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The golden section in the art of ancient Greece: an anthropometric study of the young warrior of Riace

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More than half a century after their discovery in the Ionian Sea near Riace Marina (Italy, 1972 CE), the Riace Bronzes continue to fascinate and interest. They are a pair of statues most likely sculpted in Greece in the mid-5th century and are considered masterpieces of Classical Greek sculpture. Many studies have been conducted, yet there are no morphometric studies that delve into their facial features. The aim of this study is to determine dentofacial characteristics and the presence of golden ratios in the so-called "young-man" warrior statue, equipped with a silver foil to represent the upper teeth. The interpupillar, interalar, intercanthal distances and mesio-distal diameter of the upper central incisors were subjected to virtual photogrammetry (GIMP-GNU Image Manipulation Programme). The presence of vertical and horizontal golden ratios between different landmarks (intercanthal distance: centre of the nose; centre of pupils: base of nose: base of upper incisors; centre of pupil-base of nose: base of upper incisors-base of chin; centre of pupils- base of upper incisors: base of the chin) were analysed using a dedicated software (PhiMatrix). The face and teeth satisfy most of golden canons, as also corroborated by the one-way ANOVA-statistic test ($p < 0.05$) for repeated measurements by independent expert operators. The virtual golden facial grid should confirm an ovoid face, which should match the same dental shape. The statue design stems from the search for beauty as divine proportion, and a relatively small detail such as teeth seems to have been carefully programmed.

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Introduction

The Riace Bronzes, also known as the Riace Warriors, currently housed in the National Archaeological Museum of Reggio Calabria, were retrieved from the Ionian Sea off Riace Marina (near Reggio Calabria) in August 1972. They are a pair of bronze statues most likely sculpted in Greece in the mid-5th century BCE (Gullì and Partenope, 1997). They have been dated between 460 and 450 BCE. The sculptor(s) are not known, even if many are somewhat inclined to attribute the figures to the famous Greek sculptors Phidias or Myron (Dontas, 1988). In 1998 and later in 2006, analysis of the materials making up the statues suggested that their origins were in Attica or Argos (Schenal Pileggi, 2014). They probably were lost at sea between the 1st to 2nd century BCE when the ancient Romans looted large quantities of Greek artwork and transported them to present-day Italy (Rolley, 1984; Micheli and Vidale, 2013). The Riace bronzes are made of cast bronze, the favourite metal of Greek artists. No more than 12 ancient Greek bronze statues have survived because the bronze was always in demand for reuse in later periods (Boardman, 1985). The most common bronze statue production used the lost wax technique. The two nude male figures, slightly larger than life size, represent two warriors. According to the principles of Greek art, heroes and athletes should necessarily be represented without clothing, to differentiate from everyday life (Dafas, 2019).

The debate centres on their origins and identity. Fifty years after their discovery, they have been defined as kings, warriors, priests, strategists, heroes, founders of ancient cities, or athletes of the first Olympics (Moreno, 1998; Hoffman and Konstam, 2002). A more recent theory identifies them in the brothers Eteocles and Polynice and wants them to be part of a complex group of five elements, which reconstructs the final duel between the two for the throne of Thebes. Experts believe that together with the two warriors was their mother Euryganeia, with her arms outstretched and desperate as she tries to dissuade her sons from the duel, and between them Antigone and the soothsayer Tiresias (Castrizio, 2019). The position of the body and the expression of the statue's face seem to convey the moment before a fight, in which the statue A (the one who has been labelled "the young man" for his physical characteristics) continues with a threatening attitude toward statue B (dubbed "old man") and a grin on his face that reveals a precious and unique detail of its kind: silver foil teeth (Sabbione, 1984; Moreno, 1998; Castrizio, 2019; Dafas, 2019). This comprises the central upper incisors, a small portion of the lateral incisors, and of one of the upper canines. Although the teeth are not completely exposed, they are sufficiently visible to evaluate some dimensional characters useful for further analysis. In the official literature dedicated to the Riace Bronzes, there are no studies that delve into the rationale according to which the face of the statues and the teeth in the case of statue A, were designed in the proportions used. However, at the time of the estimated origin of the statues, the golden proportion principles which would guide classical art and concepts of aesthetic balance of the modern age (still current today), were known and used (Della Monica et al., 2013; Papagrigorakis et al., 2011; Mantelakis et al., 2018; Thapa and Thapa, 2018; Kaya et al., 2019; Mamidi et al., 2020) (Fig. 1). The derived golden ratio establishes the value of 1:1.6181, considered the ideal and the most widely used proportion. The Golden Proportion principles also apply to the face and teeth. The ideal ratio between the length and width of the upper incisors is 1 to 0.8 (Liao et al., 2019). The factors that most affect smile aesthetics are the relationship of upper incisors in terms of respective length and width and the ratio of visible width of central incisors to lateral incisors and canines (Koidou et al., 2018). This golden aesthetic proportion plays an important role in dentistry, as well as in figurative arts. For a harmonious smile, the teeth must be proportional to each other but they must also be proportional to the face. Both the size and the shape of the teeth affect a good and balanced aesthetic appearance in relation to the face (Aldegheishem et al., 2019; Swelem and Al-Rafah, 2019; Putrino et al., 2021; Wang et al., 2021). This is the reason why it is important to know the mean widths and heights of the crowns of anterior teeth to achieve aesthetically pleasant results during restorations (in conservative, prosthetic, and orthodontic treatments). The shape of teeth, in frontal view, can be classified as square, triangular (or tapered), and oval (Hussain et al., 2016; Mahn et al., 2018). In most cases there is at least some degree of similarity between dental and facial morphology (ever classified as square, triangular, or oval shaped), as well as a great deal of similarity in terms of shape and size among teeth present in the same mouth and dental arch (Pedrosa et al., 2011; Mehndiratta et al., 2019). The symmetry is involved in these evaluations. It should be observed between teeth, with respect to midline, and in the natural curvature of the teeth, with respect to the smile line (Cesario and Latta, 1984; La Vere et al., 1992). Furthermore, dental and facial morphodimensional relations arise from gender and ethnic traits, although the value of the latter relationship is now more downsized, which confirms the role of a dental-facial genetic pattern in the individual appearance (Cesario and Latta, 1984; La Vere et al., 1992; Papagrigorakis et al., 2011; Pedrosa et al., 2011; Jin et al., 2016; Nguyen et al., 2016; Al-Kaisy and Garib, 2018; Koidou et al., 2018; Mantelakis et al., 2018; Aldegheishem et al., 2019; Mehndiratta et al., 2019). The most widely used anthropometric approach in clinical and scientific fields that allows us to evaluate dentofacial relationships and also the possible presence of golden ratios is traditionally the so-called "photogrammetry". Photogrammetry operates without direct contact with the object to be analysed and uses photographic images of the object measured at a known and constant distance to guarantee both the respect of the stereoscopic observation and the real position in the three-dimensional space of the points used as a reference. Its advantage is that it allows the same photographic source to be used for repeated measurements that are not affected by operator-method dependent errors.

