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Equivalent spin-orbit interaction in the two-polariton Jaynes-Cummings-Hubbard model

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A cavity quantum electrodynamics (cavity-QED) system combines two or more distinct quantum components, exhibiting features not seen in the individual systems. In this work, we study the one-dimensional Jaynes-Cummings-Hubbard model in the two-excitation (two-polariton) subspace. We find that the centre momentum of two-excitation induces a magnetic flux piercing the equivalent Hamiltonian H_k in the invariant subspace with momentum k , which can be described as a 4-leg ladder in the auxiliary space. Furthermore, it is shown that the system in π -centre-momentum subspace is equivalent to a lattice system for spin-1 particle with spin-orbit coupling. On the basis of this concise description, a series of bound-pair eigenstates which display long-range polaritonic entanglement is presented as a simple application.

A cavity quantum electrodynamics (cavity-QED) system combines two or more distinct quantum components, exhibiting features are not seen in the individual systems. Such a system offers a promising platform from which to study novel quantum phenomena. The Jaynes-Cummings-Hubbard (JCH) model is an archetype of such hybridization, which consists of the Jaynes-Cummings (JC) model and the coupled cavities. The JCH model was proposed to exploit the atom-light interaction in coupled microcavity arrays to create strongly correlated many-body models¹⁻⁵, though it has been studied in a variety of directions. In the context of quantum simulation, several good quantum simulators have been proposed that realize the JC model, a vital component of the JCH model, such as superconducting circuits (see the review⁶ and references therein).

Previous studies have mainly focused on the ground state phase of many-particle systems and the dynamics in a single-particle system. The Mott insulator phase and superfluid phase are identified by the traditional order parameter. For example, the average of the annihilation operator³ and observable quantities such as atomic concurrence and photon visibility⁷. Studies of single-particle dynamics suggests that this hybrid architecture can parallel a coherent quantum device to transfer and store quantum information as well as to create laser-like output⁸⁻¹⁰. Recently, the few-body problem for the JCH Hamiltonian has also been investigated^{11,12}, postulating the existence of two-polariton bound states when the photon-atom interaction is sufficiently strong.

In this work, we study the one-dimensional JCH model in the two-excitation (two-polariton) subspace. In each invariant subspace, the sub-Hamiltonian is equivalent to a 4-leg ladder with an effective flux which is proportional to the centre momentum of two excitations. It is shown that in π -centre-momentum subspace, the ladder system can be reduced to a lattice system of spin-1 particles with spin-orbit coupling. On the basis of this concise description, a series of bound-pair eigenstates, which display long-range polaritonic entanglement is presented as a simple application.

Results

JCH Model. The JCH model describes a cavity array doped with two-level atoms in which every cavity is embedded by a single two-level atom. In this model, the dipole interaction leads to complex

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dynamics involving photonic and atomic degrees of freedom, which is in contrast to the widely studied Bose-Hubbard model. Such a cavity-QED system can be implemented with a defect array in a photonic crystal^{13,14} or a Josephson junction array in a cavity^{8,15,16}. The Hamiltonian of a cavity-QED system, or indeed a lattice atom-photon system

$$H = H_{AP} + H_{JC} + H_C \tag{1}$$

can be written as three parts: free Hamiltonians of the atom and photon,

$$H_{AP} = \omega_a \sum_{l=1}^N a_l^\dagger a_l + \omega_b \sum_{l=1}^N |e\rangle_l \langle e|_l, \tag{2}$$

the JC-type cavity-atom interaction in the l -th defect,

$$H_{JC} = \lambda \sum_{l=1}^N (a_l^\dagger |g\rangle_l \langle e|_l + \text{H. c.}), \tag{3}$$

(with strength λ) and the photon hopping between nearest neighbour cavities,

$$H_C = -\kappa \sum_{l=1}^N (a_l^\dagger a_{l+1} + \text{H. c.}), \tag{4}$$

where κ is the hopping integral constant for the tunnelling between adjacent cavities. Here $|g\rangle_l$ ($|e\rangle_l$) denotes the ground (excited) state of the atom placed at the l -th cavity, and a_l^\dagger and a_l are the creation and annihilation operators of a photon at defect l . The rotating-wave approximation, which requires that $\omega_a + \omega_b \gg |\omega_a - \omega_b|$ and $\omega_a \gg \lambda$, is satisfied automatically in the JCH model. Obviously the total excitation number

$$\hat{N} = \sum_{l=1}^N \hat{N}_l = \sum_{l=1}^N \left(a_l^\dagger a_l + \frac{1}{2} \sigma_l^z + \frac{1}{2} \right), \tag{5}$$

