Optimal decision bounds for probabilistic population codes and time-varying evidence

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Introduction

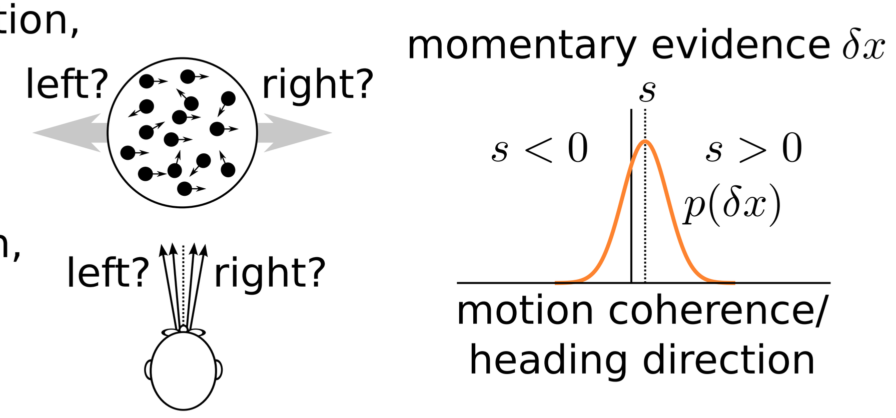
Decision making under time constraints requires trading off between making quick, inaccurate decisions and gathering more evidence for more accurate, but slower decisions. Under rather general settings, optimal behavior can be described by a time-dependent decision bound on the decision maker's belief of being correct[1]. Such a bound corresponds to a bound on the most active neuron in simple neural models with two perfectly anti-correlated neurons, but only if the reliability of the sensory evidence is time-invariant. In more realistic neural population codes[2] we show that the optimal decision bound is on the activity of all neurons rather than the previously populated bounds on its maximum activity. The theory predicts that the bound on the most active neurons would appear to shift depending on the firing rate of other neurons in the population, a puzzling behavior under the drift diffusion model as it would wrongly suggests that subjects change their stopping rule across conditions. This theory also applies to the case of time varying evidence, a case that cannot be handled by the simple two-neuron model.

Optimal Decision Making

Two-alternative forced choice tasks

Coarse discrimination, e.g. random dot kinetogram

Fine discrimination, e.g. heading discrimination



For both: integrate momentary evidence to find sign of s

Optimal integration/decision time[1]

Objective function: maximize reward rate / temporally discounted reward / number of correct decisions / ...

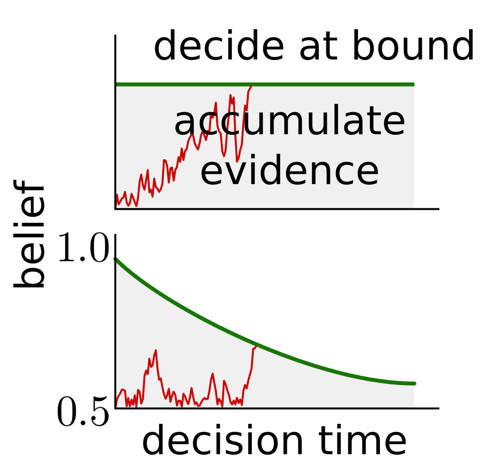
is function of ∘probability of correct choice
∘decision time

Maximising objective by **time-dependent bound on belief of choosing correctly**

Optimal bound examples

flat belief bounds
e.g. single evidence strength

collapsing belief bounds
e.g. repeated trials
multiple evidence strengths



Prerequisits for optimal decision making

- Time-dependent bound on belief of being correct
- Belief of being correct corresponds to probability of being correct

"Two Neural Pools" Model

Setup

Correlated noisy inputs

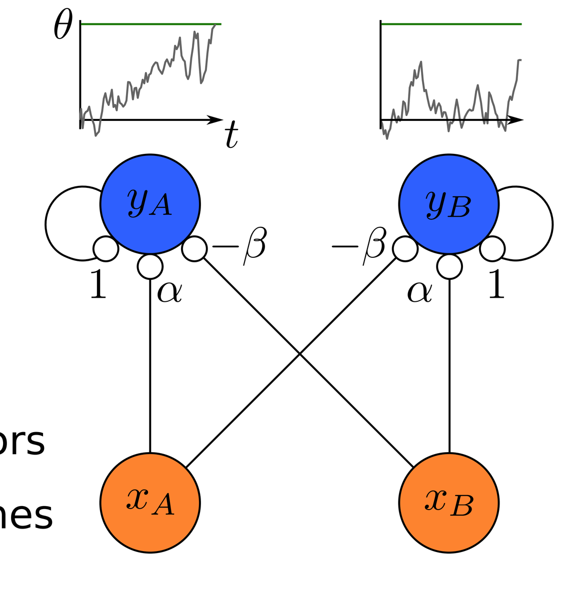
$x_A = d + ks + \eta_A\sqrt{1-\nu} + \eta\sqrt{\nu}$
$x_B = d - ks + \eta_B\sqrt{1-\nu} - \eta\sqrt{\nu}$

