

REVIEW

Electrodynamics of transformation-based invisibility cloaking

Baile Zhang^{1,2}

The existing knowledge on the electrodynamics of invisibility cloaking based on transformation optics is reviewed from an integrated science and engineering perspective. Several significant electromagnetic problems that have resulted in intense discussions in the past few years are summarized in terms of propagation, scattering, radiation and fabrication. Finally, the road ahead toward invisibility cloaking and transformation optics is discussed from the viewpoint of the author.

Light: Science & Applications (2012) 1, e32; doi:10.1038/lsa.2012.32; published online 12 October 2012

Keywords: invisibility cloaking; metamaterials; transformation optics

INTRODUCTION

The majority of optical/electromagnetic devices have historically been developed according to the following procedure:

material → function

This conventional ‘material-oriented’ procedure, which is usually the only choice because of the wide barrier from ‘material’ to ‘function’, has at least two limitations: (i) limited imagination; many innovative optical functions that can potentially be created are unknown; and (ii) slow research and development progress; the properties of a specific material are often studied using a time-consuming try-and-error methodology, whereas in many cases, the best solution with respect to light or electromagnetic waves is unintentionally missed.

Transformation optics,^{1,2} which is a recently growing field in optical and materials sciences, describes a material as the geometrical space where a desired optical function occurs, thereby bridging the barrier between ‘material’ and ‘function’ in design principles. This field naturally results in an inverse development procedure, as follows:

material ← function

The desired optical function is first described geometrically, and then the required geometrical space for that function to occur is converted to the required properties of the materials while preserving the effect of ‘wave bending’. This revolutionary application not only revisits the fundamental physics of light-matter interactions (e.g., the negative refractive index metamaterial and its ‘superlens’ effect³ can be intuitively explained in an inverse optical space⁴), but also opens the door to many unprecedented optical functions, including invisibility cloaking,^{1,2} which were previously only observed in science

fiction. Note that the underlying mechanism of transformation optics, i.e., the ‘metric invariant’ property of Maxwell’s equations, has been known for decades.^{5–11} However, this ‘metric invariant’ property only began to be applied in 2006 for designing specific optical/electromagnetic devices in a systematic inverse design procedure from ‘function’ to ‘material’, which subsequently spawned a novel research field—transformation optics. This explosively developing field has attracted worldwide attention, especially in its representative application for invisibility cloaking. Related transformation techniques are now being continuously extended and applied to considerably more applications, such as imaging,¹² manipulation of surface plasmons,¹³ field enhancement and light harvesting.¹⁴ There have already been several excellent reviews^{15–17} and introductions^{4,18–21} that have provided a general overview of transformation optics.

Despite the exciting inverse design procedure of transformation optics, the limitations from the perspective of materials have become the primary bottleneck for practical applications at this stage. A possible approach to overcome this bottleneck, from the author’s point of view, is to combine both inverse and forward design procedures, and finally form a ‘loop’ of development, as follows:

material ⇌ function

which will hopefully converge to a useful solution in practice. Therefore, it is very important to summarize the current knowledge of a specific transformation optics function from the perspectives of both science and engineering in such a way that both scientists and engineers can benefit from it, and more importantly, can finally achieve an applicable solution in cooperation.

In this review, we will specifically focus on the electrodynamics of transformation-based invisibility cloaking, and we will attempt to provide some hindsight regarding several electromagnetic problems

¹Division of Physics and Applied Physics, School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore 637371, Singapore and ²Singapore-MIT Alliance for Research and Technology (SMART) Centre, Singapore 117543, Singapore
Correspondence: Dr BL Zhang, Division of Physics and Applied Physics, School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore 637371, Singapore

E-mail: blzhang@ntu.edu.sg

Received 12 May 2012; revised 6 July 2012; accepted 10 July 2012

on this topic with the author's own research experiences. As an alternative approach to invisibility cloaking, the recent exciting development of plasmonic cloaking,^{22–24} which is based on scattering cancellation, deserves another special review, and thus, is not covered here. The recent, ingenious proposal of 'cloaking at a distance'²⁵ utilizing media with a negative refractive index has been reviewed in detail in Ref. 15 from the perspective of folded geometrical properties; therefore, this topic is not repeated here. The list of references at the end of this review is only representative and by no means exhaustive.