is a conserved quantity for the Hamiltonian H , i.e. $[H, \hat{N}] = 0$, where $\sigma_l^z |e\rangle_l = |e\rangle_l$ and $\sigma_l^z |g\rangle_l = -|g\rangle_l$. Here the excitation refers to a combination of photonic and atomic excitations, termed as polaritons². Therefore \hat{N} is the excitation number of the polaritons. For each cavity, the basis state can be expressed as $\{|n\rangle_l |e\rangle_l, |n\rangle_l |g\rangle_l\}$, where the basis state of the Fock space for the l -th cavity is $|n\rangle_l = (a_l^\dagger)^n / \sqrt{n!} |0\rangle_l$. In this paper, we consider the invariant subspace with $\hat{N} = 2$, which is spanned by a basis in the form

$$\begin{aligned} |2\rangle_i &\equiv |2\rangle_i \langle 0|_i |G\rangle \\ |1\rangle_i |1\rangle_{i+j} &\equiv |1\rangle_i |1\rangle_{i+j} \langle 0|_{i+j} \langle 0|_i |G\rangle \end{aligned} \tag{6}$$

$$\begin{aligned} |e\rangle_i |e\rangle_{i+j} &\equiv |e\rangle_i |e\rangle_{i+j} \langle g|_{i+j} \langle g|_i |G\rangle \\ |e\rangle_i |1\rangle_{i'} &\equiv |e\rangle_i |1\rangle_{i'} \langle 0|_{i'} \langle g|_i |G\rangle \\ i' &= i \text{ or } i + j \end{aligned} \tag{7}$$

where $j \geq 1$ and $|G\rangle \equiv \prod_{i=1}^N |g\rangle_i |0\rangle_i$ denotes the empty state with zero \hat{N} . We denote the matrix representation of the Hamiltonian of Eq. (1) in the basis of Eq. (6) as \underline{H} . In the case of real values of κ and λ , we have $\underline{H}^* = \underline{H}$, which indicates that \underline{H} has time-reversal symmetry.

4-Leg Ladder with flux. The system is translational invariant¹⁷. In the two-particle Hilbert space, the Hamiltonian H can be written as $H = \sum_k H_k$ with periodic boundary conditions, where

$$\begin{aligned} H_k &= \sum_{j=1}^4 \sum_{m=1}^4 (J_m |j, m, k\rangle \langle j+1, m, k| + \lambda |j, m, k\rangle \langle j, m+1, k| \\ &+ \text{H. c.}) + \sum_{j=1}^4 \sum_{m=1}^4 (\mu_m |j, m, k\rangle \langle j, m, k|) + h_k, \end{aligned} \tag{8}$$

and

$$\begin{aligned}
 h_k = & \sum_{j=0, m=1,3} J_m |j, m, k\rangle \langle j+1, m, k| \\
 & + \sqrt{2} \lambda |0, 1, k\rangle \langle 0, 2, k| + \sqrt{2} J_2 |0, 2, k\rangle \langle 1, 2, k| \\
 & + \sum_{j=0, m=1,2} \mu_m |j, m, k\rangle \langle j, m, k| + \text{H. c.}
 \end{aligned} \tag{9}$$

Here the set of states $\{|j, m, k\rangle\}$ is defined as following: For $j \geq 1$, it reads

$$\begin{pmatrix} |j, 1, k\rangle \\ |j, 2, k\rangle \\ |j, 3, k\rangle \\ |j, 4, k\rangle \end{pmatrix} = \sum_l \frac{e^{ik(l+j/2)}}{\sqrt{N}} \begin{pmatrix} |1\rangle_l |e\rangle_{l+j} \\ |1\rangle_l |1\rangle_{l+j} \\ |e\rangle_l |1\rangle_{l+j} \\ |e\rangle_l |e\rangle_{l+j} \end{pmatrix}, \tag{10}$$

and

$$\begin{pmatrix} |0, 1, k\rangle \\ |0, 2, k\rangle \end{pmatrix} = \sum_l \frac{e^{ikl}}{\sqrt{N}} \begin{pmatrix} |1\rangle_l |e\rangle_l \\ |2\rangle_l \end{pmatrix}, \tag{11}$$

where we have taken $|j, 5, k\rangle \equiv |j, 1, k\rangle$ for $j \geq 1$, and $|0, 1, k\rangle \equiv |0, 3, k\rangle$. The parameters read

$$J_{1,2,3,4} = (-\kappa e^{ik/2}, -2\kappa \cos(k/2), -\kappa e^{-ik/2}, 0), \tag{12}$$

and

$$\mu_{1,2,3,4} = (\omega_a + \omega_b, 2\omega_a, \omega_a + \omega_b, 2\omega_b). \tag{13}$$

