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Catch-up growth
growth velocity

A Note on the Mathematics of "Catch-up" Growth

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Extract

The concept of "catch-up" growth is presented in terms of growth velocity. Mathematically speaking, size function is the integral of velocity function, regardless of the form of the function. The process of "catch-up" growth will be complete only when the integrated velocity excess during recovery matches the previous velocity deficit.

Speculation

An appreciation of the importance of growth velocity, *i.e.*, change in body size with respect to time (dS/dt), may help in understanding the magnitude of the processes involved in recovery from undernutrition, and perhaps overnutrition as well.

The phenomenon of "catch-up" growth has intrigued physicians and nutritionists for many years, and a number of authors (2, 3, 7, 8, 11) have commented on this process which takes place when undergrown infants and children are properly rehabilitated. Provided the growth deficit is not too great or of too long duration the deficit can be made good. The term "developmental canalization" was used by Prader *et al.* (8); distortions in growth patterns tend to be corrected when adequate nutrients are available, in an attempt to conform to a predetermined growth potential.

My thesis is that this phenomenon can best be understood in the context of growth velocity, *i.e.*, change in size per unit time (4). Velocity standards have been published by Tanner and his associates (9).

Figure 1 shows a hypothetical situation, in which an infant grew normally for a time, then suffered a period of undernutrition from which he recovered, and thereafter grew normally. Area A represents the velocity deficit, the magnitude of which is determined by the duration of subnormal weight gain and the degree to which velocity is reduced below normal.

Since velocity (V) is the differential (dW/dt) of the weight-time function, it follows that weight is the integral of the velocity function:

$$W(t) = W(t_0) + \int_{t_0}^{t} V(t)dt \qquad (1)$$

Hence weight at time t is equal to weight at some arbitrary time t_0 plus the integral of the velocity function between limits t_0 and t. By the fundamental theorem of the integral calculus this is the area under the velocity curve, an area which is bounded by these limits and the horizontal zero coordinate. Area A is thus the difference between the area under the dotted velocity curve in Figure 1 and that under the solid velocity curve from 6 months of age to 2 years. Areas B and C, which have to do with recovery, can be similarly visualized, except that velocity is now supranormal.

The relation described by Equation 1 holds regardless of the form of the differential (velocity) function, provided, of course, that the function is a continuous one. Nor does one need to worry about the composition of either the lost weight or that gained during recovery, as long as one assumes a normal body composition before onset of malnutrition and again after recovery is complete (15).

Obviously, then, complete recovery from growth failure, *i.e.*, precise catch-up growth, will occur only if area B (or area C, a slower rate of recovery) equals area A. Since weight velocity fell below normal as a result of undernutrition, it must perforce rise above normal, and remain so throughout the recovery period, if the child is ever to catch up; and velocity must eventually return to normal if the desired weight is not to be exceeded.

The average weight curve during recovery from infantile malnutrition published by Kerr *et al.* (Reference 7, p. 475) appears to be exponential, and I have estimated the half-time at 2.5–3 weeks. Hence a suitable equation would be

$$W(t) = W(i) + W(d)\,(1 - e^{-kt}) \qquad (2)$$

where W(i) is initial weight, *i.e.*, at onset of recovery, W(d) is the weight deficit, t is time in days, *e* is the base of the natural system of logarithms, and k is a rate constant which equals 0.036 $days^{-1}$, or ln 2 divided by the half-time of 19 days.

The rate of gain, or velocity of the "catch-up" phrase, is found by differentiating Equation 2

$$V = dW/dt = +k\,W(d)\,e^{-kt} \qquad (3)$$

This includes only the excess weight velocity needed for the "catch-up" process.

The upper bounds of areas B and C in Figure 1 were calculated from Equation 3, using the value 0.036 for k (corresponding to a half-time 19 days) for area B and the value of 0.018 (half-time 38 days, a slower rate of recovery) for area C.

The practical consequences of this formulation are obvious. The greater the velocity deficit, *i.e.*, the larger area A is by virtue of either its height or breadth, the greater must be area B, or area C. If growth acceleration is rapid during recovery the deficit is quickly made up; if the spurt is less intense, as in area C, a longer period is required.

