N semiconductor quantum wells, each characterized by an individual radiation decay rate, γ, and stacked in a periodic linear array whose spacing corresponds to half the characteristic exciton wavelength λ_x (Bragg condition) to ensure that the wells couple in phase[3]. For such a structure, there is one superradiant mode that radiates N times faster than a single well and N–1 optically inactive modes. For Bragg spacing the reflections of a weak probe (even one photon at a time) from all of the quantum wells add in phase resulting in a one-dimensional photonic crystal; theory predicts that the linewidth of the reflectivity stopband should be proportional to $N\gamma$, which has been verified experimentally[4] for InGaAs wells for N up to 100. The corresponding $1/N$ shortening of the radiative lifetime of the superadiant mode has been observed using a technique involving resonant coherent pulses.

Collective Dicke states can also be observed by placing N atoms in a resonant cavity of small enough volume and high enough Q (quality factor) to see a normal-mode splitting — dubbed the vacuum Rabi splitting — in the eigenvalue spectrum of the atom-cavity system[5]. As it is not known which of N identical atoms is excited by a single photon, then the effective dipole moment for the ensemble will be $\sqrt{N}$ larger than for a single atom. This cooperative enhancement makes the vacuum Rabi splitting (which is directly proportional to the total dipole moment) $\sqrt{N}$ larger for N atoms than for a single atom, and that is why it took so much longer to see vacuum Rabi splitting with a single atom[6]. Likewise in the semiconductor case, vacuum Rabi splitting is much harder to see with a single quantum dot than with a single quantum well[6].

To this point we have discussed superradiance of N indistinguishable ground-state oscillators probed one photon at a time. Their cooperation arises from a fundamental rule of quantum mechanics — that if in a given experiment it is impossible, even in principle, to distinguish between the paths that lead from a given initial state to a given final state, then the net transition amplitude will be given by the sum of the individual transition amplitudes for each path. Such many-atom cooperation not only gives rise to atomic superradiance[7], but also to atomic four-wave mixing, and matter-wave amplification previously attributed to the bosonic statistics of a Bose–Einstein condensate[8]. And it is used in a three-level scheme to prepare a many-atom 'superatom' in a phased array for very directional emission by detecting a photon in the desired direction — and thereby is a means of generating atomic-ensemble photons on demand[9,10].

But what if initially all N atoms are in the excited state? This is the situation considered in the case of Dicke's superradiant optical bomb[1,11], an unsuccessful proposal for generating short coherent pulses of light without a cavity, in the years before the invention of the laser. If all of the atoms are much closer than a cubic wavelength, the spontaneous emission from the atoms is cooperative — the N atoms return to their ground states by spontaneously emitting a pulse of coherent light in a decay time decreasing as $1/N$, making the peak intensity scale as N^2. This represents the ultimate superradiance condition in which each and every excited dipole in the system decays simultaneously, radiating all the energy stored in the system[11]. However, the closest experiments have come to demonstrating such optical superradiance is through a related effect known as superfluorescence, in systems in which inverted atoms are spread out over a distance of many wavelengths (making dipole–dipole dephasing negligible) in a pencil-shaped geometry; propagation

SEARCH ALGORITHMS

Generality found

Knowing how to derive the shape of a protein from X-ray diffraction data is one thing — but could it also help solve a Sudoku puzzle? Yes, say Veit Elser and colleagues (*Proc. Natl Acad. Sci. USA* **104,** 418–423; 2007). They argue that their variant of a strategy used for decades in image reconstruction provides a general approach to tackling the kind of problem for which a solution can usually only be found by searching through a (typically large) set of possibilities. They have explored some of these problems — from protein folding, to identifying the ground state of spin glasses, to placing numbers in the correct squares of a Sudoku grid — and found that their general-purpose algorithm can keep up with those developed specifically for the respective applications.

Elser *et al.* realized that, in many search problems, two constraints can be formulated, such that the solution can be found in the set of elements that satisfy these two constraints simultaneously. Solving the overall problem might be hard, but finding a solution satisfying one constraint can be easy, as is finding a solution consistent with the other constraint. Take Sudoku for example: one constraint is that each block has to contain a permutation of the numbers 1 to 9, and the other is that each number appears only once in each row and column. Beginning with a random set of numbers on the Sudoku grid, it is relatively easy for a computer to find a valid solution that is consistent with either constraint — but it takes some artistry to arrive at a set of numbers that satisfies both at the same time.

And this is where the techniques of image reconstruction come into play. In the case of X-ray diffraction data, so-called iterated-map algorithms have been developed and refined since the 1980s. These algorithms do exactly what is required: with high computational efficiency, they direct a random 'point' in a particular search problem on a 'path' towards a point that satisfies simultaneously two competing constraints. Elser and colleagues' survey demonstrates, at last, the potential broader scope of iterated-map algorithms. For example, newspaper Sudokus can be solved using the algortihm in a matter of milliseconds; the puzzle pictured — contributed by 'Nick70' to the site www.setbb.com/phpbb, and the trickiest test of the approach so far — was solved in three seconds. How fast can you solve it?

3	7		6					
		9						
	6			2		1	8	
					5			
	2			1			9	
			4					
	1	6		9			7	
						5		
					7		4	2

Andreas Trabesinger