The expression of H_k in Eq. (8) has a clear physical meaning: $|j, m, k\rangle$ denotes the site state for the j -th site on the m -leg of a 4-leg ladder system with the effective magnetic flux piercing the plaquette. The flux is proportional to the centre momentum of two excitations. The structure of H_k is schematically illustrated in Fig. 1. We note that the matrix representation of H_k in the basis of Eqs. (10) and (11), H_k , breaks the time-reversal symmetry. Nevertheless, we still have $\sum_k H_k = \sum_k H_k^*$, as $H_k^* = H_{-k} = H_{4\pi-k}$. In essence, the nonzero plaquette flux arises from the relationship between the complex coupling constants $J_1 = J_3^* = -\kappa e^{ik/2}$. In contrast, one can see from H_k that the complex λ cannot induce a nonzero plaquette flux. We would like to stress that the effective magnetic field in the present model is intrinsic, not depending on an external control, but relying on the value of k . We note that there are two kinds of excitations, the spin-up (excited atom) state and a photon, which obey two different statistics (that for hardcore bosons and bosons). This may be the origin of the equivalent plaquette flux. Then, the underlying mechanisms for obtaining the equivalent plaquette flux in our work and that of Ref. 18, 19 are different.

In order to understand the mechanism of the effective flux, we investigate the exchange process for photon and atomic excitations beginning in the state $|\psi_{\pm}(l, l+j)\rangle$, passing through $|\psi_{\pm}(l+j, l)\rangle$, returning back to $|\psi_{\pm}(l, l+j)\rangle$, where

$$|\psi_{\pm}(l, l')\rangle = (|1\rangle_l |e\rangle_{l'} \pm |1\rangle_{l+1} |e\rangle_{l'+1}) / \sqrt{2} \tag{14}$$

are states in a different invariant subspace. The action of H provides a loop for this task:

$$\begin{aligned}
 |\psi_{\pm}(l, l+j)\rangle & \rightarrow (|1\rangle_l |1\rangle_{l+j} \pm |1\rangle_{l+1} |1\rangle_{l+j+1}) / \sqrt{2} \\
 & \rightarrow |\psi_{\pm}(l+j, l)\rangle \\
 & \rightarrow e^{i\pi(1\mp 1)/2} (|1\rangle_l |1\rangle_{l+j} \pm |1\rangle_{l+1} |1\rangle_{l+j+1}) / \sqrt{2} \\
 & \rightarrow e^{i\pi(1\mp 1)/2} (|1\rangle_l |1\rangle_{l+j+1} \pm |1\rangle_{l+1} |1\rangle_{l+j}) / \sqrt{2} \\
 & \rightarrow e^{i\pi(1\mp 1)/2} (|1\rangle_l |e\rangle_{l+j+1} \pm |1\rangle_{l+1} |e\rangle_{l+j}) / \sqrt{2} \\
 & \rightarrow e^{i\pi(1\mp 1)/2} |\psi_{\pm}(l, l+j)\rangle,
 \end{aligned} \tag{15}$$

which correspond to a ring network with six vortices. It shows that this round trip acquires a phase 0 or π , which is equivalent to the effect of a flux piercing the loop. We note that the flux depends on the sign \pm in each of the states. On the other hand, the sign \pm in state $|\psi_{\pm}(l, l')\rangle$ indicates the symmetry or