The aim of this study is to evaluate whether these gold standards have been applied to the face and teeth of the statue of the Warrior of Riace A and whether the dentofacial parameters detectable on it are sufficient to establish both dental and facial morphology (the lower third of the face is hidden by a considerably thick beard, which keeps us from viewing any point by which to determine the overall facial shape).

Methods

The study on the bronze of Riace A began in January 2022 with a search of bibliographic sources useful for knowing in detail the statue history from his discovery to the most recent studies. Studies applying the principles of golden section to dental and facial aesthetics were searched. In both cases, databases such as PubMed, Scopus, and Google Scholar were consulted. The grey literature and printed texts available in online libraries were also also examined. Historical bibliographic research on the statue was carried out by an expert archaeologist, while the search for official ministerial and museum sources of the statues for which authorisation was required to use the images and data contained therein was carried out by an expert lawyer. The bibliographic research on ~~the~~ studies on the golden section of dental interest was carried out independently by two specialists in orthodontics. No time and language restrictions were applied. The search strategy for both fields of research used free terms and Mesh terms with the Boolean operators "AND" and "OR" (Table 1).

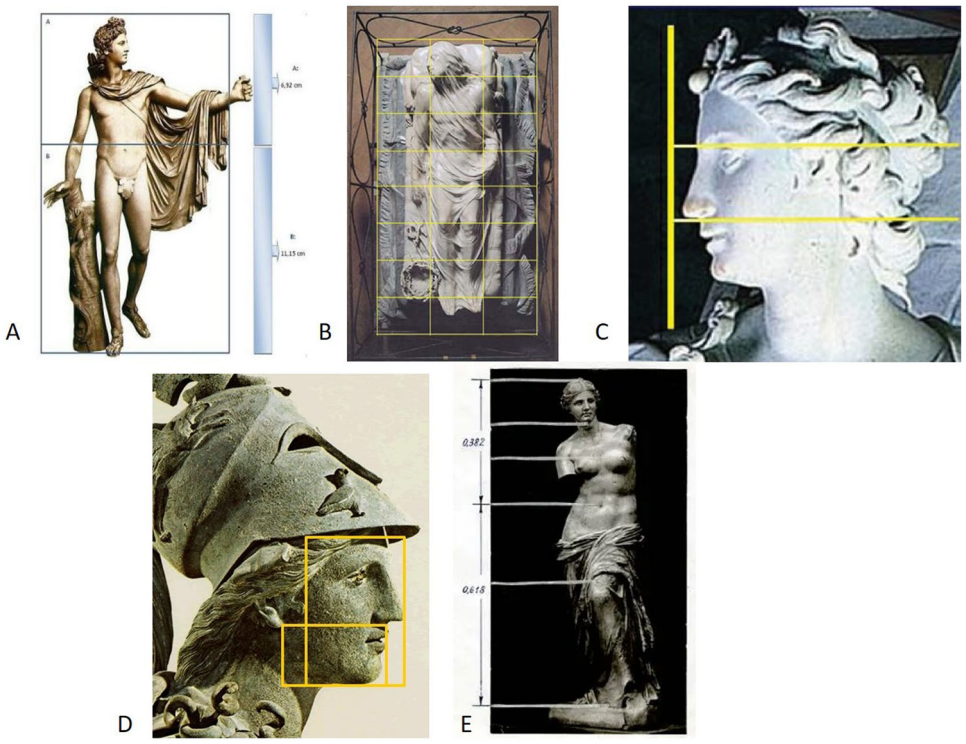

Fig. 1 Examples of artistic expressions in which the golden proportion has been observed. A Sculpture of Apollo Belvedere (Thapa and Thapa, 2018); **B**, **C** the "Veiled Christ" and the "Liberality" (Della Monica et al., 2013); **D**, **E** statues of Athena and Venus of Milo (https://canukeepup.wordpress.com/tag/golden-ratio/).

Table 1 Search strategy for historical and dental sources on Riace bronzes statues and the Golden Ratio.

Component 1		Component 2		Component 3
Historical sources search strategy				
Riace Bronzes	AND	Classical Age	AND	Beauty
OR		OR		OR
Riace Warriors		Hellenistic Age		Aesthetics
OR		OR		OR
Ancient statues		Ancient Greek Art		Golden Ratio
OR		OR		OR
Bronze statues		Greek Sculpture		Phi
		OR		OR
		Phidias		Golden Section
		OR		OR
		Myron		Golden Proportion
				OR
				Face
				OR
				Smile
				OR
				Teeth
Dental sources about Golden Ratio search strategy				
Smile	AND	Beauty	AND	Golden Ratio
OR		OR		OR
Teeth		Aesthetics		Phi
OR		OR		OR
Mouth		Attractiveness		Golden Section
OR		OR		OR
Dental elements		Ideal		Golden Proportion
OR				
Facial				

Duplicate results from the various databases consulted have been eliminated. Then, the research sources collected through bibliographic research on both fields of historical and dental interest were useful to the operators of this research to establish which facial and dental parameters should be measured on the bronze statue of Riace A, the most appropriate and reliable indirect photogrammetric measurement method, and the method of processing the data obtained from these measurements to detect the significant data for the study.

