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Mathematicians come closer to solving Goldbach's weak conjecture

A centuries-old conjecture is nearing its solution.

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One of the oldest unsolved problems in mathematics is also among the easiest to grasp. The weak Goldbach conjecture says that you can break up any odd number into the sum of, at most, three prime numbers (numbers that cannot be evenly divided by any other number except themselves or 1). For example:

35 = 19 + 13 + 3
or
77 = 53 + 13 + 11

Mathematician Terence Tao of the University of California, Los Angeles, has now inched toward a proof. He has shown that one can write odd numbers as sums of, at most, five primes—and he is hopeful about getting that down to three. Besides the sheer thrill of cracking a nut that has eluded some of the best minds in mathematics for nearly three centuries, Tao says, reaching that coveted goal might lead mathematicians to ideas useful in real life—for example, for encrypting sensitive data.

The weak Goldbach conjecture was proposed by 18th-century mathematician Christian Goldbach. It is the sibling of a statement concerning even numbers, named the strong Goldbach conjecture but actually made by his colleague, mathematician Leonhard Euler. The strong version says that every even number larger than 2 is the sum of two primes. As its name implies, the weak version would follow if the strong were true: to write an odd number as a sum of three primes, it would be sufficient to subtract 3 from it and apply the strong version to the resulting even number.

Mathematicians have checked the validity of both statements by computer for all numbers up to 19 digits, and they have never found an exception. Moreover, the larger the number, the more ways exist to split it into a sum of two other numbers—let alone three. So the odds of the statements being true become better for larger numbers. In fact, mathematicians have demonstrated that if any exceptions to the strong conjecture exist, they should become increasingly sparse as the number edges toward infinity. In the weak case, a classic theorem from the 1930s says that there are, at most, a finite number of exceptions to the conjecture. In other words, the weak Goldbach conjecture is true for "sufficiently large" numbers. Tao combined the computer-based results valid for small-enough numbers with the result that applies to large-enough numbers. By improving earlier calculations with "lots of little tweaks," he says, he showed that he could bring the two ranges of validity to overlap—as long as he could use five primes.

Next, Tao hopes to extend his approach and show that three primes suffice in all cases. But that is not likely to help with the strong conjecture. The weak conjecture is incomparably easier, Tao says, because by splitting a number into a sum of three, "there are many, many more chances for you to get lucky and have all the numbers be prime." Thus, a quarter of a millennium after Goldbach’s death, no one even has a strategy for how to solve his big challenge.

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