Transformations to a Rotating Frame

In a recent article on transformations to rotating coordinates, Atwater[1] has discussed an apparent paradox. The purpose of this communication is to deny that such a paradox exists.

Ehrenfest[2] first noted that the geometry of a uniformly rotating disk is non-Euclidean and the matter was fully discussed by Moller[3]. The distance between two adjacent points is given by

$$ds^2 = dr^2 + \frac{r^2 d\theta^2}{1 - \frac{\omega^2 r^2}{c^2}} \quad (1)$$

where r and θ are the usual polar coordinates, ω is the angular speed of the disk, and c is the speed of light. The departure of the second term from the usual Euclidean form is due to the contraction of standard measuring rods moving with the disk.

Atwater envisages an experiment in which a transparent disk is marked (Fig. 1*a*) with four equal arcs separated by four equal unmarked arcs. If the disk were set into uniform rotation about its centre and light were shone through it, what would an inertial observer see? Atwater claims that the result of such an experiment with $\omega r = 0.86c$ would be as in Fig. 1*b*. Although it is not clear why this should be so, presumably it is because the distance between arcs 1 and 2, 2 and 3, 3 and 4, would be contracted as well as the arcs themselves. This, of course, is paradoxical because there is no asymmetry in the

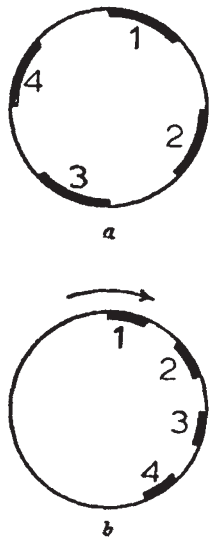

Fig. 1 Atwater's disk paradox. *a*, $\omega r = 0$; *b*, $\omega r = 0.86c$.

physical situation. After reasoning in this way, Atwater rejects equation (1) and with it the notion of the non-Euclidean geometry of a rotating disk. The answer is, of course, that the whole circumference of the disk contracts uniformly and, although each arc would be observed to be shorter, no asymmetry would result. The very lack of the variable θ in equation (1) ensures that this equation can introduce no asymmetry.

Atwater's article contains another misconception. He proposes that if a burst of light were sent through the transparent disk, the recorded result on a photographic film would look like Fig. 1*b*. Terrell[4] has shown that although the Lorentz contraction may be observed (as in the Michelson–Morley experiment) it may not be seen with either eye or camera. This result, together with the symmetry mentioned above, predicts that the outcome of the experiment proposed by Atwater would be exactly as in Fig. 1*a*.

It should be stressed that within the framework of general relativity coordinates are an almost arbitrary labelling of events. Their choice is restricted only by the conditions that distances be positive and the causal relationship of events be retained. Although the coordinate system corresponding to the readings of standard clocks and measuring rods may seem the most physical, it has no priority in nature and may be replaced when inconvenient. Although Atwater objects to it, Schiff's use of Galilean transformations[5] to define the coordinate system for the rotating frame is perfectly valid. The resulting equations are also valid but must be used and interpreted with caution. The time in these equations is not to be read from standard clocks in the rotating frame but from clocks in the inertial frame. Distances are not to be measured with standard rods, but are to be read off an *xyz* grid which corresponded to the inertial grid at $t = 0$.

As further proof that no paradox exists one need only calculate the expression for the distance between two points in the rotating frame, as measured by standard rods, directly from the transformation equations used by Schiff. The expression is that given in equation (1).

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Anomalous Redshifts Reviewed

In a survey of the agreement between reliable optical and 21 cm systemic velocities of 130 galaxies, Roberts[1] finds that the regression line has a slope of one and passes within 1 km s^{-1} of the origin. There is therefore exact statistical agreement between the two rays of determining the systemic velocity, confirming the form of the Doppler relation for wavelengths differing by a factor of 3×10^5 over a velocity range of 5,600 km s^{-1}. But because the optical velocities are usually measured from emission lines in the nuclei, whereas the 21 cm velocities come from the mean velocity of gas spread over the whole plane, Roberts's result also shows that in general the nuclear systemic velocity agrees with that of the disk. It is therefore unlikely that any component of a non-Doppler redshift is present in the nuclear systemic velocities. It is difficult to reconcile Arp's[2] suggested anomalous redshift of small galaxies with Roberts's result. In view of the importance of a substantial non-Doppler redshift, were it to be confirmed, Arp's test for a redshift anomaly is repeated here using a larger sample of data.

Arp[2] looks for the existence of an anomalous redshift by examining the distribution of redshifts of the smaller "companion" galaxies about that of the most massive or "dominant" galaxy in a pair or group. It is implicitly assumed that the dominant galaxy is unaffected by the action of the anomaly, and that to some extent the size of the potential anomaly is a function of the difference in magnitude or mass between the companion and the dominant galaxy. The existence of an anomaly will be shown by an asymmetry in the distribution of redshifts about that of the dominant galaxy, and the existence of a significant displacement of the mean of the distribution to the red.

Testing his hypothesis on the small companion galaxies found in the vicinity of larger objects, Arp[2] looked at only a small sample of the available data. This consisted of the four presumed companions of M31, the NGC 5128 and M81 groups, together with six pairs of galaxies with apparent connexions between the objects. Arp states that these limited data: "essentially exhaust those cases where redshifts are known in groups