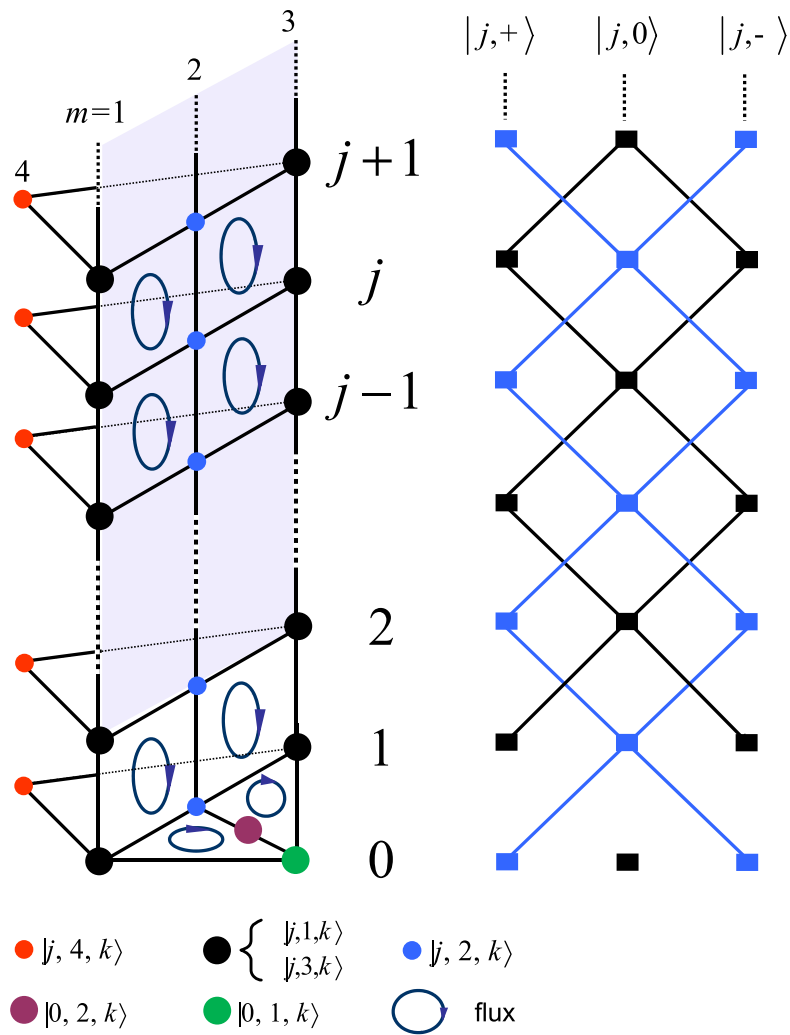


Figure 1. Schematic of the structures of equivalent Hamiltonians for the one-dimensional JCH model with two polaritons. (a) In the invariant subspace with centre momentum k , the equivalent Hamiltonian H_k describes a 4-leg ladder with k -dependent flux. The shadow indicates the semi-infinite uniform ladder. (b) For $k = \pi$, H_k is equivalent to a spin-1 chain with spin-orbit interaction. The graph of H_π consists of two unconnected subgraphs, characterized by the parity $\Pi = \pm 1$. H_k indicates that H_π can be further decomposed into two independent parts H_o (dark) and H_e (blue).

antisymmetry of the state under the transformation $|1\rangle_l |e\rangle_{l'} \rightleftharpoons |1\rangle_{l+1} |e\rangle_{l'+1}$. This investigation implies that the origin of the effective magnetic field may be the special statistical properties of two quasi-particles in each invariant subspace.

Equivalent Hamiltonian in π -momentum subspace. We focus on the case $k = \pi$ and $\omega_a = \omega_b$, which leads to $H_{AP} = \omega_a \hat{\mathcal{N}}$. It is a simple but non-trivial case, since the hopping along leg 2 is switched off but the plaquette flux still exists. We note that the on-site potentials μ_l of different legs are identical, which allows us to ignore the diagonal terms in H_π .

Introducing the three-dimensional vector bra and ket for

$$\begin{aligned}
 |j\rangle &= (|j, +\rangle, |j, 0\rangle, |j, -\rangle), \\
 \langle j| &= \begin{pmatrix} \langle j, +| \\ \langle j, 0| \\ \langle j, -| \end{pmatrix},
 \end{aligned} \tag{16}$$

the Hamiltonian H_π in the π -momentum subspace can be expressed as

$$H_\pi = H_{\text{SO}} + 0 \sum_{j=1} |\psi_j\rangle\langle\psi_j|, \quad (17)$$

with $[H_{\text{SO}}, \sum_{j=1} |\psi_j\rangle\langle\psi_j|] = 0$, which indicates that H_π is block-diagonal.

Here $|j, S_z\rangle$ represents a spin-1 particle at the j -th site with spin polarization $S_z = 0, \pm 1$ defined as

$$\begin{cases} |j, \pm\rangle = (|j, 1, \pi\rangle + |j, 3, \pi\rangle \\ \quad \pm |j, 2, \pi\rangle \pm |j, 4, \pi\rangle)/2, \\ |j, 0\rangle = (|j, 1, \pi\rangle - |j, 3, \pi\rangle)/\sqrt{2}, \end{cases} \quad (18)$$

for $j \geq 1$, and

$$|0, \pm\rangle = (|0, 1, \pi\rangle \pm |0, 2, \pi\rangle)/\sqrt{2}. \quad (19)$$

In addition, the state $|\psi_j\rangle$ ($j \geq 1$) is defined as

$$\begin{aligned} |\psi_j\rangle &= \frac{1}{\sqrt{2}} (|j, 2, \pi\rangle - |j, 4, \pi\rangle) \\ &= \sum_l \frac{e^{iml}}{\sqrt{2N}} (|1\rangle_l |1\rangle_{l+j} - |e\rangle_l |e\rangle_{l+j}), \end{aligned} \quad (20)$$

which, together with states $\{|j, \pm\rangle, |j, 0\rangle\}$, constructs the complete orthogonal set. Of particular interest, $|\psi_j\rangle$ is the eigenstate of H with energy $2\omega_a$. In Eq. (17), the zero-energy term represents this point, where we have ignored a constant shift $2\omega_a$.