A digital official front photograph of Riace Bronze A was searched as a source for experimental evaluations (Fig. 2). Subsequent calibration of the image with parameters of known size constituted the necessary step to proceed with measurements and verification of the application of dentofacial golden proportions (Kaya et al., 2019). If the image of the Riace Bronze A, chosen to conduct the study, is not in digital format, but on paper, a scanner (HP Officejet J4850 All In One) has been set up for its digitisation. Before conducting experimental evaluations of the selected image, five experts in the fields of archaeology (Operator 1), computer science (Operator 2), forensic science (Operator 3), oral surgery (Operator 4), and orthodontics (Operator 5), completed a two-month independent remote training on the two software chosen for the experimental study(GIMP-GNU Image Manipulation Programme, version 2.10.18, Free Software Foundation, Inc., Boston-USA; PhiMatrix™ 1.618 Pro, PhiPoint Solutions, LLC, Mount Juliet, TN 37122 USA). The two software were then used independently by the operators, and they did not influence each other. The software GIMP, was used to perform random linear measurements on the image between different points of the face of the statue. Subsequently, in order to analyse the presence of the golden proportions, the "grids" tool of the PhiMatrix™ software was applied to the image in which the landmarks were those identified with the GIMP software. To reach a reliable level of agreement between examiners and inter examiners, evaluated

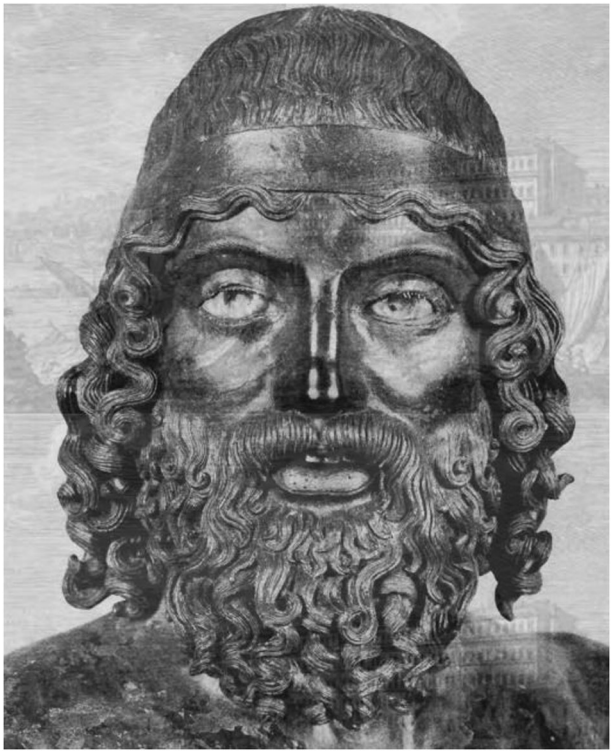

Fig. 2 Head of Statue A. Official front photograph (Sabbione, 1984).

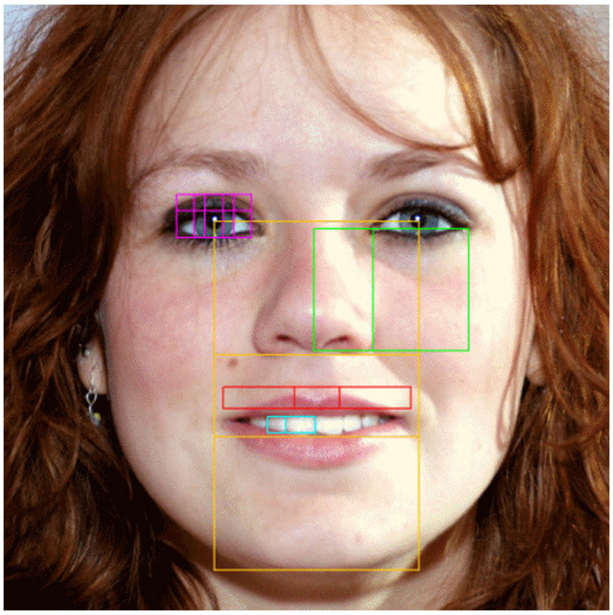

Fig. 3 The photo illustrates the main proportions of golden ratios on the human face detected with the PhiMatrix™ software. In orange colour the ratio "centre of pupil: bottom of teeth: bottom of chin"; in green colour "outer & inner edges of the eye: centre of nose"; in red, "outer edges of the lips: upper ridges of lips"; in light blue, "width of centre tooth: width of the second tooth"; in purple, "width of the eye: width of the iris". (Source of the explanatory image: https://www.phimatrix.com/face-beauty-golden-ratio/).

of using the Kappa statistical test, no less than 98%, each operator, repeated the procedures three times with an interval of 10 days. The results of their own previous measurements and those of other operators were not made available to any of the investigators, to avoid influence and bias. Only at the end of the experiment were the results of image manipulation and the measurements compared and verified by the level of correspondence, and then presented in this study as average measurements for each operator and like unified mean values. In case of discrepancy, the evaluations were discussed among the operators until all doubts were dispelled and agreement was reached. In order to establish the presence of divine proportions, it is necessary for golden ratios to be respected in various facial measurements. Since in statue A not all ~~the~~ linear measurements for the determination of dento-facial morpho-dimensional characteristics are detectable, as some points are hidden by hair or beard, the following were accounted for by the GIMP-GNU software and then used for the purpose of this study:

1. interpupillary distance (IPD);
2. distance between the external and internal canthus of each eye, called respectively ICDr and ICDl;
3. distance between centre of the nose and each lateral nasal point called, respectively, CNLNPr and CNLNPl;
4. inner or internal canthal distance (ICD)
5. distance between each lateral nasal point called LND corresponding to the base of the nose (BNL);
6. mesio-distal diameter of each upper central incisor (considered the longest distance between the areas of contact on the proximal crowns of teeth) called, respectively, DCRUI and DCLUI;
7. base of the upper anterior incisors (BTL).

