First proof that infinitely many prime numbers come in pairs

Mathematician claims breakthrough towards solving centuries-old problem.

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It's a result only a mathematician could love. Researchers hoping to get '2' as the answer for a long-sought proof involving pairs of prime numbers are celebrating the fact that a mathematician has wrestled the value down from infinity to 70 million.

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Mathematician Yitang Zhang has outlined a proof of a 'weak' version of the twin prime conjecture.

"That's only [a factor of] 35 million away" from the target, quips Dan Goldston, an analytic number theorist at San Jose State University in California who was not involved in the work. "Every step down is a step towards the ultimate answer."

That goal is the proof to a conjecture concerning prime numbers. Those are the whole numbers that are divisible only by one and themselves. Primes abound among smaller numbers, but they become less and less frequent as one goes towards larger numbers. In fact, the gap between each prime and the next becomes larger and larger — on average. But exceptions exist: the 'twin primes', which are pairs of prime numbers that differ in value by 2. Examples of known twin primes are 3 and 5, or 17 and 19, or $2{,}003{,}663{,}613 \times 2^{195{,}000} - 1$ and $2{,}003{,}663{,}613 \times 2^{195{,}000} + 1$.

The twin prime conjecture says that there is an infinite number of such twin pairs. Some attribute the conjecture to the Greek mathematician Euclid of Alexandria, which would make it one of the oldest open problems in mathematics.

The problem has eluded all attempts to find a solution so far. A major milestone was reached in 2005 when Goldston and two colleagues showed that there is an infinite number of prime pairs that differ by no more than 16 (ref. 1). But there was a catch. "They were assuming a conjecture that no one knows how to prove," says Dorian Goldfeld, a number theorist at Columbia University in New York.

The new result, from Yitang Zhang of the University of New Hampshire in Durham, finds that there are infinitely many pairs of primes that are less than 70 million units apart without relying on unproven conjectures. Although 70 million seems like a very large number, the existence of any finite bound, no matter how large, means that that the gaps between consecutive numbers don't keep growing forever. The jump from 2 to 70 million is nothing compared with the jump from 70 million to infinity. "If this is right, I'm absolutely astounded," says Goldfeld.

Zhang presented his research on 13 May to an audience of a few dozen at Harvard University in

Cambridge, Massachusetts, and the fact that the work seems to use standard mathematical techniques led some to question whether Zhang could really have succeeded where others failed.

But a referee report from the *Annals of Mathematics*, to which Zhang submitted his paper, suggests he has. “The main results are of the first rank,” states the report, a copy of which Zhang provided to *Nature*. “The author has succeeded to prove a landmark theorem in the distribution of prime numbers. … We are very happy to strongly recommend acceptance of the paper for publication in the *Annals*.”

Goldston, who was sent a copy of the paper, says that he and the other researchers who have seen it “are feeling pretty good” about it. “Nothing is obviously wrong,” he says.

For his part, Zhang, who has been working on the paper since a key insight came to him during a visit to a friend’s house last July, says he expects that the paper’s mathematical machinery will allow for the value of 70 million to be pushed downwards. “We may reduce it,” he says.

Goldston does not think the value can be reduced all the way to 2 to prove the twin prime conjecture. But he says the very fact that there is a number at all is a huge breakthrough. “I was doubtful I would ever live to see this result,” he says.

Zhang will resubmit the paper, with a few minor tweaks, this week.

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References

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