Correlated integrators

$\dot{y}_A = \alpha x_A - \beta x_B$
$\dot{y}_B = \alpha x_B - \beta x_A$

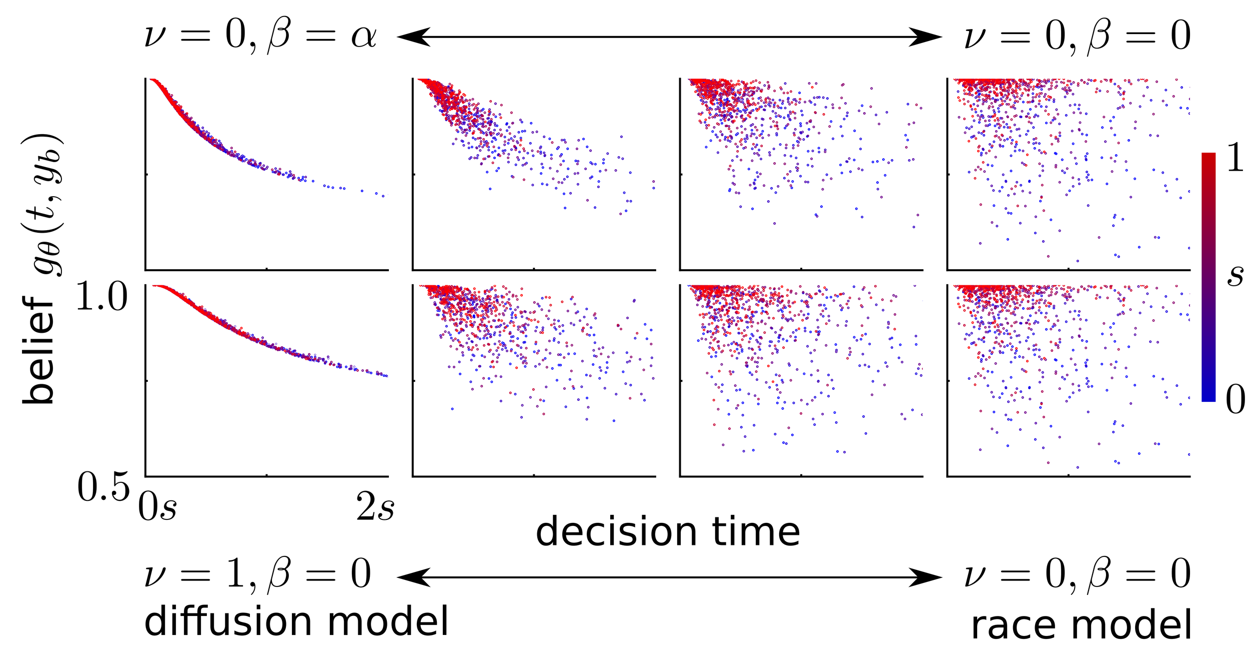
Aim: find sign of s given integrators

Appoach: choose $s>0$ if y_A reaches bound first, $s<0$ otherwise



Is belief at decision a function of only time?