Figures 2, 3, and 4 illustrate three patients whom I have observed personally. Weight is depicted for two of these, and height for the third. Each returned (approximately) to his or her original size percentile once the cause of the condition was recognized and treated. The curves are not as smooth as the hypothetical example in Figure 1, yet in each instance the area representing velocity excess during recovery equalled, as it inevitably must, the area of velocity deficit.

"Catch-up" growth requires energy, and since velocity must exceed normal throughout the entire recovery period food must be provided in excess of normal requirements until recovery is complete. Indeed, both Waterlow (11) and Kerr *et al.* (7) found that calories were relatively more important than protein in facilitating recovery from kwashiorkor. Weight gain during recovery was proportional to caloric intake in excess of maintenance needs: for every 6 calories 1 g weight was gained (7). Hence if a malnourished child is returned too quickly to an environment where the food supply is marginal, he may never catch up. Conversely, a short period of subnormal growth, or even of weight loss, can be easily made up since area A is small.

Since growth velocity is high in infancy and in adolescence a given degree of malnutrition at these ages will incur a larger velocity deficit than in the midchildhood years, so that the "catch-up" process will have to be more intense, or more prolonged. A similar line of reasoning suggests that "catch-up" growth may more easily be achieved by the adolescent girl because her growth velocity is normally less than that of the boy. Indeed, Williams *et al.* (12, 13) have shown that "catch-up" growth in female rats is a little faster than in males.

This general formulation applies to adults also, but with an important quantitative difference, namely that the velocity of

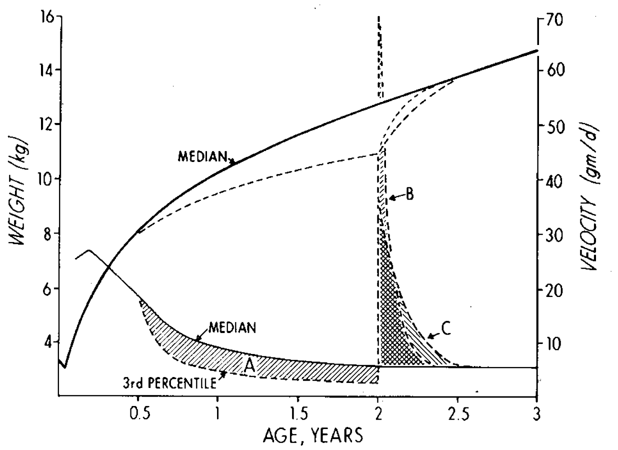

Fig. 1. Idealized plots of body weight (*upper lines*) and weight velocity (*lower lines*) —, 50th percentile (9); – – –, hypothetical example of episode of malnutrition and recovery. For explanation of areas A, B, and C, see the text.

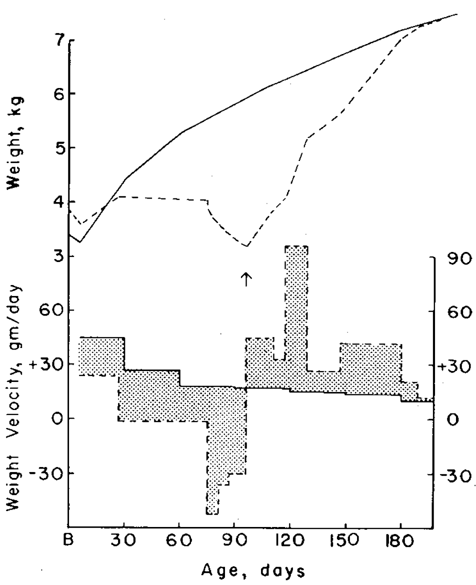

Fig. 2. Weight and weight velocity for infant with trypsinogen deficiency. —, 50th percentile (9); – – –, patient (10). Arrow shows beginning of treatment.

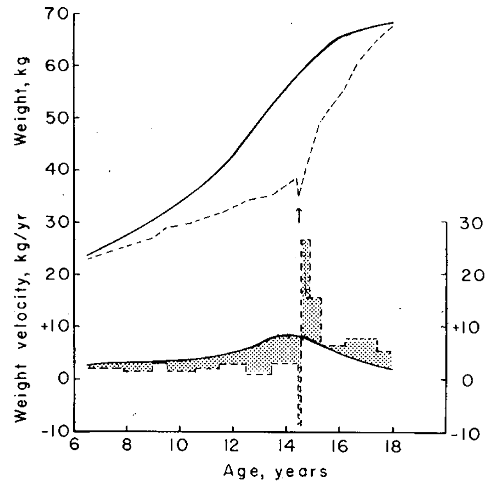

Fig. 3. Weight and weight velocity for a boy with intestinal lymphangiectasia. —, 75th percentile (9); – – –, patient (1). Arrow shows beginning of treatment.

