

# Heated debate in different dimensions

As every physicist knows, Fourier's law of heat conduction states that the flow of heat increases with the thermal conductivity and the temperature gradient. In 1811, Fourier's work on this equation and its solutions won him the mathematics prize of the Paris Institute. Yet the prize committee, including Laplace, Lagrange and Legendre, expressed reservations: "the manner in which the author arrives at these equations is not exempt of difficulties and ... his analysis to integrate them still leaves something to be desired on the score of generality and even rigour".

Nearly 200 years later, no one doubts the empirical standing of Fourier's law in ordinary circumstances. Yet a contemporary committee could echo the reservations of Fourier's original judges. No one has yet managed to derive Fourier's law from truly fundamental principles. Indeed, in only one or two dimensions the relationship doesn't always hold — although precisely when it does, or doesn't, no one can yet say.

One thing we do know is that Fourier's law is a macroscopic consequence of ordinary diffusion at the microscopic level. That is, whatever the detailed microphysics, if the carriers of heat in a statistical

description follow ordinary brownian diffusion, Fourier's relation is the natural macroscopic result. That's comforting, as it justifies a number of famous results: Maxwell's (and others') use of kinetic theory to derive the conductivity of a dilute gas, for example, and Debye's result for crystalline solids at low temperature.

But starting out with a statistical description simply bypasses the task of showing how 'normal' diffusion emerges — if it does — from the real microdynamics of a physical system. Recent efforts to tackle this problem through studying (supposedly) simpler one- or two-dimensional systems have stumbled into rich forest of theoretical confusion.

Consider a simple model for a set of interacting particles. Fixing the system between thermal baths at different temperatures, one then tries to calculate analytically, or explore in simulations, how the heat flux through the system grows with its length  $L$ . If Fourier's law is to hold, the thermal conductivity,  $\kappa$ , should approach a finite limit as  $L \rightarrow \infty$ .

Does it? That depends. Studies of dynamical chains of the Fermi–Pasta–Ulam type (chains of masses linked pairwise by nonlinear springs) find that  $\kappa$  grows as  $L^\beta$ , with  $\beta$  very



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roughly equal to 1/3. Many other similar models also show anomalous behaviour, suggesting that, in one dimension, the thermal conductivity diverges as the system grows larger — and heat doesn't flow as Fourier thought it would.

However, some one-dimensional systems do seem to confirm Fourier's law, in particular those that do not conserve momentum because, for example, their particles are held in an external confining potential. So maybe momentum conservation is the key? Apparently not: just to keep things interesting, some models that conserve momentum also satisfy Fourier's relation. And in two-dimensional systems, research suggests yet another twist: anomalous conductivity, but with a weak logarithmic divergence.

Unfortunately, for the case of most compelling interest — that of three dimensions — no theoretical result has been demonstrated, except for a trivial model of non-interacting particles. Here, of course, we have overwhelming empirical evidence that Fourier's law really does hold. But even today, like Laplace, Lagrange and Legendre, we still don't know why.

Mark Buchanan

# Why should physicists care about Dover?

There is once again a controversy brewing in the United States over the teaching of evolution in schools. Scientists, both in the US and abroad, might be tempted to dismiss this as short-term hysteria, instigated by far-right religious zealots. Physicists in particular might feel happily insulated from the argument: an amusing article in satirical web magazine *The Onion* poked fun by suggesting that a theory of 'intelligent falling' should be introduced in physics classes as a competing theory with Newton's law of gravity.

Thankfully, there is little chance that physics teachers will be required to discuss religion-based alternatives to newtonian gravity in high-school classes. Nevertheless, there are dangerous implications for all of science arising from

a court case in process in Harrisburg, Pennsylvania. Eleven families have sued the board of the Dover Area School District over its ruling that 14-year-old biology students be read a statement by their teachers saying that evolution is a controversial theory, that intelligent design is a competing theory, and that the latter is discussed in a textbook (written by a lobbying group) available in the public library.

In the many places in which the teaching of evolution has come under attack, other curricular changes have also been proposed, such as removing discussions of the Big Bang. The issue is not so much the teaching of evolution, or even the separation of church and state, but that the notion of



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what constitutes a viable scientific theory in public education can be determined by popular demand and media campaigns.

Even if the intelligent-design bullet is dodged this time, similar attacks on science are likely in the future. Physicists have as much at stake in reaching out to the public as biologists do in the current battle. Hosting public lectures at universities and museums is not enough. We need to recognize that, in the public arena, the rules are different from those of scientific journals. Simply having empirical evidence on your side seems not enough to win the public-relations campaign against those who would replace science with ideology.

Lawrence M. Krauss