Bubble trouble

A fine nozzle immersed in a tank injects air slowly into water. As a bubble grows, it is pulled upwards and stretched by buoyant forces. Surface tension acts to thin down the thread of air linking the bubble to the nozzle until finally its radius, at a point, drops abruptly to zero. A free bubble 'pinches off' and breaks loose.

A flat solid disc hits a liquid surface from above at high speed. The impact drives water downwards and outwards, creating an air cavity that extends into the liquid. In this case, it's hydrostatic pressure rather than surface tension that causes the inflow of the cavity walls, leading to collapse and — again — the pinching off of a bubble at a point about halfway down the cavity.

These two processes seem superficially similar, although the forces driving initial collapse and bubble creation (surface tension and hydrostatic pressure) differ in the two cases. Nearly twenty years ago, theorists conjectured that the similarities might be very deep, and that bubble creation (or pinch-off, at least in low-viscosity fluids) in these cases and many others might follow dynamics having certain universal features. In particular, a simple theory predicted that the neck radius, in the run up to the final pinch-off, should vanish in a non-smooth way, decaying in proportion to $\sqrt{(t - t_c)}$, with t_c being the moment of pinch-off.

Since then, this appealing idea — which would reveal a striking simplicity in one of the most basic processes of fluid dynamics, the creation of bubbles — has had an uncertain history. At first, a number of experiments and simulations showed it to be roughly accurate, but definitively incorrect. However, a recent refinement of the original theory has found strong empirical support. What's emerging is a beautifully unified description of relatively simple physics behind a wide range of pinch-off phenomena.

Naively, one might guess by analogy that the pinch-off of a gas bubble in water might follow a scaling law akin to that for a liquid drop pinching off in air, for which it's known that the minimum radius of the connecting filament dwindles in proportion to $(t - t_c)^{2/3}$. This formula follows from a simple analysis of the interplay of surface tension (which furthers pinch-off) and inertia (which resists it). But this simple analogy turns out to be misleading. Rather, many experiments — with injected bubbles, or those created by impacts, or in other settings — find the scaling exponent α for bubble pinch-off to be close to 1/2, not 2/3.

As always, the identification of what is universal demands a clear understanding of what is not.

In an effort to explain that result, the initial hypothesis of universality for gas-bubble pinch-off — suggested independently in the early 1990s by Michael Longuet-Higgins and colleagues and by Hasan Oguz and Andrea Prosperetti — rested on the supposed irrelevance of anything but inertial forces close enough to the pinch-off event. Surface tension and fluid pressure may initially drive bubble collapse during gas injection or following an impact, but in the ultimate approach to pinch-off, so the idea went, conservation of mass alone should control the acceleration of liquid around the collapsing neck. Analysis in this framework yielded the pleasantly simple result that the radius should shrink to zero as $\sqrt{(t - t_c)}$, very close to what is observed.

But not quite exactly what is observed. A wide range of experiments and numerical simulations since the early 1990s have found exponents differing systematically from 1/2 — often being slightly larger, around 0.56 or so — and results also differ with the details of the experiment in question. In the case of gas injection, some theorists suggested, surface tension might in fact never become negligible near pinch-off, spoiling the sole control of inertial forces and, with it, the universal behaviour.

But this conclusion now seems to have been a little too pessimistic. Two years ago, revisiting the original analyses of pinch-off, two different groups (J. M. Gordillo and M. Pérez-Saborid, *J. Fluid Mech.* **562,** 303–312; 2006, and J. Eggers, M. A. Fontelos, D. Leppinen and J. H. Snoeijer, *Phys. Rev. Lett.* **98,** 09450; 2007) hit on a way to improve the initial theory, which had approximated the bubble shape as a long cylinder. The more accurate theory, based on a delicate scaling relationship between the axial and longitudinal dimensions of the bubble, predicted that pinch-off should indeed be universal and dominated by inertia, but that the bubble radius near pinch-off wouldn't go to zero as a simple power law. Rather, it would vanish as $(t - t_c)^\alpha$, with α only becoming constant asymptotically as $\alpha(\tau) = 1/2 + 1/(4\sqrt{\tau})$, where $\tau = -\ln(t_c - t)$. This implies that any experimental attempt to measure the exponent would generally have to wait a long time until the bubble gets deep into a universal regime where α is roughly constant just before pinch-off.

Stephan Gekle and colleagues have now tested this idea in detailed simulations (*Phys. Rev. E* **80,** 036305; 2009). Their results seem to confirm the universality predicted by the new analyses, while also explaining the variation of exponents measured in previous experiments. It's a vindication for the initial hypothesis of universality, which — in a slightly modified form — seems to be true.

Gekle and colleagues studied bubbles created not only in gas injection or by impact, but also for other situations — such as a bubble being gradually elongated and torn apart by shear forces in an inhomogeneous fluid flow — and were able to probe the fluid dynamics over some 12 orders of magnitude in time. For an impacting disc of about 3 cm diameter, with impact speeds ranging from 1 to 20 m s^{-1}, the simulations showed that the time to pinch-off varied from about 1 to 6 ms, and that bubbles created in the higher-velocity impacts reached the universal regime later. This makes sense, the authors point out, as the impact creates higher hydrostatic pressure, and so the influence of non-inertial forces persists longer.

The behaviour of injected bubbles also shows a clear universal regime. In this case, Gekle and colleagues found that universality set in at times ranging from 5 μs down to 10 ns depending on the conditions (gas flow rate, fluid pressure and so on). Significantly, the universal regime for this situation lasted for a time at least three orders of magnitude shorter than for the impacting disc, which probably explains why no physical experiment has yet detected the universal regime for this system.

The important point in both cases is that the universal regime only sets in when the inertial driving of the collapse becomes dominant over the external driving force, and this depends on lots of details of the system in question. As always, the identification of what is universal and general demands a clear understanding of what is not, and of how to control for it. What's gained is a gratifying glimpse of order lying behind messy details. ❐

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