This procedure makes it possible to verify compliance with the following horizontal and vertical golden proportions using the adequate Phimatrix™ grids (Fig. 3) and to hypothesise the shape of the face (deriving the position of the chin and base of the chin called "Me" alias Menton):

1. Outer and inner edge (canthi) of the eyes (ICDr and ICDl): centre of nose
2. Centre of the pupil: base of the nose (BNL): base of anterior upper incisors (BTL)
3. Centre of pupil base of the nose (BNL): base of anterior upper incisors (BTL)-base of the chin (Me)
4. Centre of pupil- base of anterior upper incisors (BTL): base of chin (Me)

If these proportions ratios are respected, it could be possible to establish the most probable dental shape in accordance to the measurements of the visible incisors. Mean measurements taken by each operator will be presented and differences will be subjected to statistical analysis to verify if they are significant (Microsoft Excel 2021 version, Microsoft, Redmond, WA, USA). Comparison of the averages of the measurements reported by each operator was subjected to ANOVA one – way testing. The final measurements represent the mean values of the measurements performed by each operator involved in the experiment.

Results

The historical and scientific literature on the Riace bronzes has not yielded any scientific descriptive studies on the face and teeth of the statues. The scientific literature concerning dental and facial golden proportions is instead rich in studies, including recent ones, and based on a conceptual and methodological point of view on historical reference works with mostly prosthetic, aesthetic, and epidemiological applications (Table 2). The official front photograph of Riace Bronze A (Fig. 2) contained in the pdf version of the collection of documents published by the Ministry of Cultural and Environmental Heritage in 1984 (Sabbione, 1984), was used as the source for the dento-facial measurements. In the same ministerial document, the presence of a graphical representation of the face of the statue in

Table 2 Bibliographic sources bibliographic sources found in the consulted databases.

Fields of Bibliographic Research	Results
Scientific studies relating the face and teeth of the Riace Bronze statues	None
Scientific studies related to dental and facial golden proportions	52 articles (1978–2023)

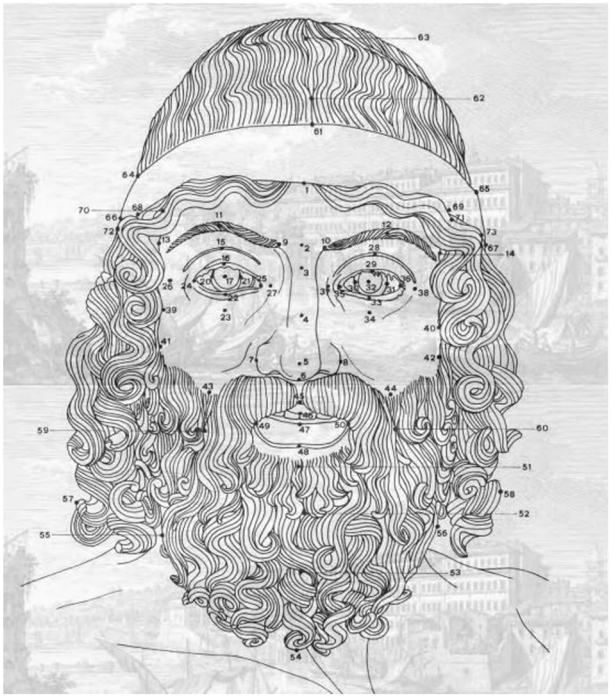

Fig. 4 The image illustrates the graphical representation of the face of Statue A. This image is superimposable on the official front view photograph (Sabbione, 1984).

frontal projection (superimposable on the official front photograph) (Fig. 4) with a relative anthropometric table whose points are visible and original measurements in centimetres of facial traits already calculated except dental ones and those of projection for the determination of golden ratios (Table 5), allowed to determine image dimensions, and thus work out the linear measurements. Permission was requested for the use of this documentation in its original form and subsequently granted by regulating authorities.

Both the photograph and the graphical representation were opened in GIMP software (GIMP-GNU Image Manipulation Programme, version 2.10.18, Free Software Foundation, Inc., Boston-USA). The points of the ministerial anthropometric table, represented in the graphic representation of the frontal image (Fig. 4), selected for this study, were transferred by superimposition on the frontal photo (Fig. 5A, B) without the need to calibrate the dimensions of the two images. Inter- and intra-examiner calibration allowed to exclude subjective errors from experimental measurements. In fact, an index greater than expectations has been reached ($K = 100$). With the exception of dental and projected measurements, which were not included in the original ministerial anthropometric reference table, the other mean individual and unified measurements detected experimentally match the original ones. It was decided to take into account for dental measurement the largest mesio-distal diameter of only the two central incisors, excluding the lateral incisors and the canine, whose crowns were not clearly visible in their widths. First, the dental elements appear to be centred with respect to the face, with the upper interincisal midline falling in correspondence with the midline of the face (Fig. 5A, B). The mean measurements taken by each operator after the training phase were reported (Tables 3 and 4). The measurements are mostly clearly matching between operators, but they were nevertheless subject to statistical testing to assess with the ANOVA- One Way (Analysis of Variance) the significance of the differences found (Table 5) (Microsoft Excel 2021 version, Microsoft, Redmond, WA, USA). The value of F-ratio is 1.45035. The p-value is 0.227794. The result is not significant at $p < 0.05$. This means that no significant differences were found in the measurements based on individual experience and that the method used in this study is highly reliable and repeatable. Most of the measurements considered in this study correspond to preexisting values (Table 6). The interpupillary distance (IPD) is equal to 7 cm. The intercanthal distances of the right eye and left eye (ICDr and ICDl) are equal to 3.6 cm. The distance between the lateral edges (outer edge or external canthi) of both eyes called the LCD is equal to 10.2 cm. The distance between the internal canthi of both eyes called IntCD is equal to 3.5 cm. The distances between the centre of the nose and each lateral nasal point are equal to 2.1 cm on the right (CNLNPr) and 1.8 cm on the left (CNLNPl). In the original table, for both values equal to 3.7 cm, is reported. The distance between each lateral nasal point called LND, hereinafter called BNL in the determination of golden ratios, is equal to 4 cm according to the original table, while in our measurement it is equal to 3.9 cm. (Fig. 5A, B). Since as specified in the original ministerial document "the photogrammetric measurements do not take into account the curvature of the surfaces and thus there may be a methodological limitation" (Sabbione, 1984), the measurements referred to the distance between the centre of the nose and lateral points of the nasal wings (CNLNPr and CNLNPl) on the ministerial document are likely to be inaccurate, also in light of the fact that the distance between the lateral nasal points (LND or BNL) reported by them (4 cm), which represents the sum, is much closer to our measurement (3.9 cm). Among the measures obtained by the operators (Table 5, Fig. 5A, B), which are not taken from the ministerial anthropometric table, there is first of all the distance between the projection of the centre of the nose on the line passing through the internal canthi of each eye, which is 1.89 cm. The distance between the line passing through the pupils (IPD) perpendicular to the base of the teeth line (BTL), called IPD-BTL, is equal to 7 cm, as is the interpupillary distance (IPD) itself. The perpendicular distance between the line passing through the internal canthi of both eyes (IntCD) line and the line defined between the two lateral nasal points (LND), called IntCD-LND is equal to 3.5 cm, and such a value matches the internal canthi of the distance between the two eyes (IntCD) itself. The distance between the IPD line and BNL called IPD-BNL is 4 cm and corresponds to the LND itself. The distance between the BNL and the BTL called BNL-BTL equals 2.64 cm. The distance between BTL and the base of the chin (Me) is then supposed to be 4 cm corresponding to the lower base of the golden rectangle. All these linear measurements represent horizontal and vertical golden ratios (Fig. 5C–H). The widths of each central incisor (the maximum diameter detected between the distal and mesial interproximal surfaces of each tooth) are 0.75 cm and are called, respectively, DCRUI (mesio-distal diameter of the central right upper incisor) and DCLUI (mesio-distal diameter of the central