The sub-Hamiltonian H_{so} is in the form

$$\begin{aligned} H_{\text{SO}} &= \sqrt{2} i\kappa |0\rangle S_x (1 - S_z^2) \langle 1| \\ &\quad + i\kappa \sum_{j=1} |j\rangle S_x \langle j+1| + \text{H. c.} \\ &\quad + \sqrt{2} \lambda |0\rangle S_z \langle 0| + 2\lambda \sum_{j=1} |j\rangle S_z \langle j|, \end{aligned} \quad (21)$$

where the Pauli spin matrices for a spin-1 particle are given by

$$S_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (22)$$

Consequently, within a specific invariant subspace, a system made of N -cavity array with a single two-level atom embedded in each cavity appears to be equivalent to a tight-binding chain of spin-1 particle with spin-orbit interaction. The structure of H_{so} is schematically illustrated in Fig. 1. Intuitively, the graph of H_{so} consists of two unconnected subgraphs. This can be clarified by observing that the parity operator

$$\hat{\Pi} = (-1)^{j+S_z+1} \quad (23)$$

where $\hat{\Pi}|j, S_z\rangle = \Pi|j, S_z\rangle$ and $\Pi = \pm 1$ characterize the two subgraphs.

We can thus conclude that the equivalent Hamiltonian H_{so} can be decomposed into two independent parts

$$H_{\text{SO}} = H_o + H_e, \quad (24)$$

with $[H_o, H_e] = 0$, and $[\hat{\Pi}, H_e] = [\hat{\Pi}, H_o] = 0$. The sub-Hamiltonians are defined as

$$\begin{aligned} H_o &= i\kappa \sum_{j=1,3,5,\dots} |j\rangle S_x (1 - S_z^2) \langle j+1| \\ &\quad + i\kappa \sum_{j=2,4,6,\dots} |j\rangle S_x S_z^2 \langle j+1| + \text{H. c.} \\ &\quad + 2\lambda \sum_{j=1,3,\dots} |j\rangle S_z \langle j|, \end{aligned} \quad (25)$$

and

$$\begin{aligned}
 H_e = & \sqrt{2} i\kappa |0\rangle S_x (1 - S_z^2) \langle 1| \\
 & + i\kappa \sum_{j=2,4,6,\dots} |j\rangle S_x (1 - S_z^2) \langle j+1| \\
 & + i\kappa \sum_{j=1,3,5,\dots} |j\rangle S_x S_z^2 \langle j+1| + \text{H.c.} \\
 & + \sqrt{2} \lambda |0\rangle S_z \langle 0| + 2\lambda \sum_{j=2,4,6,\dots} |j\rangle S_z \langle j|.
 \end{aligned}
 \tag{26}$$

The subscripts o and e represent the contributions associated with the sites of odd and even parity Π . The structures of H_o and H_e are schematically illustrated in Fig. 1. This figure indicates that the invariant space with $k=\pi$ is split in two unconnected subspaces. This allows us to investigate the Hamiltonians $H_{o,e}$ separately.

Exact bound-pair states. Based on the above analysis, besides states $|\psi_j\rangle$, one can also construct a series of bound-pair states of the form

$$\begin{aligned}
 |\varphi_j\rangle = & \frac{1}{\sqrt{\Omega_j}} [a_j(|j, +\rangle - |j, -\rangle) \\
 & + i2\sqrt{2}(\lambda/\kappa)|j+1, 0\rangle \\
 & - (|j+2, +\rangle - |j+2, -\rangle)],
 \end{aligned}
 \tag{27}$$

where the normalization factor Ω_j and amplitudes a_j are given as

$$\Omega_j = 2(a_j)^2 + 8(\lambda/\kappa)^2 + 2,
 \tag{28}$$

and

$$a_j = \begin{cases} 2, & j = 0 \\ 1 & j \geq 1 \end{cases}.
 \tag{29}$$

A straightforward derivation shows that

$$\begin{aligned}
 H_e |\varphi_j\rangle &= 0, \text{ even } j \\
 H_o |\varphi_j\rangle &= 0, \text{ odd } j
 \end{aligned}
 \tag{30}$$

i.e. $|\varphi_j\rangle$ is an eigenstate of H_{s_o} . This is a direct application of the bound state theorem given in²⁰, which states that any eigenstate of a sub-graph is also an eigenstate of the whole, if the nodes cover all the joint points. We are interested in the expression of these states in the atom-photon basis, given by

$$\begin{aligned}
 |\varphi_j\rangle = & \sum_l \frac{(-1)^l}{\sqrt{N\Omega_j}} \left[a_j (|1\rangle_l |1\rangle_{l+j} + |e\rangle_l |e\rangle_{l+j}) \right. \\
 & - 2(\lambda/\kappa) (|1\rangle_l |e\rangle_{l+j+1} - |e\rangle_l |1\rangle_{l+j+1}) \\
 & \left. + (|1\rangle_l |1\rangle_{l+j+2} + |e\rangle_l |e\rangle_{l+j+2}) \right]
 \end{aligned}
 \tag{31}$$

for $j \geq 1$, and

$$\begin{aligned}
 |\varphi_0\rangle = & \sum_l \frac{(-1)^l}{\sqrt{N\Omega_0}} [a_0 \sqrt{2} |2\rangle_l \\
 & - 2(\lambda/\kappa) (|1\rangle_l |e\rangle_{l+1} - |e\rangle_l |1\rangle_{l+1}) \\
 & + (|1\rangle_l |1\rangle_{l+2} + |e\rangle_l |e\rangle_{l+2})].
 \end{aligned}
 \tag{32}$$

Alternatively, a direct derivation can check our conclusion for the original Hamiltonian of a lattice atom-photon system in Eq. (1) that

$$H |\varphi_j\rangle = 2\omega_a |\varphi_j\rangle.
 \tag{33}$$

The formation mechanism of these bound-pair eigenstates can be understood as the result of quantum interference, which is presented in the Methods section.

