# Experimental bound entanglement?

**To the Editor** — Entangled quantum states can be classified into two types: those that can be distilled[1] into pure entangled states using local operations and classical communication and those that cannot[2]. Undistillable entangled states are called bound entangled. A recent claim of experimental bound entanglement[3] by Amselem and Bourennane is unfounded. In their work, they aimed to produce the Smolin state[4],

$$\rho = \frac{1}{4}\sum_{\mu=0}^{3} |\Psi_\mu\rangle\langle\Psi_\mu|_{AB} \otimes |\Psi_\mu\rangle\langle\Psi_\mu|_{CD} \quad (1)$$

in the polarization of four photons, where A to D label the parties and $|\Psi_\mu\rangle$ are the Bell states, $|\phi^\pm\rangle = (1/\sqrt{2})(|00\rangle \pm |11\rangle)$ and $|\Psi^\pm\rangle = (1/\sqrt{2})(|01\rangle \pm |10\rangle)$. This state is entangled and undistillable[4] and is, therefore, bound entangled. Undistillability for the experimental implementation of this state is guaranteed if the partially transposed density matrix is non-negative across each two–two bipartite cut, (AB):(CD), (AC):(BD) and (AD):(BC) (ref. 2). In other words, one considers the partial transposition of the density matrix across each cut and calculates the eigenvalues. If all of the eigenvalues are positive across these three cuts, the state is positive under partial transposition (PPT) and undistillable.

Amselem and Bourennane demonstrated entanglement in their state using a witness[5]. They applied the PPT test and reported five negative eigenvalues across the three cuts: one instance of $-0.02 \pm 0.02$ and four instances of $-0.01 \pm 0.01$, where the uncertainty is assumed to be one standard deviation.

Just one negative eigenvalue means undistillability has not been demonstrated. Considering the uncertainties and assuming normal distributions, the probability that any one of these five eigenvalues is non-negative is only $\frac{1}{2}\,\mathrm{erfc}(1/\sqrt{2}) \approx 0.159$. Although this is already insufficient evidence of undistillability, if we assume the eigenvalues are uncorrelated, the probability that all five are non-negative is $(0.159)^5$, about 1 in 10,000. Furthermore, Amselem and Bourennane also report eleven 0 eigenvalues with uncertainties and another fourteen that are positive by one standard deviation; under the same assumptions, these lower the probability that all eigenvalues are non-negative by an additional factor of 20,000. Correlations could change these estimates considerably, but they cannot be calculated from the published data.

The Smolin state in equation (1) is not an experimentally robust form of bound entanglement. The state and, most importantly, its partial transpose are not full rank and thus the PPT condition is very sensitive to experimental errors. More robust forms of bound entanglement can be created using states with full- or nearly full-rank partial transposes. This approach was used for the observation of pseudo-bound entanglement in liquid-state NMR[6] and for the first experimental demonstrations of bound entanglement using optics[7] and trapped ions[8]. Recently, a different approach was used to demonstrate bound entanglement in continuous-variable quantum optics[9].

Undistillability is the one property that makes an entangled state bound. Amselem and Bourennane's results fail to show undistillability and thus they cannot claim to have produced bound entanglement. ❒


References

1. Bennett, C. H., DiVincenzo, D. P., Smolin, J. A. & Wootters, W. K. *Phys. Rev. A* **54,** 3824–3851 (1996).
2. Horodecki, M., Horodecki, P. & Horodecki, R. *Phys. Rev. Lett.* **80,** 5239–5242 (1998).
3. Amselem, E. & Bourennane, M. *Nature Phys.* **5,** 748–752 (2009).
4. Smolin, J. *Phys. Rev. A* **63,** 032306 (2001).
5. Gühne, O. & Tóth, G. *Phys. Rep.* **474,** 1–75 (2009).
6. Kampermann, H., Bruβ, D., Peng, X. & Suter, D. *Phys. Rev. A* **81,** 040304 (2010).
7. Lavoie, J., Kaltenbaek, R., Piani, M. & Resch, K. J. *Phys. Rev. Lett.* **105,** 130501 (2010).
8. Barreiro, J. T. *et al. Nature Phys.* doi:10.1038/nphys1781 (2010).
9. DiGuglielmo, J. *et al.* Preprint at http://arxiv.org/abs/1006.4651 (2010).



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**Amselem and Bourennane reply** — The goal of our experiment was to prepare a quantum state as close as possible to the pure four-qubit bound entangled (BE) Smolin[1] state $\rho_S$ and show its entanglement properties.

To investigate the properties of the Smolin state $\rho_S$ experimentally and fully, we have evaluated the four-photon $16 \times 16$ density matrix $\rho_S$ by the quantum state tomography method. The separability across the bipartite AB|CD, AC|BD and AD|BC cuts was tested using the PPT criterion. Our results have shown that all the eigenvalues are positive or zero within experimental error, indicating separability across these cuts. To rule out one–three qubit separability and three-party entanglement, a stabilizer witness was constructed. We have also demonstrated the distillation protocol and violation of the Bell inequality[2].

It is well known that noisy Smolin states of the form $(1-p)\rho_S + p\mathbb{1}^{\otimes 4}/16$ — a mixture of Smolin state and depolarizing noise, where $p$ is the amount of noise, $0 \le p < 2/3$ — are bound entangled states[3]. In recent experiments[4,5], the search for bound entanglement was done by scanning over the family of these noisy Smolin states. Iterative quantum state tomography and Monte Carlo simulations over obtained data, and elaborated error analysis, were then used. Therefore these refinements show the bound entanglement more clearly.

After your comment, we have reanalysed our data and we have found that the errors on the eigenvalues of the partial transpose on the bipartite AB|CD, AC|BD and AD|BC cuts are correlated, thus giving less evidence for the separability criterion. By adding depolarizing noise to our experimental $\rho_S^{\exp}$, we have reinvestigated the entanglement properties of this noisy Smolin state. Our results give strong evidence for bound entanglement for an amount of depolarizing noise with $0.38 \le p < 0.58$. The witness value for these states is negative and the eigenvalues of the partial transpose of all bipartite cuts are all positive. For example, for the less noisy state ($p = 0.38$), the lowest eigenvalue for the PPT criterion is $0.014 \pm 0.009$ and the witness value for three-party entanglement is $-0.54 \pm 0.01$. ❒


References

1. Smolin J. A. *Phys. Rev. A* **63,** 032306 (2001).
2. Amselem, E. & Bourennane, M. *Nature Phys.* **5,** 748–752 (2009).
3. Augusiak, R. & Horodecki P. *Phys. Rev. A* **73,** 012318 (2006).
4. Lavoie, J., Kaltenbaek, R., Piani, M. & Resch, K. J. *Phys. Rev. Lett.* **105,** 130501 (2010).
5. Barreiro, J. T. *et al. Nature Phys.* doi:10.1038/nphys1781 (2010).



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