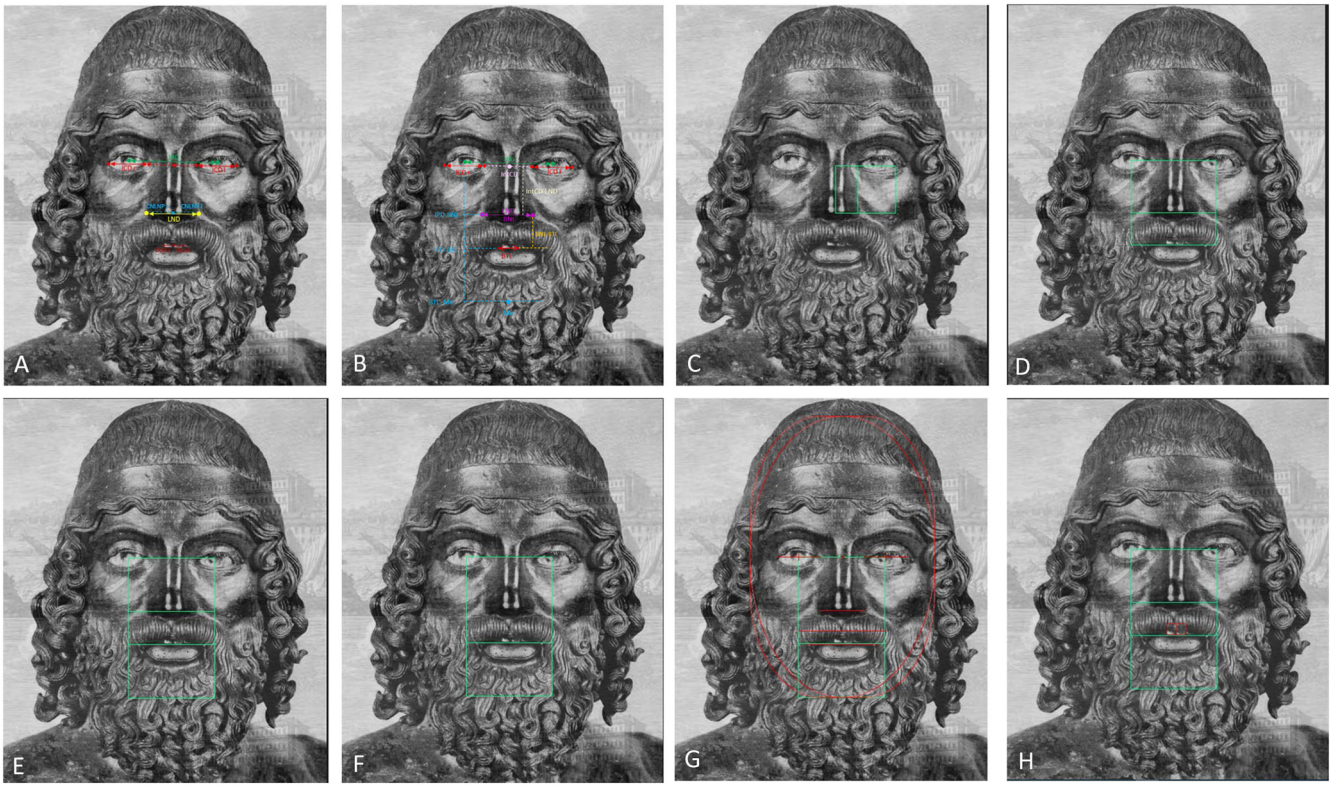

Fig. 5 Representation of the experimental evaluations on the official front photograph of the Statue A. A, **B** Linear distances marked by GIMP-GNU software on the Statue. Golden ratios found with the PhiMatrix™ software: **C** Outer and inner edge (canthi) of the eyes (ICDr and ICDl): centre of the nose; **D** Centre of the pupil: base of the nose (BNL): base of anterior upper incisors (BTL); **E** Centre of pupil-base of the nose (BNL): base of anterior upper incisors (BTL)-base of chin (Me) ; **F** Centre of pupil- base of anterior upper incisors (BTL): base of chin (Me) ; **G** Face grid golden ratio; **H** central upper incisors shape detected with golden rectangle tool.