Long-range polariton-polariton entanglement. We now study the features of the obtained eigenstates. It is apparent that the pair states $|\psi_j\rangle$ and $|\varphi_j\rangle$ are entangled states. In the strong coupling limit $\lambda \gg \kappa$, we have

$$|\varphi_j\rangle \approx \sum_l \frac{(-1)^l}{\sqrt{2N}} \left(|1\rangle_l |e\rangle_{l+j+1} - |e\rangle_l |1\rangle_{l+j+1} \right), \quad (34)$$

which is the superposition of entangled states between two cavities at distance $j+1$. States

$$\left(|1\rangle_l |e\rangle_{l+j+1} - |e\rangle_l |1\rangle_{l+j+1} \right) / \sqrt{2} \quad (35)$$

in $|\varphi_j\rangle$ and

$$\left(|e\rangle_l |e\rangle_{l+j} - |1\rangle_l |1\rangle_{l+j} \right) / \sqrt{2} \quad (36)$$

in $|\psi_j\rangle$ are both maximally entangled states of the l -th and $(l+j)$ -th (or $(l+j+1)$ -th) cavities for the two modes: excited cavity fields and excited atom modes. To demonstrate this concept in a precise manner, we introduce lower branch and upper branch exciton-polariton states,

$$|\downarrow\rangle_l = \frac{1}{\sqrt{2}} \left(i|1\rangle_l - |e\rangle_l \right), \quad (37)$$

$$|\uparrow\rangle_l = \frac{1}{\sqrt{2}} \left(i|1\rangle_l + |e\rangle_l \right), \quad (38)$$

the superposition of which yields a polariton qubit state at cavity l . As $|\downarrow\rangle_l$ and $|\uparrow\rangle_l$ are a basis, it is given that

$$|\varphi_j\rangle \sim \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_l |\uparrow\rangle_{l+j+1} - |\downarrow\rangle_l |\downarrow\rangle_{l+j+1} \right), \quad (39)$$

$$|\psi_j\rangle \sim \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_l |\uparrow\rangle_{l+j} + |\downarrow\rangle_l |\downarrow\rangle_{l+j} \right), \quad (40)$$

which are standard Bell states. We see that the entanglement does not decrease as the distance j increases. This entanglement is one of great importance in the new field of quantum information theory. Polaritons²¹, as quasiparticles of light and matter, are the most promising solution for the interface between electronic and photonic qubit states. However, there is another form of entanglement: the atomic entanglement, which is based on the atomic qubit, referring to ground and excited atomic states. This is different from the entanglement discussed above, two atoms with state $|\psi_j\rangle$ (or $|\varphi_j\rangle$) in the l -th and $(l+j)$ -th (or $(l+j+1)$ -th) cavities do not entangle with each other, which means that atomic entanglement does not exist in our model. A demonstration of this will be given in the Methods section.

Discussion

In summary, we have established the link between the two-excitation JCH model and the single-particle 4-leg ladder with an effective flux, which has proven to be equivalent to a chain system of spin-1 particle with spin-orbit coupling. This study also introduces a mechanism to construct a series of bound-pair eigenstates which display long-range polaritonic entanglement. This finding reveals that cavity-QED systems can offer rich features and useful functionality, which will motivate further investigation.

However, it is a great challenge to realize the predictions in experiment. Although the obtained results do not require a special range of system parameters, several issues should be concerned for an experimental realization of our findings. Firstly, JCH model is obtained under the rotating wave, single-mode, and narrow band approximations. Then, parameters in a real system should be in the range to meet the condition of such approximations. Secondly, one needs a scheme for the preparation of the bound states. Below there is a possible way based on the fact that the proposed eigenstates have the same energy $2\omega_a$, which is identical to the eigen energy of an uncoupled system, i.e., $\kappa = \lambda = 0$. (i) Generating extended state of two photons for the system with $\lambda = 0$ but $\kappa \neq 0$. This is relatively accessible since all the eigenstates of such a system are extended states. (ii) Taking $\kappa \rightarrow 0$. Then all two-photon extended states have energy $2\omega_a$. (iii) Switching on and increasing λ and κ slowly. The initial state should evolve to the states

with excited atomic state. There should be a large probability for the transition between states with the same energy, including our target states. This scheme requires a temporal control of parameters λ and κ in experiment. At this stage, this is just a qualitative analysis, but will motivate further quantitative investigation for the procedure of entangled state preparation in a cavity-QED system.