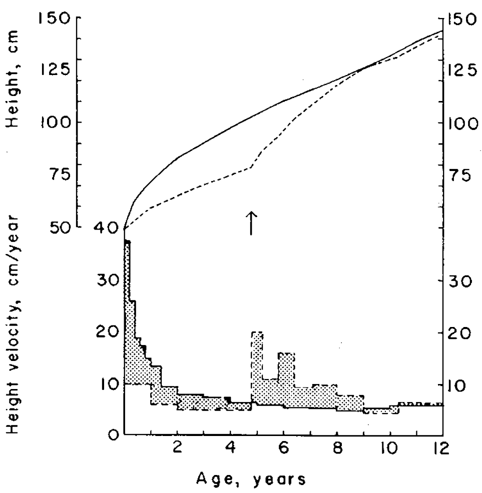

Fig. 4. Height and height velocity for girl with hypothyroidism. —, 25th percentile (9); – – –, patient. Arrow shows beginning of treatment.

growth at this time of life is 0 (the very slow decline in adult height and lean body mass (6) can be neglected for purposes of the present argument). Even so, recovery from undernutrition requires that velocity exceed this normal (*i.e.,* 0) value. But the adult has an advantage: if weight is lost and then remains stationary at a subnormal value, the velocity deficit does not increase with time, as it inevitably does in the child.

The formulation applies equally well to the overweight child, but in converse fashion; the excess area under the velocity curve must be matched by an equivalent area of velocity deficit if normal weight is ever to be achieved.

Ashworth (2), who allowed her undernourished child subjects to eat *ad libitum,* noted that appetite was voracious at first, and then fell off as normal weight for height was approached. This suggests that appetite is geared in some mysterious fashion to the needs of the "catch-up" process, just as it seems to be geared to normal needs throughout life, needs which range from those of the active adolescent boy to those of the sedentary adult. Wilson and Osborn (14) have also commented on the role of appetite in other species during recovery from undernutrition.

Although velocity curves are not always easy to construct, the mathematical formulation presented here may help to conceptualize the quantitative nature of the "catch-up" process, and to make vivid what is already obvious: undernoruished children are at a disadvantage with adults.

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15. This statement needs qualification for that time of life, such as early infancy, when certain changes in body composition are normally occurring, yet the magnitude of these normal compositional changes are small in comparison to the body weight changes under consideration here.

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Burns
emotional trauma

Long Range Emotional Sequelae of Burns: Effects on Children and Their Mothers

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Extract

Twelve children ranging between 8 and 13 years of age who had been hospitalized for burns, their mothers, 12 control children, and 12 control mothers matched for age, sex, socioeconomic status, and the experience of hospitalization were administered a battery of psychologic tests 1.5–5 years after the burn accidents. This effort produced few differences between burned children and their matched control children. However, the mothers of burned children were found to be significantly more distressed than their matched controls on 10 of 28 study variables.

Speculation

It is hypothesized that either burn victims' mothers were disturbed prior to the accident and possibly contributed to it or that the emotional sequelae to burns are more extensive with parents (mothers) than with children. It is suggested that more medical and/or emotionally supportive care be considered for the parents of burn victims, with a particular emphasis on restoring appropriate self and role perceptions. Consideration should also be given to feelings of parental insufficiency and unconscious hostility toward the child.

Several articles in the medical and behavioral literature have dealt with emotional reactions and other sequelae to burns and the resulting disfigurement in children. Most have been intuitive or impressionistic in nature, and very few have utilized even minimally rigorous research methods such as quantifiable measures and/or control groups. Although most authors agree that there is emotional trauma associated with being burned and/or disfigured, the precise nature of what burn victims experience (as compared with the behavioral concomitants of other diseases) remains speculative. In particular doubt is the question of how much of the emotional disturbance is an antecedent rather than a consequence of being burned. The purpose of this investigation was to