Then the operator for P_1, for example, is the Pauli spin matrix σ_z, and one can show that $P_1 = \mathrm{tr}(\rho\sigma_z)$. This formalism only yields percentage polarization ratios, but the intensity can in the partially polarized case be shown to transform in the same way as in the monochromatic case, equation (1). Thus we need only show how the percentage polarizations transform.

The effect of a general proper Lorentz transformation on the state vector $|a\rangle$ of a quantum system[5] is given by a unitary operator $\mathbf{T}$; that is, in the moving frame, $|a'\rangle = \mathbf{T}|a\rangle$, where $\mathbf{T}^{-1} = \mathbf{T}^{+}$. We now wish to find how the density matrix transforms, and to do this we inquire first about the transformation properties of a Hermitian observable R. We denote by R' the Lorentz-transformed observable, in the sense that the measuring apparatus has not been lifted into the moving frame, but left in the stationary frame. In this case, the expectation value of R is invariant; i.e., $\langle a|R|a\rangle = \langle a'|R'|a'\rangle$, and therefore $\langle a|R|a\rangle = \langle a|\mathbf{T}^{+}R'\mathbf{T}|a\rangle$, for all $|a\rangle$. Hence, $R = \mathbf{T}^{+}R'\mathbf{T}$, or $R' = \mathbf{T}R\mathbf{T}^{+}$.

Now let us apply the same reasoning to the density matrix case: $R = \mathrm{tr}(\rho R) = \mathrm{tr}(\rho' R') = \mathrm{tr}(\rho'\mathbf{T}R\mathbf{T}^{+}) = \mathrm{tr}(\mathbf{T}^{+}\rho'\mathbf{T}R)$, since $\mathrm{tr}(AB) = \mathrm{tr}(BA)$. Therefore, $\mathrm{tr}(\rho R) = \mathrm{tr}(\mathbf{T}^{+}\rho'\mathbf{T}R)$ for all Hermitian operators R, and therefore $\rho = \mathbf{T}^{+}\rho'\mathbf{T}$ or $\rho' = \mathbf{T}\rho\mathbf{T}^{+}$.

But what is T? We have shown in the preceding section that in the monochromatic case, which corresponds to the "pure" quantum mechanical state, the percentage polarizations do not change under a Lorentz transformation, and that the ratios U/I and Q/I are affected only by rotations in the plane perpendicular to the direction of the beam. Therefore, except for such special rotations, **T** is the unit matrix, and hence the density matrix is also unchanged by Lorentz transformations, except for the above mentioned special rotations. However, it can be shown[3] that in the partially polarized case, the effect of rotations is the same as in the completely polarized case. Therefore the Stokes parameters for partially polarized light also transform according to equations 1 to 4.

Our results indicate that the observed percentages of polarization and the position angle of the polarization ellipse do not depend on any relative motion between the observer and the source. Polarization measurements may thus be made of any object without concern about relativistic effects on the polarization parameters.

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Discrepancies in Measurements of the Jupiter Atmospheric Scale Height

J. C. BHATTACHARYYA has presented observations[1] of the occultation of Beta Scorpii by Jupiter on May 13, 1971, from which he concludes that the scale height of the Jupiter atmosphere is 3 km. In his discussion he mentions work by Baum and Code[2] referring to the 1952 observations of the occultation of Sigma Arietis and by Fairall[3] reporting cinematography of the 1971 events. The observation by Baum and Code gave a scale height of 8 km and raised serious questions because such a small value indicated a high molecular weight for the Jupiter atmosphere, supposedly due to a high helium content. If Bhattacharyya's result is correct, then the anomaly is even more pronounced.

Other observations of this same event have appeared in the literature[4], with results implying mean scale heights in the range 24 to 31 km, with a possible dependence on Jovian latitude. Were one to compare the results so far published uncritically, the great discrepancy might tend to discredit the method entirely. We wish to point out several factors and independent lines of evidence which indicate that some results for the mean Jovian scale height are far more reliable than others.

The effective temperature of Jupiter is 134±4 K[5]; the upper atmosphere temperature must also be of this order of magnitude or greater. A scale height of 3 km implies an upper atmosphere temperature of $\sim 10\mu$ K, where μ is the mean molecular weight. Consequently, such a small scale height would require an atmosphere of methane or ammonia (the only constituents other than hydrogen so far identified in the Jovian spectrum). But it is well known that hydrogen is approximately 1,000 times more abundant in the Jovian atmosphere than either methane or ammonia[6], and the currently-accepted hydrogen abundance of ~60 km-atmosphere is entirely consistent with a scale height of 25–30 km and a cloud layer at a pressure of about 1 atmosphere. Indeed, the presently available Jovian abundances (deuterium excepted[7]) are consistent with solar composition, and interior models of the planet[8] show that it is predominantly of hydrogen, with the hydrogen well mixed into the atmosphere.

Having dealt with the matter of consistency of the observational results with independent evidence, we turn to the observations themselves. First, we note that nine different light curves were published by the Texas group alone[4], two of these using an entirely different observational technique at Hartbeespoort (Stevenson and Pel). All are consistent with the quoted result, one order of magnitude greater than Bhattacharyya's result for the scale height. This is also true of the preliminary results from the four light curves obtained by the Meudon group[9]. It can be shown[4] that improper monitoring of the Jovian background can produce wildly erroneous results; we suggest that this is the main origin of scale-height values which are discrepant with plausible models of the Jovian atmosphere.

Some observations of the Beta Scorpii occultation remain to be published[10]; we suggest that the results be weighed with attention to the soundness of the observational technique and the plausibility of the derived values.

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