Methods

Construction of bound-pair eigenstates. The formation mechanism of these bound-pair eigenstates can be understood as the result of quantum interference in the following three process.

(i) We start with the case of switching off the JC interaction such that $\lambda = 0$. The atoms are decoupled from the cavity array. It is uncomplicated to check that

$$\left[\eta_j, H - \omega_a \sum_l a_l^\dagger a_l \right] = 0, \quad (41)$$

where the operator η_j is defined as

$$\eta_j = \sum_l (-1)^l a_l^\dagger a_{l+j}. \quad (42)$$

According to a similar analysis in Ref. 22, it is found that state

$$|\Psi_n\rangle = (\eta_j)^n |G\rangle \quad (43)$$

is an eigenstate of H ,

$$H|\Psi_n\rangle = 2n\omega_a |\Psi_n\rangle. \quad (44)$$

Furthermore, it is worthy to note that even for a Bose Hubbard model, which involves the on-site interaction

$$H_{\text{BH}} = -\kappa \sum_{l=1}^N (a_l^\dagger a_{l+1} + \text{H.c.}) + \frac{U}{2} a_l^\dagger a_l (a_l^\dagger a_l - 1), \quad (45)$$

we still have

$$[\eta_j, H_{\text{BH}}] |G\rangle = 0, \quad (46)$$

which leads to the conclusion that $|\Psi_n\rangle$ is an eigenstate of H_{BH} .

The essence of the construction of $|\Psi_n\rangle$ is due to the destructive interference between the two transitions from states $|1\rangle_l |1\rangle_{l+j}$ and $|1\rangle_{l+1} |1\rangle_{l+j+1}$

$$\left. \begin{array}{l} H|1\rangle_l |1\rangle_{l+j} \\ H|1\rangle_{l+1} |1\rangle_{l+j+1} \end{array} \right\} \rightarrow |1\rangle_{l+1} |1\rangle_{l+j} + |1\rangle_l |1\rangle_{l+j+1}, \quad (47)$$

which results in

$$H \left(|1\rangle_l |1\rangle_{l+j} - |1\rangle_{l+1} |1\rangle_{l+j+1} \right) \rightarrow 0. \quad (48)$$

Here the contribution of H_0 is ignored. We refer to this as a Hubbard-type process.

(ii) Now we consider the case of switching off the tunnelling between cavities, $\kappa = 0$. Each cavity becomes separated from its neighbours. We have the identity

$$\left. \begin{array}{l} H|e\rangle_l |n-1\rangle_l |n\rangle_{l+j} \\ H|n\rangle_l |e\rangle_{l+j} |n-1\rangle_{l+j} \end{array} \right\} = \lambda n |n\rangle_l |n\rangle_{l+j}, \quad (49)$$

which results in

$$H \left(|e\rangle_l |n-1\rangle_l |n\rangle_{l+j} - |n\rangle_l |e\rangle_{l+j} |n-1\rangle_{l+j} \right) = 0. \quad (50)$$

