A New General Allometric Biomass Model

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Abstract: To implement monitoring and assessment of national forest biomass, it is becoming the trend to develop generalized single-tree biomass models suitable for large scale forest biomass estimation. Considering that the theoretical biomass allometric model developed by West et al. [1,2] was statistically different from the empirical one, the two parameters in the most commonly used biomass equation $M=aD^b$ were analyzed in this paper. Firstly, based on the knowledge of geometry, the theoretical value of parameter b was deduced, i.e., b=7/3(≈2.33), and the comparison with many empirical studies conducted throughout the globe indicated that the theoretical parameter could describe soundly the average allometric relationship between aboveground biomass M and D (diameter on breast height). Secondly, using five datasets of aboveground biomass which consisted of 1441 M D pairs of sample trees, the new general biomass allometric model was validated. Finally, the relationship between parameter a and wood density p was analyzed, and the linear regression was developed. The new model, which is not only simple but also species-specific, offers a feasible approach on establishment of generalized biomass models for regional and national forest biomass estimation.

Key words: allometric model; biomass estimation; geometry; theoretical parameter; wood density

1. Introduction

Since forest ecosystem plays an irreplaceable role in regulating global carbon balance and mitigating global climate change, the forest biomass monitoring is becoming more important all over the world [3]. It is becoming the trend for implementing the monitoring and assessment of national forest biomass to develop generalized single-tree biomass models suitable for large scale forest biomass estimation. In fact, a lot of efforts for large scale forest biomass estimation have been made in the world, and many researchers have attempted to establish generalized single-tree biomass models suitable for national, regional even global forest biomass estimation [4-12].

It is worth special attention that based on branching networks and biomechanics of trees or vascular plants, West et al. [1,2] presented a general allometric model, and derived such a formula $D \propto M^{3/8}$ (D-diameter of tree, M-mass of tree) from complicated derivations based on several assumptions: (1) the branching network is volume filling; (2) the leaf and petiole size are invariant; (3) biomechanical constraints are uniform; and (4) energy dissipated in fluid flow is minimized. Appling the formula above to aboveground biomass estimation, it meant the power parameter b=8/3 (≈2.67) in the allometric biomass model $M=aD^b$. Thereafter, the theoretical model (simply called WBE model) attracted broad attention. Chojnacky [5] thought the methods for biomass estimation in the Forest Inventory and Analysis (FIA) of USFS were different among FIA regions, and the WBE model might offer a possible approach for improving the biomass estimation. Zianis & Mencuccini [13] compared three methods, including WBE model, for simplifying allometric equations of aboveground biomass, and the results showed that the average b value calculated from the 279 compiled studies was statistically different from the theoretical one (2.67) and equals 2.37. Zianis & Radoglou [14] validated the WBE model against a pooled dataset which consisted of 764 M D pairs compiled from empirical studies conducted throughout the globe and for several tree species, and the results indicated that the WBE model failed to describe the shape in *M* *D* allometry for the empirical datasets. Pilli et al. [15] analyzed the *a* and *b* values of different stages of forest development, and found that all the *b* values estimated for the adult stage were not statistically different from 2.67 in the WBE model while 14 out of 30 values estimated for the mature stage were significantly different from the theoretical one, and the *a* values were highly related to wood density.

In this paper, a new general allometric biomass model was presented based on the knowledge of geometry, and was validated against a pooled dataset which consisted of 1441 *M* *D* pairs from destructive sampling and against many references compiled from empirical studies conducted throughout the globe. The new model may provide a feasible approach on simplifying regional and national forest biomass estimation.

2. Data

The data used in this study include two parts. The 1st part is aboveground biomass data from destructive sampling, including three datasets: (i) 447 sample trees collected by the National Biomass Modeling Program in 1997 from two regions of north-east and south of China. In the north-eastern region, 295 trees for eight tree species (or species groups) were sampled; and in the southern region, 152 trees for three tree species were sampled. (ii) 694 sample trees collected in Guizhou province in 2007 for establishment of forestry tables for Chinese fir (*Cunninghamia lanceolata*) and Masson pine (*Pinus Massoniana*). The numbers of trees for Chinese fir and Masson pine were 399 and 295 respectively. (iii) 300 sample trees collected by the National Biomass Modeling Program for Continuous Forest Inventory in 2009 from two regions of north-east and south of China for two tree species of larch *(Larix)* and Masson pine, and 150 trees for each species. The total number of sample trees in the three datasets is 1441. Diameter at breast height of each sample tree was measured in the field. After the tree was felled, the total length of tree (tree height) and length of live crown were also measured. The trunk was divided into 11 sections on the points of 0, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 tree height, and the base diameters of all sections were measured from which the tree volume was computed using Smalian's formula. In addition, the fresh weights of stem wood, stem bark, branches, and foliage were measured respectively, and subsamples were selected and weighed in the field. After taken to the laboratory, all subsamples were oven dried at 85℃ until a constant weight was reached. According to the ratio of dry weight to fresh weight, each compartment biomass could be computed and the above-ground biomass of the tree was obtained by summation. The 2nd part is the parameter estimates of aboveground biomass equations compiled from empirical studies conducted throughout the globe, including the parameter estimates of 146 equations in North American and 61 equations in the Europe, and parameter estimates from references in USA, Canada, Mexico and China.