Belief when y_A at bound: $g_\theta(t, y_B) = p(s>0|y_A=\theta, y_B, t)$

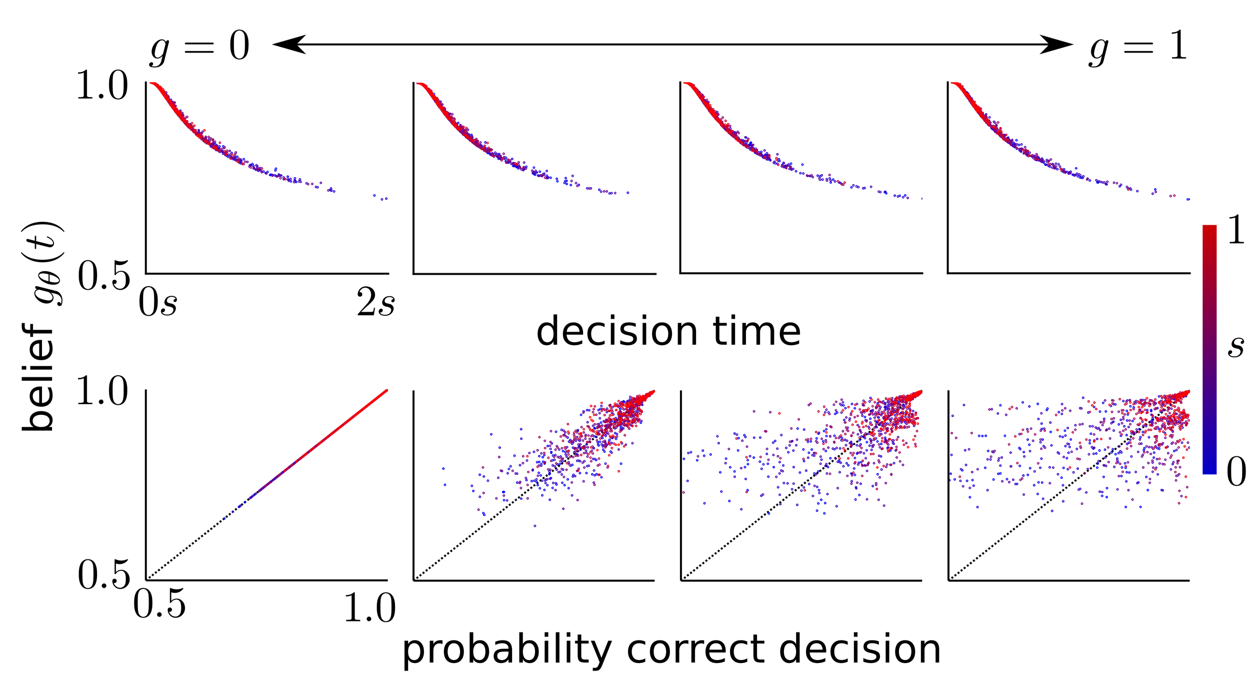


Belief function of only time, $g_\theta(t, y_B) = g_\theta(t)$, only if integrators completely anti-correlated (diffusion model), which requires well-tuned network with $\alpha=\beta$, or anti-correlated input, $\nu=1$

Otherwise $y_B \neq -y_A = -\theta$ at decision, such that belief also depends on state y_B of "losing" race.

Does belief of being correct equal probability correct?

Assume $\alpha=\beta$ and time-varying evidence strength, $k(t) = k(1+g\sin(2\pi t f))$ with $f=5\text{Hz}$ and varying gain g



Belief in diffusion model only corresponds to performance if evidence strength is constant over time.

Otherwise diffusion model incapable of tracking correct sufficient statistics (see next panel).

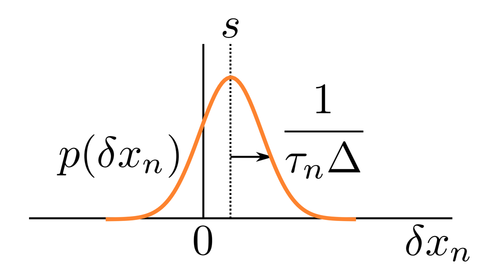
Inference with Gaussian Evidence

Integrating Gaussian Evidence

Momentary evidence

$p(\delta x_n|s) = \mathcal{N}\left(\delta x_n | s, \frac{1}{\tau_n\Delta}\right)$

precision/
evidence strength

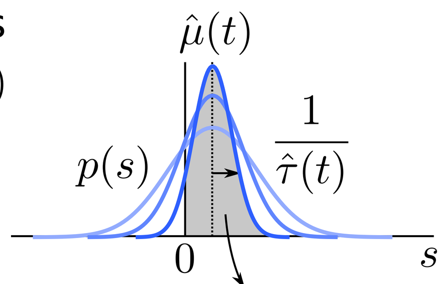


Posterior natural sufficient statistics

$p(s|\delta x_1, \delta x_2, \dots) = \mathcal{N}(s|\hat{\mu}(t), 1/\hat{\tau}(t))$

with $\hat{\mu}(t)\hat{\tau}(t) = \sum \tau_n \delta x_n$

$\hat{\tau}(t) = \sum \tau_n \Delta$



Belief is function of sufficient statistics, $g(t) = \Phi\left(\hat{\mu}(t)\sqrt{\hat{\tau}(t)}\right)$