Table 3 Mean measurements for each operator for each anthropometric parameter considered in the experimental study.

Anthropometric Measurements	Operator 1	Operator 2	Operator 3	Operator 4	Operator 5
Interpupillary Distance (IPD) (points 17-32)	7	7	7	7	7
Distance between the external and internal canthus of the right eye (ICDr) (points 24-25)	3.59	3.56	3.6	3.58	3.6
Distance between the external and internal canthus of the left eye (ICDl) (points 35-36)	3.6	3.6	3.57	3.6	3.57
Distance between the lateral edge (outer edge or external canthi) of both eyes called LCD (points 24-36)	10.18	10.21	10.22	10.2	10.19
Distance between the internal canthi of both eyes called IntCD (points 25-35)	3.51	3.49	3.48	3.5	3.5
Distance between the centre of the nose and the right lateral nasal point (CNLNPr) (points 5–7)	2	2.1	1.98	2.1	2.1
Distance between the centre of the nose and the left lateral nasal point (CNLNPl) (points 5–8)	1.79	1.8	1.81	1.79	1.8
Distance between left and right lateral nasal points (points 7 and 8) called LND	3.9	3.88	3.9	3.88	3.91
Distance between the vertical projection of the centre of the nose (point 5) and the internal canthus of the right eye called (point 25)	1.87	1.88	1.89	1.89	1.89
Distance between the vertical projection of the centre of the nose (point 5) and the internal canthus of the left eye called (point 35)	1.89	1.88	1.87	1.9	1.89
Distance between the internal canthi of both eyes (IntCD) line and the line defined between the two lateral nasal points (LND), called IntCD-LND	3.48	3.47	3.51	3.52	3.5
Distance between IPD and perpendicular base of nose line (BNL, passing through point 5) called IPD-BNL	4	4.1	4	3.98	4
Distance between base of nose line (BNL) and base of teeth line (BTL) called BNL-BTL	2.7	2.6	2.6	2.8	2.6
Distance between the IPD line and perpendicular base of teeth line (BTL) called IPD-BTL	6.9	7.1	6.9	7	7
Distance between base of teeth line (BTL) and base of the chin (Me)-supposed	4	4	3.98	3.99	4
Mesio–distal diameter of the central right upper incisor (DCRUI)	0.74	0.75	0.74	0.75	0.75
Mesio–distal diameter of the central left upper incisor (DCLUI)	0.75	0.75	0.75	0.75	0.74

Table 4 Summary of data for each operator who participated in the study.

Summary of Data	Operator 1	Operator 2	Operator 3	Operator 4	Operator 5	Total
N	17	17	17	17	17	85
$\sum X$	61.9	62.17	61.8	62.23	62.04	310.14
Mean	3.6412	3.6571	3.6353	3.6606	3.6494	3.649
$\sum X^2$	320.4042	323.9769	320.3798	323.0329	321.9934	1609.7872
Std.Dev.	2.4369	2.4574	2.4459	2.4397	2.4442	2.3859

Table 5 Details reporting the ANOVA one-way test results.

Source	*SS*	*df*	*MS*	
Between-operators	0.0076	4	0.0019	$F = 1.45035$
Within-operators	478.1699	80	5.9771	
Error	0.084	64	0.0013	

left upper incisor) (Fig. 5A, B, H). In our experimental evaluation, the reduced exposure of the entire dental upper frontal group and lip (since the subject is not smiling, but rather exhibits a kind of grin) prevented us from using the PhiMatrix Software tool called "Levin dental grid", designed to superimpose this golden ratio grid between upper anterior teeth (from canine to canine), upper lip, and bottom of the nose. However, the mesiodistal dimensions at the base of each central incisor (0.75 cm) remain the same throughout the visible height of these teeth (Fig. 5A, B, H). Based solely on the measurements of the incisal margins, it cannot be ruled out that the shape of the teeth was conceived as tapered, or ovoid or square. The presence of non-rounded corners at the base of the incisal margins may be indicative of a shape more compatible with the tapered one, but this semblance could instead be due to the wear of the silver foil, and therefore cannot be taken as a decisive element. However, since the facial and dental parameters considered are in golden proportion (Fig. 5C–H), it was possible to indirectly determine the position of the chin (Fig. 5B, E–G), and consequently, hypothesise the shape of the face (Fig. 5g) and teeth referring to the upper central incisors (Fig. 5h). The application of PhiMatrix™ face grid function on the basis of the golden rectangle that defines the interpupillary position as the upper limit and the position of the chin as the lower limit, and as intermediate the position of the bottom of the upper central incisors gives us a facial type ovoid to which could correspond, since all golden parameters are fulfilled, an equally ovoid tooth shape (Fig. 5G, H). If we accept the hypothesis that the design of the face has taken into account the verified golden ratios, since the golden ratio between the length and width of the upper incisors is 1 to 0.8, we could accept the hypothesis pointing to the total height of the incisors being 0.93 cm, i.e., a ratio compatible with the ovoid shape estimated above based on the other parameters.

Discussion

The Riace bronzes are an artistic, historical and cultural heritage much loved by Italians and foreigners alike. Fame and history, laden with mystery, still surround them (Hoffman and Konstam, 2002; Paterlini, 2017; Scalco and Salvadori, 2017; Castrizio, 2019; Dafas, 2019). Recent research proposed in-depth studies on some of their anatomical traits, with interesting results (Bonafini and Pozzilli, 2012; Muzzupappa et al., 2012). Although the statues have been preserved very well, they have come to us without some of their details (spears, shields, and helmets) (Rolley, 1984; Micheli and Vidale, 2013). Other details are preserved quite well, such as ivory and marble eyes, pink glue for the lacrimal glands, eyelashes, lips, and nipples, all in copper.

The teeth, visible only in statue A, are in silver. They would have been separately produced and subsequently applied from the outside, with indirect lost wax casting technique, with the exception of some parts (such as the cap of statue B) (Formigli, 1999; Hoffmann and Konstam, 2002). Moustaches, beards, and hair were fused separately and applied to the bare heads. The beard also rests on the face and partially covers the lips, which, however, are perfectly and completely structured. The upper and lower lips constitute a single piece connected at the level of the commissures (other similar models were found in the workshops of the bronze workers in Olympia) (Hoffmann and Konstam, 2002).

