The value of the future

Economists have often been criticized — by physicists, among others — for being overly formal with their mathematics. Journals are full of proofs and theorems, and for some economists the aim seems to be to give economics an axiomatic basis akin to quantum field theory. Even so, mathematical formalism has its uses, and can be an invaluable guide in confusing terrain.

Consider, for example, the matter of determining the relative values of things at different times, say, now versus the future. What would you pay today to get £100 in one year? You could put £100 today into an investment and earn, perhaps, 5% compound interest, getting £105; equivalently, just over £95 invested now would give you £100 in a year. So, it seems, £95 is what that year-delayed £100 is worth now.

Of course, there are complications. Inflation matters too, making the £100 still less valuable, as does uncertainty about the future; to be blunt, you might be dead in a year. To find some stable ground on this matter — the proper form of discounting, as it is called — economists have turned to formal logic. For all the competing factors, they note, surely one thing should be true: we should treat time consistently so that subsequent periods of time should all contribute to the discounting in an equal way. This condition of 'time consistency' implies that a discount over a time A+B should be equal to a discount over time A multiplied by a discount over time B.

If this is true, and it seems sensible that it should be, then it's possible to show that exponential discounting is the only possibility. It is, using economists' favourite word, the 'rational' way to discount the future. The right exponent to use may be in doubt, but the exponential form itself is not.

Exponentials, of course, grow (or shrink) very quickly, and this kind of discounting rapidly reduces the value of the future. If the human race has another one million years to exist, it would seem, naively, that the second half of that period should be as valuable as the first half. But exponential discounting with even a small rate gives the latter half-million years essentially zero value.

This effect is hugely important in analyses of things like possible responses to climate change. For example, when the UK government's Stern Report in 2006 suggested that fast action should be taken to

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reduce greenhouse-gas emissions because of the negative long-term consequences, some economists countered that this was an overreaction based on insufficient devaluation of the future. William Nordhaus argued that the value of future economic losses attributed to climate change (or any other concerns about the environment) should be discounted at about 7% per year (*Science* **317,** 201–202; 2007) — far higher than the value of 1.4% used in the Stern review.

As he said, "the key economic variable is the real return on capital, *r*, which measures the net yield on investments in capital, education, and technology. In principle, this is observable in the marketplace. For example, the real pre-tax return on US corporate capital over the last four decades has averaged about 0.07 per year. The return on capital is the 'discount rate' that enters into the determination of the efficient balance between the cost of emissions reductions today and the benefit of reduced climate damages in the future."

Obviously, squabbles over the proper value of *r* make a big difference. But there may also be another more subtle flaw in the logic of discounting, which goes back to the assumption of time consistency. It needn't imply the exponential form, as earlier theorists have thought.

A physicist and an economist, Doyne Farmer and John Geanakoplos, have recently revisited the problem in the case in which the discount rate isn't just a fixed number, but varies randomly through time (as interest rates indeed do in the market). It's tempting to suppose that if discount rates vary, the overall discount factor over a time T should be something like $e^{-<r>T}$, with $<r>$ being the average discount rate over the period. In other words, the average of the product of many exponential factors for bits of time along the way can be expressed as the exponential of an average overall discount rate.

But if the discount rate *r* varies in the right way, it turns out, this simply isn't true. Farmer and Geanakoplos illustrate the effect with several simple models. You might take the discount rate at any moment to be the current interest rate, for example, which moves, according to a standard model in finance, as a geometric random walk. In one simple model, this means that the rate gets multiplied or divided at each moment by a number (say, 1.1) to determine the next rate; a bigger number means more volatility. In such a (realistically) uncertain world, Farmer and Geanakoplos show the exponential discounting function no longer satisfies the time consistency condition. Instead, a different mathematical form is the natural one for discounting, with the discount factor over a time T being proportional to $1/(1 + \alpha T)^{\beta}$, where α and β are constants.

For long times, this form has a power-law tail proportional to $T^{-\beta}$, which falls off far more slowly than an exponential. Hence, the value of the future isn't discounted nearly as strongly. For example, taking the average interest rate as 4%, with a volatility of 15%, Farmer and Geanakoplos show that in 500 years' time the exponential is already discounting values about one million times more strongly than the random process, and it gets worse after that.

This implies a potentially enormous revision in economic analyses performed to date on climate policy (or steps to counter other problems where costs come in the future). Farmer and Geanakoplos don't claim that this geometric random walk model is the only correct one, or even the best one, it's only illustrative of a broad class of statistical processes. But it also isn't obviously unreasonable. The point is that everything about discounting depends very sensitively on the kinds of assumptions made, not only about the rate of discounting but the very process it follows through time.

As they put it, "no fixed discount rate is really adequate", and, as a consequence, "the proper discounting function is not an exponential."

It seems to me this is a finding of potentially staggering importance. What are currently considered the best analyses of some of the world's most pressing problems hinge almost entirely on quite arbitrary techniques for discounting the future. ❐

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