3. The New Model

According to the viewpoint of classical geometry, the dimension of a regular object is integer. However, the natural objects are generally irregular, whose dimension can be described by fractal geometry [16]. One of the characteristics of fractal geometry is that they can be used to describe the irregular objects by a non-integer dimension [17]. Theoretically, tree shapes can be described as hybrid objects of surface and volume, since they are neither three dimensional solids, nor two dimensional photosynthetic surfaces, so the dimension should be between two and three. If extended to biomass estimation of single tree, then the parameter b in model $M=aD^b$ can be regarded as the dimension whose value is between 2 and 3; if the model is formed as $M=aD^bH^c$, then $2<b+c<3$ [13]. For the commonly used one variable model, West et al. [2] presented that the theoretical value of b was equal to 8/3, but Zianis & Mencuccini [13] and Zianis & Radoglou [14] validated the WBE model against large numbers of data, and concluded that the theoretical value failed to describe the shape in M D allometry, and was statistically different from empirical value and positively biased. Based on the knowledge of geometry, the value of parameter b was analyzed as follows.

Firstly, let us look at the shape of a trunk. The comparison of straight lines and stem taper of Chinese fir [18] is showed in Fig.1. The irregular trunk of a tree can be described approximately by a cone. We know that the area of cross section at the base of trunk scales as $A \propto D_0^2$ where D_0 is the diameter of cross section which is two-dimensional; and the volume of cylinder composed by the cross section and tree height H scales as $V \propto D_0^3$ (if $H \propto D_0$) where the cylinder is three-dimensional. To be deduced by analogy, if volume of the cone composed by cross section and tree height H scales as $V \propto D_0^b$ (if $H \propto D_0$), then b value can be regarded as the non-integer dimension of the cone. Since volume of the cone is equal to 1/3 volume of the cylinder, from intuitive derivation, we can result b=2+1/3=7/3 or b=3-2/3=7/3. Because a cone can be regarded as an approximate description of the stem, the non-integer dimension of the stem is about 7/3 (≈2.33). The crown may possess a similar fractal dimension as stem [19], and the stem contributes about 70%   80% of total aboveground biomass, thus the dimension of overall shape of a tree (stem and crown) may be equal to 7/3, that is to say, for the biomass model $M=aD^b$, we have b=7/3.

(Fig.1 Stem taper vs. line)

Secondly, the value of parameter a was analyzed. On the one hand, since a significant negative relationship between a and b was existed [13,15], even it was not completely appropriate to determine a constant b value (such as 2.33), but the effect could be compensated from value a in large extent. On the other hand, since value a was highly related to wood density p [15], a regression between parameter a and wood density p could be established. Because wood density is one of the important properties of tree species, the differences of aboveground biomass between various tree species may be reflected mainly through parameter *a*. It implies that the biomass models are the same for tree species with the same wood density. Therefore, the tree species with close wood densities may be grouped to establish aboveground biomass models.

4. Validation of the Model

From the analysis above, we can result the general aboveground biomass model $M=aD^b$, where b=7/3 (≈2.33). Whether the new model can describe the biomass data or not, two approaches will be taken for validation: (i) previous study results and parameter estimates from references available all over the world were used for comparison and analysis; (ii) aboveground biomass data from destructive sampling were used for modeling and examination.

