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The maths of the pop-up tent

Single parameter shows the easiest way to pack up camp.

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Ever wrestled with a pop-up tent, trying to flatten it to fit into the bag? Help is at hand, in the form of a mathematical theory to describe the shapes adopted by the types of flexible ring from which these tents are made. “We have found the best way to fold rings,” says Alain Jonas, a materials scientist at the French-speaking Catholic University of Leuven in Belgium, who led the research.

Ref. 1

The authors created a creased origami sculpture that illustrates the most efficient way of buckling a ring-shaped object.

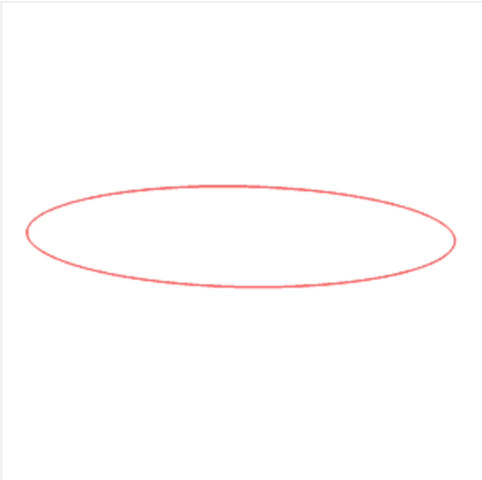
Buckled rings crop up in many everyday contexts: Jonas and his colleagues give examples of their use not just for tents, but also for laundry baskets, pop-up football goals and sculptures made of wood or paper. The rings are too curved to lie flat in a normal circle, so they either buckle into a three-dimensional 'saddle' shape or save space by coiling up to form what looks like a stack of interlinked loops.

Round and round

The researchers show in *Nature Communications*[1] that these shapes can be predicted accurately using a theory that invokes a single key mathematical concept: 'overcurvature', or the amount by which a ring is made more curved than a perfect, unfolded circle of the same length.

The results can be used to work out the easiest folding pathways to collapse a single overcurved ring into a small coil — the problem of folding a pop-up tent. “It’s not trivial to find this pathway empirically,” says Jonas. “You naturally start by deforming the ring in two lobes, since this is easiest. But then you have to deform the ring further into shapes that require a lot of energy.”

By contrast, he says, “if you take the pathway we propose, you have to use more energy at the start, but then have to cross lower energy barriers to reach the energy valley of the ring coiled in three” — meaning that you don’t get trapped by starting off following the path of least resistance. The researchers provide a detailed route for how best to reach the three-ring compact form (see animation).



Alain M. Jonas

They also show that such a ring can be made even more compact, folded into five loops instead of three. “This is more difficult, because the energy barriers are higher,” says Jonas, adding that for a tent, it would be best to have three people on the job. He sees no reason why his group's proposed folding route shouldn’t work for real tents, provided that the pole material is flexible and strong enough.

From camping to chromosomes

Jonas thinks that the results might also apply on the molecular scale for understanding the shapes of some relatively stiff molecular rings, such as plasmids — circles of DNA found inside organisms such as bacteria — and other ring-shaped polymers[2].

“There is a lot of interest currently in this kind of fundamental mechanical problem,” says Basile Audoly, a mathematician at the Jean le Rond d'Alembert Institute at the Pierre and Marie Curie University in Paris, who points out that rather similar and related findings have been reported by several others[3, 4, 5, 6, 7]. For example, he says, the same question has been related to the buckled fringes at the edges of some plant leaves[3, 4].

Jonas says that compared with earlier work, his finding that the single parameter of overcurvature will describe the mechanical problem “has the virtue of allowing us to find general laws and provide easy-to-use designing tools”.

References

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