STATISTICAL MECHANICS

The physics of where to go

Humans tend to explore unknown locations, but preferentially return to familiar places. The interplay between these two basic behaviours accounts for many of the scaling relations observed in human-mobility patterns.

Dirk Brockmann

Interdisciplinary theoretical physics is a difficult business. Frequently physicists who work outside of the traditional realm of physics are ignored or misunderstood by those whose fields they plough — biologists, economists, social scientists, to name but a few. Similarly, all too often their work is labelled as 'nice, but not physics' by conservative peers in the physics community. But the candid model for human mobility that Chaoming Song and colleagues report[1] in *Nature Physics* is unlikely to suffer this fate. Starting from random-walk processes, and taking on board elements from the theory of scale-free networks, they arrive at a conceptually simple model that accounts for a broad range of scaling patterns seen in data that capture human-mobility patterns. Their model, in combination with straightforward back-of-the-envelope scaling arguments, is of distinct statistical-physics flavour, yet highly transparent to behavioural and social scientists, epidemiologists and transportation experts, among many others who rely on a better understanding of human mobility.

Modern mobility is massive and complex[2]. It has dramatically changed over the past few decades — today we can, in principle, travel to any place on the globe within a day or two. More than three billion passengers travel each year on the global air-transportation network that connects more than 4,000 airports worldwide. Hundreds of millions of commuters travel to work each day on an intricate web of highways and public transportation systems that often operate at their maximum capacity. Our mobility plays a key role in the rapid global spread of emergent infectious diseases — the latest example being the worldwide spread of pandemic influenza H1N1 in 2009 — and in human-mediated bioinvasion, which is a key factor in the global biodiversity crisis (that is, humans aid the relocation of non-endemic species to new habitats where they proliferate and potentially extinguish endemic species). In the light of these phenomena, research that advances our understanding of how we travel is vital.

A promising avenue of research emerged a few years ago when researchers began to analyse large-scale datasets tracing individual human movements. The data are typically generated either directly or indirectly by modern technologies such as mobile phones or precise GPS devices, combined with websites that collect records of individuals' locations. Pervasive data of this type reveals aspects of when and where we go with unprecedented spatiotemporal precision. One of the earliest quantitative discoveries was made by analysing the circulation of banknotes[3] (and subsequently confirmed by a more detailed study on mobile phones[4]). Looking at the statistics of distances travelled (r) and interjourney times of rest (t), it was found that the probabilities of travel distance, $p(r)$, and rest times, $p(t)$, both follow an inverse power-law: $p(r) \sim r^{-(1+\alpha)}$ and $p(t) \sim t^{-(1+\beta)}$, where $\alpha = 0.8$ and $\beta = 0.6$. This implied that human mobility had anomalous properties both spatially and temporally — it lacks a characteristic scale and is fractal as well as self-similar.

What does this mean for modelling human mobility? For instance, in continuous-time random walks, which are frequently employed to model random processes in physics and biology, the first relation typically yields superdiffusive behaviour (that is, the average squared displacement increases faster than linearly with time), whereas the second relation yields subdiffusion. If human trajectories were indeed random walks, an individual's position would scale with time according to $X \sim t^{\alpha/\beta}$. It doesn't require a degree in physics to realize that trajectories of individuals are not purely random. In fact, for random walks as described above, the expected time to revisit a point in space is infinite, which clearly is at odds with most humans' habit of returning home after work.

Song and colleagues[1] show to what extent real human-mobility patterns deviate from those expected from simple random-walk predictions. But this is not why their model is important. It is important because they propose a slightly more intricate random-walk model that, unlike the simple continuous-time random walks, can account for many of the empirical scaling relations observed in mobility data. Studying the same dataset on the trajectories of mobile-phone users that was investigated in earlier studies[4,5], Song *et al.* focus on two key quantities: the number of new locations visited as a function of time, $S(t)$, and the visitation rank frequency f_k of those locations (which measures how often an individual goes to the kth-most-visited location).

The model[1] has two basic dynamic ingredients: exploration and preferential return. More specifically, Song *et al.* assume that at every step of the process an individual can explore unvisited locations with a probability $p \sim \rho S^{-\gamma}$, where the pre-factor ρ and the (strictly positive) exponent γ are the two parameters of the model. This means that the more sites there are in a person's individual 'network of places', the less likely it is that he or she will explore new places. With the complementary probability a person returns to previously visited places, choosing between the set of known places according to their rank probability — a person is more likely to return to places already visited many times. Song and colleagues refer to this behaviour as 'preferential return'. The main principle behind preferential return is the same as in 'preferential attachment', a mechanism for the growth of scale-free networks. Preferential return generates a strong heterogeneity in the set of locations that a person visits. Combined with the exploratory component, the mobility model of Song *et al.*[1] accounts for many of the observed scaling laws.