4.1 Comparison with previous study results

Zianis & Mencuccini [13] reported the average *b* value resulting from the 279 compiled studies be equal to 2.3679 which was statistically different from the WBE model (2.67) but very close to the theoretical value 2.33 presented above. Resulting from the 146 aboveground biomass equations for 65 tree species in North American compiled by Ter-Mikaelian & Korzukhin [20], the mean value *b* equals to 2.33, and the median value is 2.35. Resulting from the 61 aboveground biomass equations for 39 tree species in the Europe compiled by Zianis et al. [21], the mean value *b* equals to 2.30, and the median value is 2.33. Resulting from the aboveground biomass equations for 24 tree species in Canada established by Fournier et al. [22], the average *b* value is 2.33. Resulting from the generalized aboveground biomass equations for 10 tree species groups in USA developed by Chojnacky [5], the average *b* value is 2.33. Resulting from the generalized aboveground biomass equations for 7 tree species groups in the Europe developed by Muukkonen [10], the average *b* value is 2.27. Resulting from the aboveground biomass equations for 7 kind of pines (n=721) in Mexico developed by Návar [12], the average *b* value is 2.29 (because the number of trees for each kind of pine was very different, ranging from 27 to 384, if calculated by weight of tree numbers, the average *b* value is 2.33). Resulting from the aboveground biomass equations for 10 tree species of north-east in China developed by Chen & Zhu [23], the average *b* value is 2.33. Obviously, the average *b* values from the studies above are very close to the theoretical value presented in this paper, and most of them even have no differences. The results for comparison are listed in Table 1.

Table 1 Comparison of *b* values of aboveground biomass model on one variable

No	Region	Data	Mean of *b*	Range of *b*	Reference source
1	North American	146	2.33	1.35 2.87	Ter-Mikaelian & Korzukhin (1997)
2	USA	10	2.33	1.70 2.48	Chojnacky (2002)
3	Canada	24	2.33	2.13 2.63	Fournier et al. (2003)
4	Globe	279	2.37	1.16 3.32	Zianis & Mencuccini (2004)
5	Europe	61	2.30	1.83 2.81	Zianis et al. (2005)
6	Europe	7	2.27	2.12 2.41	Muukkonen (2007)
7	Mexico	7	2.33[a]	2.16 2.43	Návar (2009)
8	China	10	2.33	1.66 2.79	Chen & Zhu (1989)

[a] Because the number of trees for each kind of pine was very different, so the average *b* value was calculated by weight of tree numbers.

4.2 Validation against observed biomass data

Using the data of aboveground biomass from destructive sampling, the biomass model $M=aD^b$ was fitted by weighted regression where the weight function was $W \quad 1/f(x)^2$ [24]. The fitting results are listed in Table 2. It is obvious that the estimates of parameter *b* are very close to the theoretical value 2.33 presented in this paper, but they are statistically different from the theoretical value 2.67 in the WBE model.

When parameter *b* was set to be 2.33, the five datasets were fitted again using the same method, and the results are listed in Table 3. From the comparison of two statistical indices with Table 2, two sets of models have similar performance, and the goodness-of-fit for NBMP data in 1997 and Masson pine of Guizhou in 2007 in Table 3 is even better than that in Table 2. Because the models in Table 3 have only one parameter, the comparison between different tree species is very simple. For example, from the parameter estimates in Table 3, we know that the difference between Masson pine models of Guizhou and south is less than 5% (the estimate from Guizhou's model is 3.68% larger than that from south's model). In addition, since parameter *a* is highly related to wood density, we can know the difference of wood density between various tree species from the estimates of parameter a. For example, we can conclude that the wood density of Masson pine is higher than that of Chinese fir, and the wood density of larch is higher than that of Masson pine.

Table 2 The fitting results of aboveground biomass model $M=aD^b$

Data	Samples	Diameter range	Parameter estimates		Statistical indices [a]	
	n	(cm)	a	b	R^2	SEE
NBMP data in 1997	447	1.6 69.7	0.1046	2.4098	0.9123	101.44
Chinese fir of Guizhou in 2007	399	4.1 36.4	0.0811	2.3815	0.9143	22.90
Masson pine of Guizhou in 2007	295	4.0 44.8	0.1028	2.4094	0.8965	45.65
Masson pine of south in 2009	150	1.5 47.2	0.1122	2.3650	0.9547	49.92
Larch of north-east in 2009	150	1.7 44.1	0.1309	2.3418	0.9594	47.31

[a] R^2 is the determination coefficient, SEE is the standard error of estimate. It is the same in Table 3.