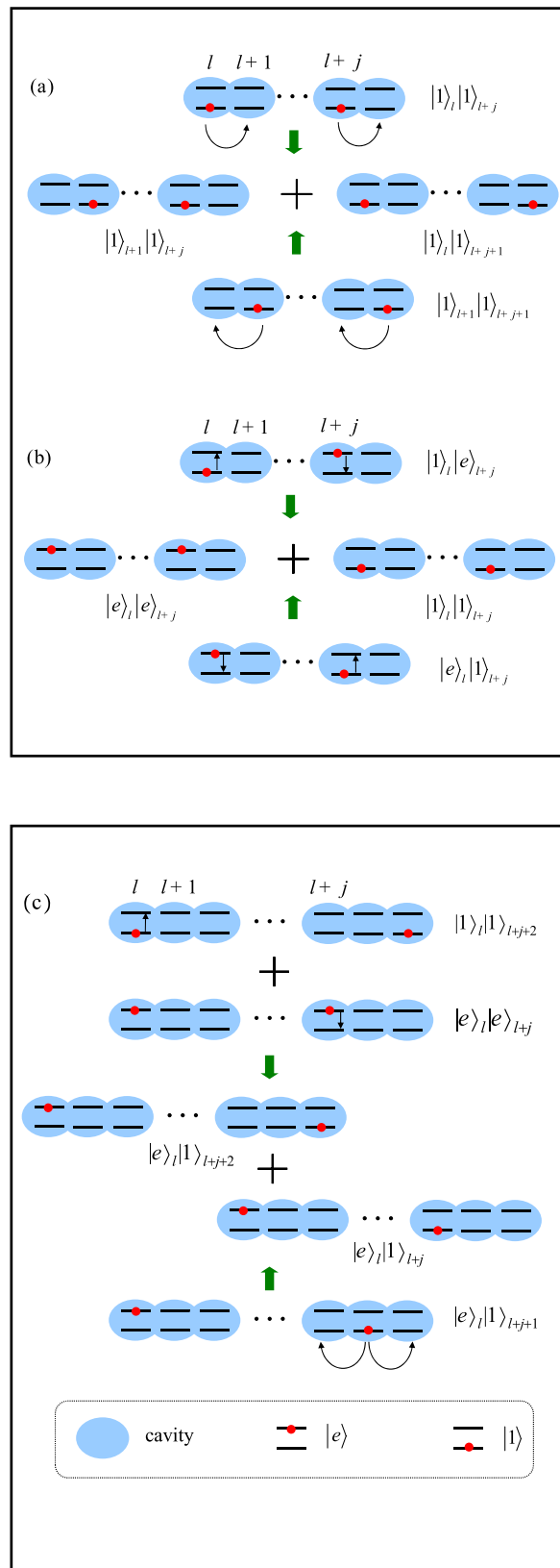


Figure 2. Schematic illustration for the mechanism of the formation of bound pair eigenstates. There are three types of destructive interference processes which result in the exact eigenstate $|\varphi_j\rangle$. (a) The Hubbard-type process represented in Eq. (47). (b) The JC-type process represented in Eq. (49). (c) The key process referred to as mixed-type in Eq. (51) shows that the cancellation of the transitions requires an optimal ratio between the parameters λ and κ .

This means that there is destructive interference between the two paths, which are the atom-photon transitions in the two different cavities l and $l+j$. It is a pure QED process in a JC model, which is referred as a JC-type process. It is easy to check that the combination of Hubbard and JC-type processes results in the formation of the eigenstate $|\psi_j\rangle$.

(iii) The crucial process that makes state $|\varphi_j\rangle$ become an eigenstate of the complete Hamiltonian is a combination of the above two processes (i) and (ii). In this case, the excitation number must be 2. The transitions which result in destructive interference are

$$\left. \begin{array}{l} (1/\lambda)(|e\rangle_l|e\rangle_{l+j} + |1\rangle_l|1\rangle_{l+j+2}) \\ (-1/\kappa)|e\rangle_l|1\rangle_{l+j+1} \end{array} \right\} \rightarrow (|e\rangle_l|1\rangle_{l+j} + |e\rangle_l|1\rangle_{l+j+2}). \quad (51)$$

We find that the cancellation occurs only if the amplitudes of the two components $|e\rangle_l|e\rangle_{l+j} + |1\rangle_l|1\rangle_{l+j+2}$ and $|e\rangle_l|1\rangle_{l+j+1}$ are properly assigned. We refer to this as the mixed-type process. In Fig. 2, three the processes for the formation mechanism of the bound pair state are schematically illustrated.

Atomic entanglement. The atomic entanglement can be characterized by concurrence⁷. The reduced density matrix for two atoms in the l -th and $(l+j+1)$ -th cavities is

$$\rho^{(l,l+j+1)} = \text{Tr}_p \text{Tr}_{(l,l+j+1)} \left(|\varphi_j\rangle\langle\varphi_j| \right), \quad (52)$$

where Tr_p denotes the trace over all photon variables and $\text{Tr}_{(l,l+j+1)}$ denotes the trace over all atomic variables except for the l -th and $(l+j+1)$ -th atoms. It has been shown in Ref. 7 that the formula for the concurrence of two quasi-spin particles in a hybrid system is the same as that for a pure spin-1/2 system^{23–25}. Then, the concurrence $C_{ll'}$ shared between two atoms l and l' is obtained as

$$C_{ll'} = 2 \max(0, |z_{ll'}| - \sqrt{u_{ll'}^+ u_{ll'}^-}). \quad (53)$$

in terms of the quantum correlations

$$z_{ll'} = \langle\varphi_j| \sigma_l^+ \sigma_{l'}^- |\varphi_j\rangle, \quad (54)$$

$$u_{ll'}^\pm = \frac{1}{4} \langle\varphi_j| (1 \pm \sigma_l^z)(1 \pm \sigma_{l'}^z) |\varphi_j\rangle, \quad (55)$$

where $\sigma_l^+ = (\sigma_l^-)^\dagger = |e\rangle_l \langle g_l|$. It is a straightforward calculation to show that $C_{ll'}$ is always zero for both states $|\varphi_j\rangle$ and $|\psi_j\rangle$.

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Author Contributions

C.L. did the derivations and edited the manuscript, X.Z.Z. revised the manuscript, Z.S. conceived the project and drafted the manuscript. All authors reviewed the manuscript.

Additional Information

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