The parameter γ in the model of Song *et al.*[1] turns out to be 0.2. This of course invites the question of why this exponent, which captures our exploratory behaviour so well, has precisely this value. The present model cannot address this question — and doesn't need to. But the simplicity of the key principles of exploration and preferential return are most likely to trigger new investigations in other contexts, as these principles are so common in human decision processes. We can therefore look forward to diverse applications of the model of Song *et al.*[1]. For example, one may wonder if the same basic mechanisms determine what restaurants we visit, what recipes we try if we decide to dine at home or what locations we pick for a summer holiday. Once data for these contexts are available we should expect to see similar patterns emerging. ❐

Dirk Brockmann is in the Department of Engineering Sciences and Applied Mathematics, Robert R. McCormick School of Engineering and Applied Science, Northwestern University, at the Northwestern Institute on Complex Systems, and in the Northwestern University Transportation Center, Evanston, Illinois, USA.
e-mail: brockmann@northwestern.edu

References
1. Song, C., Koren, T., Wang, P. & Barabási, A-L. *Nature Phys.* **6,** 818–823 (2010).
2. Vespignani, A. *Science* **325,** 425–428 (2009).
3. Brockmann, D., Hufnagel, L. & Geisel, T. *Nature* **439,** 462–465 (2006).
4. González, M., Hidalgo, C. & Barabási, A-L. *Nature* **453,** 779–782 (2008).
5. Song, C., Qu, Z., Blumm, N. & Barabási, A-L. *Science* **327,** 1018–1021 (2010).

Published online: 12 September 2010

QUANTUM CRITICALITY

Noise gets marginal

Thermal noise destroys the fragile correlations that characterize many-body systems at a quantum critical point. Theoretical work now shows that another generic form of noise acts differently: flicker noise may alter some properties of a quantum phase transition, but it can preserve the quantum critical state.

Sebastian Diehl

The quantum mechanical properties of a many-body system are most clearly revealed at zero temperature, where it is in its ground state. An interesting situation occurs when there are two competing microscopic mechanisms, each of which favours a different kind of ordering pattern, such as paramagnetic versus antiferromagnetic arrangements in a spin array. Tuning the ratio of the corresponding energy scales (by applying, for example, pressure or external fields) transforms the macroscopically distinct phases into each other through a quantum phase transition[1]. In many cases, the two phases of matter are continuously connected. The corresponding quantum critical point is then characterized by long-range correlations in both the spatial and temporal domain. One aspect that makes such points interesting is their high degree of universality — the low-energy correlations are insensitive with respect to details of the underlying microscopic model.

Despite this robustness regarding microphysics, the system at the quantum critical point is a highly fragile state of matter. In particular, finite temperature is an adversary to quantum criticality. Strictly speaking, an infinitesimal temperature immediately destroys the subtle correlations (see Fig. 1a). Writing in *Nature Physics*, Emanuele Dalla Torre and colleagues[2] argue that there are forms of noise that do not destroy quantum critical correlations. These varieties of noise are by no means exotic: conventional flicker noise — also known as $1/f$ noise and ubiquitous in (classical) electronic devices — is shown to preserve quantum criticality.

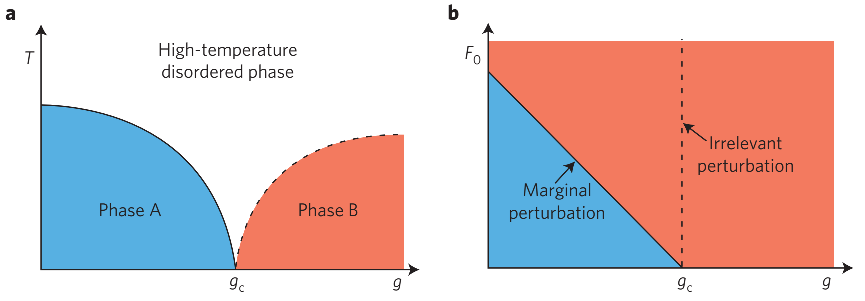

Figure 1 | Impact of thermal (equilibrium) and flicker $1/f$ (non-equilibrium) noise on a quantum critical point at the critical interaction strength g_c. **a**, A finite temperature T acts as a relevant perturbation, destroying quantum criticality: the critical point separates two distinct low-temperature phases A and B, whereas a finite temperature immediately destroys the characteristics of either phase. **b**, $1/f$ noise acts as a marginal perturbation. The distinct phases at $F_0 = 0$ remain intact at finite noise strengths F_0, but the location of the phase boundary is shifted. For an irrelevant perturbation, such shifts would be much less pronounced.

Temperature may be conceived as external thermal white noise that acts on a system due to the coupling to a heat bath in thermodynamic equilibrium. Dalla Torre *et al.*[2], in contrast, look at quantum criticality from a more general perspective, beyond the realm of thermodynamic equilibrium. They find forms of non-equilibrium noise that leave the quantum critical correlations intact, giving rise to a new scenario: non-equilibrium quantum criticality. They establish their key result in a renormalization-group framework. In this language, a (quantum) critical point corresponds to a fixed point under coarse-graining renormalization-group transformations of the system's parameters. Perturbations around a fixed point can be classified according to their degree of 'relevance'. In a quantum critical system such as the one considered in Fig. 1a, there is a single relevant perturbation that may be fine tuned to the fixed point. Other departures from the fixed point are usually 'irrelevant' (this explains the insensitivity to microscopic details). A finite temperature, however, acts as an additional relevant perturbation, thus driving the system away from criticality[1]. Flicker noise falls into another class of perturbations, termed 'marginal': in this case, the universal quantum critical correlations are preserved. However, unlike irrelevant perturbations, additional