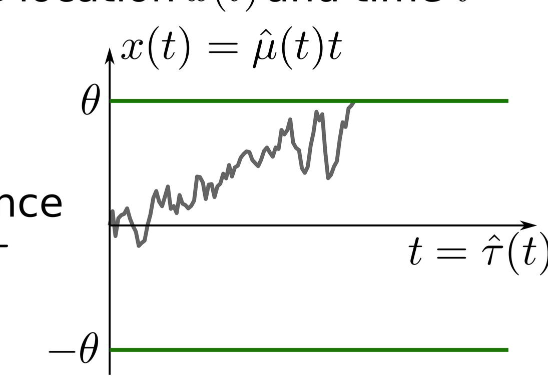
Optimal decision making by bounded integration in two-dimensional space of natural sufficient statistics

Diffusion Model representation

Sufficient statistics are particle location $x(t)$ and time t

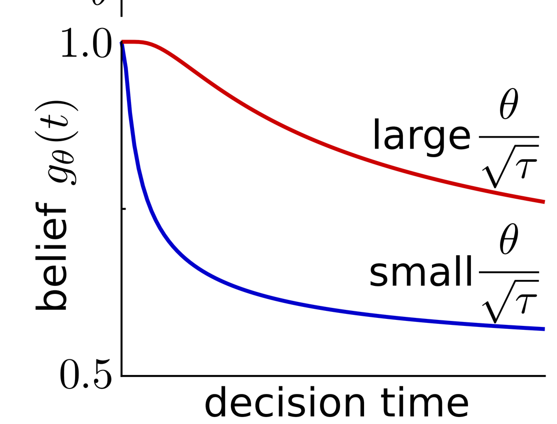
$\hat{\mu}(t)\hat{\tau}(t) = \tau x(t)$

$\hat{\tau}(t) = \tau t$ ← Only works for constant evidence strength, $\tau_n = \tau$



Constant bound on $x(t)$ equals collapsing bound on belief

$g_\theta(t) = \Phi\left(\theta\sqrt{\tau/t}\right)$



Propabilistic Population Codes[2] (PPCs) representation

PPCs assume Poisson-like encoding, $p(s|r) \propto e^{\boldsymbol{h}^T(s)\boldsymbol{r}}$

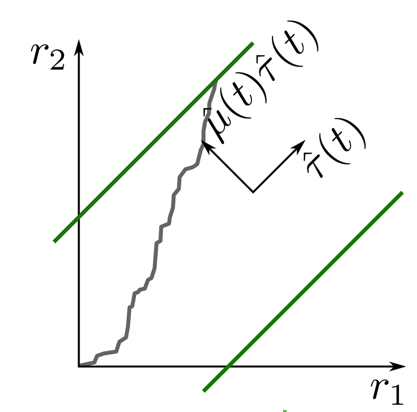
Gaussian s encoded by $\boldsymbol{h}(s) = -\frac{1}{2}\boldsymbol{b}s^2 + \boldsymbol{a}s$, resulting in

$\hat{\mu}(t)\hat{\tau}(t) = \boldsymbol{a}^T\boldsymbol{r}(t)$

$\hat{\tau}(t) = \boldsymbol{b}^T\boldsymbol{r}(t)$ ⟵ handles time-varying evidence by encoding both sufficient statistics

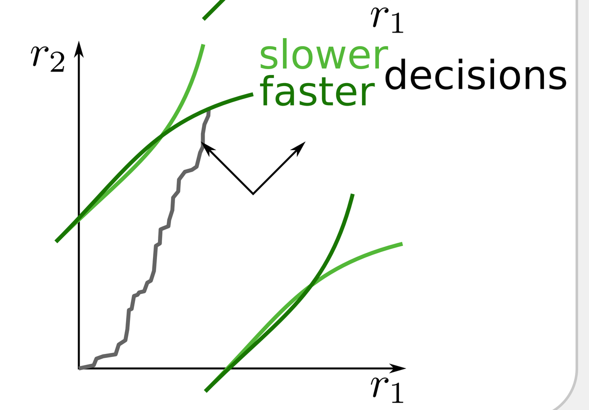
Diffusion model bound θ equals linear bound on $\boldsymbol{r}(t)$

$\boldsymbol{a}^T\boldsymbol{r}_\theta(t) = \theta\tau$



Optimal bound on belief $g_\theta(t)$ equals non-linear bound on $\boldsymbol{r}(t)$