Table 3 The fitting results of aboveground biomass model $M=aD^{2.33}$

Data	a	R^2	SEE
NBMP data in 1997	0.1304	0.9381	85.16
Chinese fir of Guizhou in 2007	0.0928	0.9080	23.70
Masson pine of Guizhou in 2007	0.1269	0.9018	44.40
Masson pine of south in 2009	0.1224	0.9478	53.43
Larch of north-east in 2009	0.1348	0.9583	47.76

Finally, using the data of 447 sample trees from NBMP in 1997, the relationship between parameter a and wood density p was analyzed. The value of wood density p (g/cm^3) of individual tree is equal to the ratio of stem biomass to tree volume outside bark, and the value of parameter a is equal to the ratio of aboveground biomass to $D^{2.33}$. From the regression result of linear model $a=b_0+b_1p$, we concluded that the intercept b_0 was not statistically different from 0, then the relationship between parameter a and wood density p could be simplified as $a=kp$. The fitting result was as follows:

$a=0.3027p$ ($R^2=0.9529$, $SEE=0.0296$, $F=9023$, P 0.0001, $n=447$) 1

If all sample trees were used, then the fitting result was:

$a=0.2967p$ ($R^2=0.9536$, $SEE=0.0269$, $F=29604$, P 0.0001, $n=1441$) 2

From the models (1) and (2), we know that if the information of wood density for some tree species was available, then the estimate of parameter a would be obtained.

In summary, the general aboveground biomass model of single tree can be expressed as $M=aD^{7/3}$, where $a≈0.3p$, and p is wood density.

5. Conclusion and Discussion

Starting from the WBE model presented by West et al. [1,2], two parameters of the general biomass model $M=aD^b$ for large scale biomass estimation were studied in this paper. Based on the knowledge of geometry, the theoretical value of parameter b was deduced, i.e., b=7/3(≈2.33), and the comparison with many empirical studies conducted throughout the globe indicated that the theoretical parameter could describe soundly the average allometric relationship between aboveground biomass M and D (diameter on breast height). In addition, five datasets of aboveground biomass which consisted of 1441 M D pairs of sample trees were used for validation to the new model. Finally, the relationship between parameter a and wood density p was analyzed, and the regression model was established. The result showed that a significant positive relationship was existed between parameter a and wood density p, and the estimate of parameter a could be obtained approximately by multiplying 0.3 to wood density p. The new model not only be simple, but also reflect the difference of aboveground biomass for various tree species, which offers a feasible approach on establishment of generalized biomass models for regional and national forest biomass estimation.

Of course, the theoretical value of parameter b presented in this paper is only based on intuitive deduction and empirical studies. Indeed, from the analogous deduction, the stem biomass model and even tree volume model on one variable should have the similar estimates of parameter b. Because the references available about stem biomass and tree volume are limited, only several study results on stem biomass are provided as follows: (i) resulting from the 134 stem biomass equations for 65 tree species in North American compiled by Ter-Mikaelian & Korzukhin [20], the mean value b equals to 2.31; (ii) resulting from the 30 stem biomass equations for 11 tree species in the Europe compiled by Zianis et al. [21], the mean value *b* equals to 2.28; (iii) using the stem biomass data of 447 sample trees for 11 species from the NBMP in China in 1997, a model on one variable was fitted where the estimate of parameter *b* was 2.32. Obviously, the values of parameter *b* are not statistically different from the theoretical one 2.33. Because the taper equations of various tree species are different in some extent, thus 2.33 is only an approximate average value of parameter *b*.

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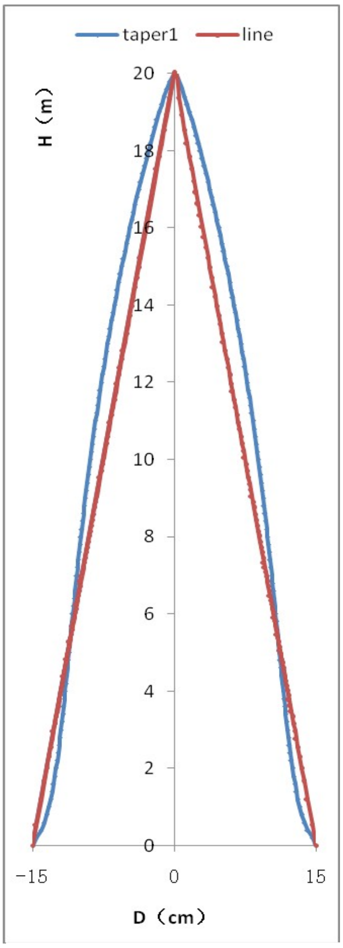

Fig.1 Stem taper vs. line