The teeth of statue A are unique: there are no other bronze statues from the same period in which the teeth were still preserved. A single silver leaf represents five upper maxillary dental elements (from the right lateral incisor to the left canine) (Sabbione, 1984). The teeth are not structurally covered by the lips. They were conceived, probably deliberately, only for the coronal portion to be visible beyond the parted lips in the provocative grin expressed by the statue.

Our main hypothesis is that the constituent elements of the statue, including teeth, are the result of an artistic production in which the search for the representation of harmony and beauty is part of the stylistic project. Taking into account the knowledge of golden proportions such as common thread, golden ratios in upper central incisors and face were searched, taking into account the original measurements taken from the official ministerial images (detected with photogrammetry), and the methods of attribution of morphostructural characters connect facial and dental parameters already known in the literature (Lavelle, 1972; Cesario and Latta, 1984; La Vere et al., 1992; Papagrigorakis et al., 2011; Pedrosa et al., 2011; Al-Marzok et al., 2013; Forster et al., 2013; Jin et al., 2016; Nguyen et al., 2016; Rawlani et al., 2017; Mantelakis et al., 2018; Kaya et al., 2019; Liao et al., 2019; Mehndiratta et al., 2019; Wang et al., 2021).

Since only the central incisors have an optimal exposure for taking measurements, the maxillary intercanine distance in relation to facial parameters could not be used as previously suggested in many comparative studies of dental aesthetic and prosthetics based on golden proportions (Papagrigorakis et al., 2011; Aldegheishem et al., 2019; Kaya et al., 2019; Swelem and Al-Rafah, 2019). The same function of the golden section analysis software with which it would have been possible to apply Levin's dental grid (Levin, 1978), to verify the golden ratio in the upper anterior teeth (canine to canine) in relation to each other and in relation to the position of the upper lip and the base of the nose, could not be used due to inadequate exposure of the upper anterior dental group.

Therefore, the maximum mesiodistal diameter of each central incisor was taken as only dental reference (Cesario and Latta, 1984; La Vere et al., 1992; Pedrosa et al., 2011; Al-Marzok et al., 2013; Al-Kaisy and Garib, 2018; Mehndiratta et al. 2019; Mamidi et al., 2020). The golden proportion described by Pythagoras is still valid to apply the mathematical balance found in nature to beauty both in art (from ancient classical art, passing through Leonardo da Vinci, up to more modern forms) and in those

Table 6 Original measurements taken from ministerial anthropometric table relating Statue A.

Original Anthropometric Widths	Values (in cm)
Interpupillary distance (IPD) (points 17-32)	7
Distance between the external and internal canthus of the right eye (ICDr) (points 24-25)	3.6
Distance between the external and internal canthus of the left eye (ICDl) (points 35-36)	3.6
Distance between the lateral edge (outer edge or external canthi) of both eyes called LCD (points 24-36)	10.2
Distance between the internal canthi of both eyes called IntCD (points 25-35)	3.5
Distance between the centre of the nose and the right lateral nasal point (CNLNPr) (points 5–7)	2.1 (vs 3.7)*
Distance between the centre of the nose and the left lateral nasal point (CNLNPl) (points 5–8)	1.8 (vs 3.7)*
Distance between left and right lateral nasal points (points 7 and 8) called LND	3.9 (vs 4)*
Anthropometric Measurements Experimentally Obtained	**Values (in cm)**
Distance between the vertical projection of the centre of the nose (point 5) and the internal canthus of the right eye called (point 25)	1.89
Distance between the vertical projection of the centre of the nose (point 5) and the internal canthus of the left eye called (point 35)	1.89
Distance between the internal canthi of both eyes (IntCD) line and the line defined between the two lateral nasal points (LND), called IntCD-LND	3.5
Distance between IPD and perpendicular base of nose line (BNL, passing through point 5) called IPD-BNL	4
Distance between base of nose line (BNL) and base of teeth line (BTL) called BNL-BTL	2.6
Distance between the IPD line and perpendicular base of teeth line (BTL) called IPD-BTL	7
Distance between base of teeth line (BTL) and base of the chin (Me)-supposed	4
Mesiodistal diameter of the central right upper incisor (DCRUI)	0.75
Mesiodistal diameter of the central left upper incisor (DCLUI)	0.75

(Sabbione, 1984) and unified derived anthropometric measurements. Values marked with an asterisk (*) refer to our measurements, as opposed to those indicated in brackets, which refer to the original anthropometric table.

scientific fields where aesthetics is associated with function, as is the case in dentistry (Lavelle, 1972; Cesario and Latta, 1984; La Vere et al., 1992; Formigli, 1999; Papagrigorakis et al., 2011; Pedrosa et al., 2011; Al-Marzok et al., 2013; Forster et al., 2013; Jin et al., 2016; Nguyen et al., 2016; Rawlani et al., 2017; Al-Kaisy and Garib, 2018; Mantelakis et al., 2018; Aldegheishem et al., 2019; Kaya et al., 2019; Mehndiratta et al., 2019; Swelem and Al-Rafah, 2019; Mamidi et al., 2020, Putrino et al., 2023).

Many researchers investigated possible morphodimensional relationships between dental elements and facial type (including ethnicity) also by verifying the applicability of the golden proportion (Lavelle, 1972; Papagrigorakis et al., 2011; Al-Marzok et al., 2013; Forster et al., 2013; Jin et al., 2016; Nguyen et al., 2016; Rawlani et al., 2017; Al-Kaisy and Garib, 2018; Mantelakis et al., 2018; Aldegheishem et al., 2019; Kaya et al., 2019; Swelem and Al-Rafah, 2019; Mamidi et al., 2020). The ratio is approximately 0.618 to 1, whereby the height of the shorter object divided by the height of the longer one is identical to the height of the longer object divided by the sum of the shorter plus the longer objects (Lavelle, 1972; Levin, 1978; Cesario and Latta, 1984; La Vere et al., 1992).