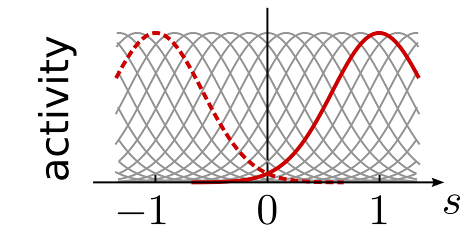
$$\frac{\boldsymbol{a}^T\boldsymbol{r}_\theta(t)}{\sqrt{\boldsymbol{b}^T\boldsymbol{r}_\theta(t)}} = \Phi^{-1}(g_\theta(t))$$

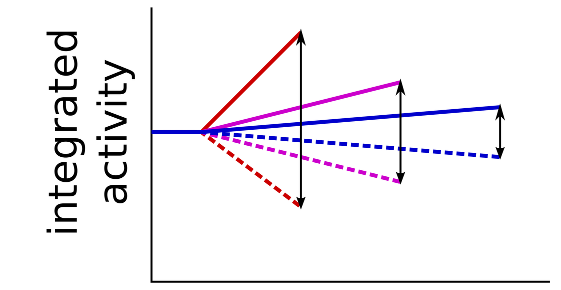


Optimal Decision Making in the Brain

The optimal strategy

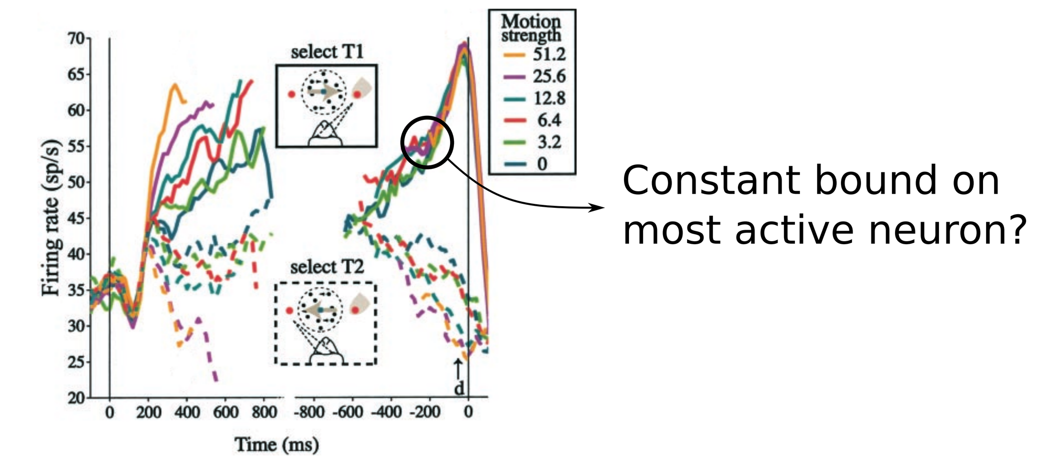
Assume neural population





Activity different between winning & losing integrator only depends on time

Does the brain use a winner-take-all strategy?

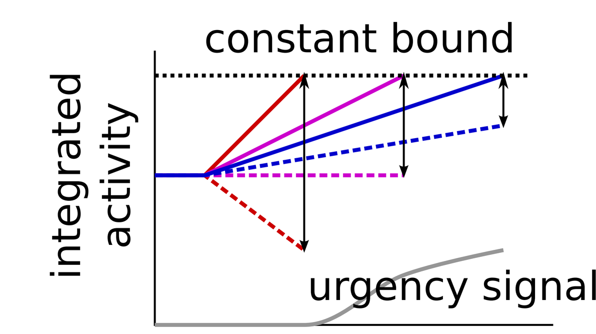


Roitman & Shadlen (2002)

Distinguishing optimal strategy from winner-take-all

Assuming time-invariant evidence strength, $\tau_n = \tau$ neural activity difference at decision time is roughly function of time

Optimal strategy well approximated by constant bound time-varying "urgency signal" to all neurons



- Is a constant bound biologically convenient, and the brain uses such approximation?
- Is the brain using the optimal strategy, and we just can't tell the difference?
- Time-varying evidence tasks should reveal the difference

Summary

- Optimal decision making requires
  - time-varying bound on belief of being correct, and
  - belief that corresponds to probability of being correct

  "Two Neural Pools" model fails in both respects
- Optimal decision making in PPCs requires the decision bound to be a function of activity of all neurons in the population, rather than a bound on maximum activity
- Hypothesis might be testable in tasks with time-varying evidence

References

[1] Drugowitsch, Moreno-Bote, Pouget (2009). Computing the cost function in decision making. COSYNE 2009.

[2] Ma et al. (2006). Bayesian inference with probabilistic population codes. Nature Neuroscience 9, 1432-1438.