)$$

where $Y_1, Y_2, Y_3, …, Y_{m2}$ are nonnegative integers. The $\Delta T'$ in the equation (15) is the result of phase jitter due to non-fixed phase relationships between two comparison signals. The $(N-1)\Delta t$ caused by system hardware and phase noise is the phase of group quantization. If the phase relationships between two comparison signals are fixed, that is to say, the phase comparison is generated between two nominal frequency signals, both $\Delta T'$ and $(N-1)\Delta t$ are equal to zero. The equation (15) will be converted into the equation (8). The measurement precision is mainly determined by the quantization error $T_{\max c}$ in the condition of the nominal frequency of two comparison signals. The phase locked-loop (PLL) technology from different frequency signals and the homologous noise canceling technology are therefore used in group quantization phase processing method.

It can be seen by equation (15) that if the Y_1, Y_2, Y_3, …, Y_{m2} are equal to 0, 1, 2, …, $m_2 - 1$ respectively, m_2 phase differences in the first $T_{\min c}$ can be obtained by the equation (16)

$$
\begin{bmatrix} \Delta p'_1 \\ \Delta p'_2 \\ \Delta p'_3 \\ \vdots \\ \Delta p'_{m2} \end{bmatrix} =
\begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ m_2 \end{bmatrix} \Delta T' + (N-1)\Delta t +
\begin{bmatrix} 0 \\ 1 \\ 2 \\ \vdots \\ m_2 - 1 \end{bmatrix} T_{\max c}.
$$

$$(16)$$

The equation (16) denotes that every phase difference in the Nth T_{minc} will vary periodically while the group quantization phase $(N-1)\Delta t$ is T_{maxc}, $2T_{maxc}$, $3T_{maxc}$, …, $(m_2-1)T_{maxc}$, T_2. While $(N-1)\Delta t = T_{maxc}$ in equation (16), m_2 phase differences in Nth T_{minc} are as follows

$$
\begin{bmatrix} \Delta p'_1 \\ \Delta p'_2 \\ \Delta p'_3 \\ \vdots \\ \Delta p'_{m2} \end{bmatrix} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ m_2 \end{bmatrix} \Delta T' + T_{\max c} + \begin{bmatrix} 0 \\ 1 \\ 2 \\ \vdots \\ m_2 - 1 \end{bmatrix} T_{\max c}
$$

$$
= \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ m_2 \end{bmatrix} \Delta T' + \begin{bmatrix} 1 \\ 2 \\ 3 \\ \vdots \\ 0 \end{bmatrix} T_{\max c}.
$$

(17)

While $(N-1)\Delta t = 2T_{maxc}$ in equation (16), m_2 phase differences in Nth T_{minc} can be obtained by the following equation

$$
\begin{bmatrix} \Delta p'_1 \\ \Delta p'_2 \\ \Delta p'_3 \\ \vdots \\ \Delta p'_{m2} \end{bmatrix} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ m_2 \end{bmatrix} \Delta T' + 2T_{\max c} + \begin{bmatrix} 0 \\ 1 \\ 2 \\ \vdots \\ m_2 - 1 \end{bmatrix} T_{\max c}
$$

$$
= \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ m_2 \end{bmatrix} \Delta T' + \begin{bmatrix} 2 \\ 3 \\ 4 \\ \vdots \\ 1 \end{bmatrix} T_{\max c}.
$$

(18)

According to the law above, while $(N-1)\Delta t = T_2$ in equation (16), m_2 phase differences in Nth T_{minc} can be just calculated by the equation (19)

$$
\begin{bmatrix} \Delta p'_1 \\ \Delta p'_2 \\ \Delta p'_3 \\ \vdots \\ \Delta p'_{m2} \end{bmatrix} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ m_2 \end{bmatrix} \Delta T' + (N-1)\Delta t + \begin{bmatrix} 0 \\ 1 \\ 2 \\ \vdots \\ m_2 - 1 \end{bmatrix} T_{\max c}.
$$

(19)

The equation (16) and the equation (18) is obviously the same. That is, while $(N-1)\Delta t = T_2$, m_2 phase differences in Nth T_{minc} have a variation of full cycle. The full cycle is called group period T_{gp} that is equal to the $(N-1)T_{minc}$.

Besides, the sensitivity of detection is also an important aspect that improves the performance index of system. It depends on the sensitivity of phase comparison in group quantization phase processing. The measurement precision of the system is determined by the sensitivity of phase comparison. The called sensitivity of phase comparison is an output voltage corresponding to the unit phase difference. Suppose $f_1 = 5\,MHz$ and $f_2 = 4\,MHz$ are two stable frequencies of phase comparison signals, $V_{PPS} = 10\,V$ is the peak to peak of output voltage of phase comparison with the same frequency, then the sensitivity of phase comparison can be calculated as follows

$$
T_1 = 1/f_1 = 200\,ns
$$

(20)

$$
T_2 = 1/f_2 = 250\,ns
$$

(21)

$$
T_{\max c} = (T_1, T_2) = (m_1 T_{\max c}, m_2 T_{\max c})
$$
$$
= (200, 250) = 50\,ns
$$

(22)

where $m_1 = 4$, $m_2 = 5$. In addition, we have

$$
T_{gp} = m_1 m_2 T_{\max c} = 4 \times 5 \times 50 = 1000\,ns
$$

(23)

$$
V'_{PPD} = \frac{V_{PPS} \cdot m_1 \cdot T_{\max c}}{T_{gp}} = \frac{10 \times 4 \times 50}{1000} = 2\,V
$$

(24)

where the V'_{PPD} is peak to peak of output voltage of phase comparison after the T_1 and the T_2 are converted into the T_{maxc}. If the generation of phase comparison with different frequency is direct, the V'_{PPD} is changed into the V_{PPD}.

Similar to the equation (24), the $V_{PPD} = 10\,V$ can be easily obtained by calculation.

From the above analysis, if the generation of phase comparison is direct, the sensitivity of phase comparison S'_D is as shown in equation (25)

$$S'_D = \frac{V_{PPD}}{T_{max\,c}} = \frac{10}{50} = 0.2\,V/ns. \tag{25}$$

If the generation of phase comparison is after the T_1 and the T_2 are converted into the T_{maxc}, the sensitivity of phase comparison S_D is as shown in equation (26)

$$S_D = \frac{V_{PPD}}{T_{gp}} = \frac{10}{1000} = 0.01\,V/ns. \tag{26}$$

Therefore,

$$\frac{S'_D}{S_D} = \frac{T_{gp}}{T_{max\,c}} = \frac{1000}{50} = 20. \tag{27}$$

It can be seen that the measurement precision with 20 times is easily improved in group quantization phase processing.

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Author Contributions

B.D. and D.F. analyzed the principle and proposed the project and wrote the paper. Y.T. and X.G. analyzed the experimental data. D.Z. and C.C. performed the experiments. M.W. and Z.Y. performed the whole system test.

Additional Information

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Corrigendum: High-Resolution Group Quantization Phase Processing Method in Radio Frequency Measurement Range

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The original version of this Article contained a typographical error in the spelling of the author Baoqiang Du which was incorrectly given as Baoqing Du. This has now been corrected in the PDF and HTML versions of the Article.