This basic principle enabled us to verify the presence of vertical and horizontal golden proportions starting from the original anthropometric measurements and then applying through dedicated software the preformed rectangles that without any difficulty have maintained golden proportion in the facial parameters of reference, and adapting perfectly to the pre-determined points.

The height of the central incisors was also defined in the scale correspondence of the golden rectangle defined by the interpupillary distance at the base of the chin (Kaya et al., 2019). The face and the teeth can be classified as square-, triangular- and oval-shaped (Pedrosa et al., 2011; Kaya et al., 2019; Mehndiratta et al., 2019; Wang et al., 2021). A much-debated theory sets forth a relationship between facial and dental shape (Williams, 1914). However, such a principle was later questioned, with many studies casting doubt on its applicability; therefore, when presenting an aesthetic analysis, dentists should weigh prosthetic or rehabilitative interventions guided by other factors (Cabello and Alvarado, 2015; Raghavendra et al., 2015; Dinesh et al., 2018; Koidou et al., 2018; Liao et al., 2019; Wang et al., 2021). These studies, and those in which golden proportions evidently represent an ideal of perceived attractiveness more than real morphostructural characters recurring to represent populations (Lavelle, 1972; Cesario and Latta, 1984; La Vere et al., 1992; Pedrosa et al., 2011; Al-Marzok et al., 2013; Forster et al., 2013; Jin et al., 2016; Nguyen et al., 2016; Rawlani et al., 2017; Al-Kaisy and Garib, 2018; Koidou et al., 2018; Mantelakis et al., 2018; Aldegheishem et al., 2019; Kaya et al., 2019; Swelem and Al-Rafah, 2019), have further encouraged us to look for a correspondence in dento-facial parameters that we consider for the statue more in the principles of the golden ratio than in modern anthropometric studies. Classical culture in art represented human features as close as possible to the concept of divinity and perfection rather than human reality (Dafas, 2019), and ours is not a work of facial reconstruction on a subject who existed in ancient times and whose body has ever been found (Papagrigorakis et al., 2011). Therefore, the position of the chin was derived from the correspondence of the golden rectangles and the "facial" mask of the golden ratio analysis software. Anthropometric comparative evaluations on ethnic basis would be farfetched from our perspective, so the hypothesis of an ovoid facial type to which an ovoid dental shape would correspond can be accepted without estimating the attribution of a racial profile. Furthermore, the racial correspondence with the shape of the face is a concept, debated in the past, which has been strongly reduced by the most recent evidence in the literature (Alam et al., 2015; Singh et al.; 2023). The ovoid shape can correspond to an ideal of beauty that meets the principles of the golden section. Famous anthropometric studies based on golden section theory, establishing the key concepts on dento-facial harmony for aesthetic and prosthetic rehabilitation purposes, have led to the following widths being deemed more reliable: interpupillary, interalar, intercanthal (Berry, 1905; House and Loop, 1939; Lavelle, 1972; Cesario and Latta, 1984; Latta et al., 1191; La Vere et al., 1992). The interpupillary and the interalar widths that in our study we called LND and BNL (depending on the software on which it was traced as a measurement) can be used reliably to select the maxillary anterior teeth width in prosthetic rehabilitation; a

consistent relationship can also be observed for sexual and racial differences (Cesario and Latta, 1984; La Vere et al., 1992). The combined width of the maxillary central incisors of the statue are in golden proportion with the interalar width (LND or BNL). Our measurements relating the intercanthal distance (ICDr, ICDl) are in golden proportion with the width of the lateral nasal portion from the outermost point to the centre of the nose (CNLNPr and CNLNPl) (Abdulaziz et al., 2021). In agreement with other authors, we found the statue inner canthal distance (ICD) of the statue is not in proportion to the combined mesio-distal width of the central maxillary incisors (Shetty et al., 2021). The interalar width was used in relation to the verification of original ministerial anthropometric measurements (then called LND) and to derive the reference line in the definition of one of the virtual golden rectangle's borders (then called BNL). Some studies state that the bizygomatic width may not be a reliable guide for estimating the width of central incisors (Al Wazzan et al., 1995; Baleegh et al., 2015). We could not have taken this parameter into consideration anyway, because the hair and high hairline of the beard would not allow the identification of reliable points. Some studies state that the intercommisural width can be used as a reliable preliminary method to determine the combined width of the maxillary anterior teeth in prosthodontic rehabilitation (Miranda and D'Souza, 2016; Bansal et al., 2019) and in the aesthetic evaluation of the natural maxillary anterior teeth (Mahmood, 2019). In our study, it was not possible to use intercommisural width. Some limitations to this study are represented by the inability to determine some other fundamental skin points on the face such as Trichion, Gnathion, and Pogonion (due to the presence of the bandage surrounds the forehead, hair, and beard) and by the lack of other landmarks useful to determine the Labial Superius, the Chelion, the Labial Inferius and the Stomion (due to the presence of the moustache and the thick beard as well as the non resting position of the lips). Such setbacks prevent a thorough evaluation in the transverse and vertical planes. Moreover, the ministerial anthropometric measurements used here could have inaccuracies arising from early methodological limits (as also stated in the ministerial document), even if our results show that they are indeed reliable. Further studies, possibly carried out with direct measurements on statues or 3D models, could confirm our photogrammetry data and analysis of golden ratios. The importance remains of having raised a question of scientific, historical, and cultural interest on the original methodological framework by which the Riace bronzes were created, which to date no one had ever considered.

Data availability

All the data generated or analysed during this study are included in this published article.

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Author contributions

A.P.: conceptualisation, research design, data acquisition, data analysis, writing-original draught preparation; M.R.: research design, data acquisition, data analysis, software; EM: data acquisition, writing—review and editing; M.C.: data acquisition, writing-original draught preparation; S.Z.: data acquisition, supervision, writing-review and editing.

Competing interests

The authors declare no competing interests.

Ethical approval

Ethical approval was not required as the study did not involve human participants.

Informed consent

Informed consent was not required as the study did not involve human participants.

Additional information

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