'Beautiful mind' John Nash adds Abel Prize to his Nobel

Mathematician made famous by Hollywood will share US$765,000 award with Louis Nirenberg for work in the field of geometric analysis.

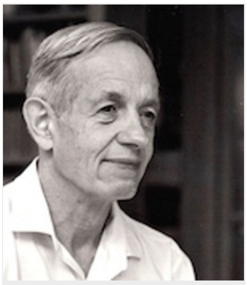
Philip Ball

25 March 2015

Marc Asnin/Redux/eyevine

John Nash.

Although some consider the Abel Prize to be the 'Nobel of mathematics', its winners are hardly ever household names. But this year's prize, announced on 25 March, includes a notable exception: John Nash, the subject of the 2001 film *A Beautiful Mind* starring Russell Crowe — and a previous winner of a Nobel prize for economics.



John Nash, 1928-2015

Nash, who spent most of his career at Princeton University in New Jersey, shares the prize with Canadian-born mathematician Louis Nirenberg of New York University's Courant Institute of Mathematical Sciences for work on partial differential equations. Nash's contributions to this field are widely considered more profound than the research in game theory that earned him a Nobel.

Partial differential equations (those that involve multiple, independent derivatives) are fundamental to pure maths and crop up throughout science, describing phenomena from the diffusion of heat to the motion of quantum particles. “Partial differential equations lie at the foundation of many areas, both within and beyond mathematics, ranging from geometry to physics,” says mathematician Robert Kohn of the Courant Institute. “Louis Nirenberg and John Nash have had huge influence on this field, not only by solving important problems, but more importantly by introducing fundamentally new methods and ideas."

Manifold mystery

Some of the prizewinners' most important work concerns an area called geometric analysis, in which partial differential equations are applied to the study of the shapes of surfaces and their higher-dimensional analogues, called manifolds. There are two approaches to thinking about manifolds. The 'extrinsic' approach views them in 'Euclidean space', rather like plotting them against the familiar axes of graphs — for example a sphere is the surface of a three-dimensional ball. The other approach focuses on 'intrinsic' geometry of a manifold: properties such as distances, angles and curvature that could be measured by an observer who 'lived' inside the space without knowing that it is part of a larger realm.

Intrinsic geometry is done using the concept of a Riemannian metric, introduced in the nineteenth century by the German mathematician Bernhard Riemann, and adapted by Albert Einstein to describe the curvature of space-time in his general theory of relativity.

Nash, now 86, showed in the 1950s that the extrinsic and intrinsic approaches are equivalent. He proved that a Riemannian manifold can always be 'embedded' as a subset of a (possibly much higher-dimensional) Euclidean space. In proving this theorem, Nash devised new methods for solving partial differential equations. "It was an unknown area," he said in a 2011 interview. "I didn't realize that, and I said, this doesn't seem so difficult".

A few years before Nash, Nirenberg had proved a result for a special case of the same problem, showing that a class of two-dimensional surfaces can be embedded as convex bodies in three-dimensional Euclidean space.

A tale of two mathematicians

These examples are only a fraction of both mathematicians' contributions, says Kohn. Nirenberg, for example, has done important work on the Navier–Stokes equations that describe fluid flow, which also involve partial derivatives. He obtained important results on the existence of 'singularities', or kinks, in the solutions to such equations. The Clay Mathematics Institute, based in Providence, Rhode Island, put the singularities of Navier–Stokes on its list of 'Millennium Prize Problems'. The institute offers US$1 million to anyone who can solve one of those problems.

Hollenshead/NYU Photo Bureau

Louis Nirenberg.

Nirenberg is particularly fond of methods involving inequalities, which often crop up with partial differential equations. "If somebody shows me a new inequality," he told an interviewer in 2002, "I say, 'Oh, that's beautiful, let me think about it,' and I may have some ideas connected with it.”

The two winners have known each other since the 1950s, when Nash visited New York University. But towards the end of that decade, Nash began experiencing paranoid delusions, which all but prevented him from pursuing his research until his near full recovery in the 1990s — a trajectory portrayed in *A Beautiful Mind*. By 1994, he had recovered enough to attend the ceremony in Stockholm in which he received his Nobel. (That award was for work in game theory, in which Nash proved the existence of situations in two-person games where neither player can derive any gain from changing their strategy — a so-called Nash equilibrium.) In 2002, Nirenberg remarked that Nash had a “remarkable mind … he thought about things differently from other people”.

Nirenberg has always preferred to work in collaboration with others, saying of mathematics that “it’s a very nice, warm family”. During his 2002 interview, he said that he enjoys the camaraderie of the subject: “That's the thing I try to get across to people who don't know anything about mathematics, what fun it is!”

In the 13-year history of the Abel Prize, Nirenberg is the fourth winner from the Courant Institute, after Peter Lax (2005), Srinivasa Varadhan (2007) and Gromov (2009).

The Abel Prize, named after the Norwegian mathematician Niels Henrik Abel, is worth 6 million kroner (about US$765,000), and is awarded by the Norwegian Academy of Science and Letters. The recipients will be presented with their awards in a ceremony in Oslo in May.

Nature | doi:10.1038/nature.2015.17179