We begin with Maxwell's equations:

$$\nabla \times \bar{E} = -\mu_0 \frac{\partial \bar{H}}{\partial t} \quad (1)$$

$$\nabla \times \bar{H} = \varepsilon_0 \frac{\partial \bar{E}}{\partial t} \quad (2)$$

If we consider the transformation in spherical coordinates, as shown in Figure 1a and 1b, as

$$r' = R_1 + r \frac{R_2 - R_1}{R_2}, \theta' = \theta, \phi' = \phi \quad (3)$$

where (r, θ, ϕ) are the coordinates in Figure 1a and (r', θ', ϕ') are the new coordinates in Figure 1b, then Maxwell's equations in the new coordinates will maintain their form because of their form-invariant (or 'metric-invariant') property and become

$$\nabla \times \bar{E}' = -\bar{\mu} \frac{\partial \bar{H}'}{\partial t} \quad (4)$$

$$\nabla \times \bar{H}' = \bar{\varepsilon} \frac{\partial \bar{E}'}{\partial t} \quad (5)$$

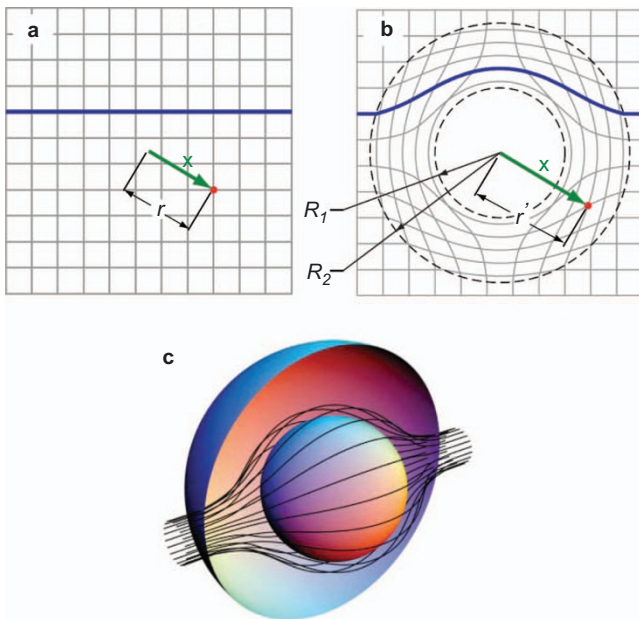


Figure 1 The path of the same ray (blue line) and a vector \mathbf{x} in the original Cartesian space in **a**, and the space after transformation in **b**. **(c)** All rays impinging on the spherical invisibility cloak go around the central region as if the space was empty. Figure reprinted with permission: **a, b**, Ref. 28, © 2006 OSA; **c**, Ref. 2, © 2006 AAAS.

where \bar{E}' and \bar{H}' are the fields in the new coordinates and $\bar{\mu}$ and $\bar{\varepsilon}$ are tensors with the following components:²

$$\frac{\varepsilon_r}{\varepsilon_0} = \frac{\mu_r}{\mu_0} = \frac{R_2}{R_2 - R_1} \frac{(r - R_1)^2}{r^2} \quad (6)$$

$$\frac{\varepsilon_\theta}{\varepsilon_0} = \frac{\mu_\theta}{\mu_0} = \frac{R_2}{R_2 - R_1} \quad (7)$$

$$\frac{\varepsilon_\phi}{\varepsilon_0} = \frac{\mu_\phi}{\mu_0} = \frac{R_2}{R_2 - R_1} \quad (8)$$

The above derivation predicts that if we can construct a physical spherical shell (as in Figure 1c) whose constitutive parameters satisfy the above set of expressions, all of the light rays impinging on the shell will be guided around the central 'hole' as if these rays were propagating in an empty space. Therefore, anything placed inside the central 'hole' will become invisible to outside observers.²

A similar transformation and similar invisibility effects were discussed in 2003 in electrical impedance tomography.^{26,27} This transformation had not received considerable attention outside of the mathematics community until the cloaking theory of light emerged in 2006.^{1,2} However, the original mathematical basis for cloaking is still beneficial and worth a careful revisit.

It should be stressed that the methods to compress the coordinates and create a 'hole' in Figure 1, as a matter of choice, are by no means limited to the linear transformation as in Equation (3). In theory, there are an infinite number of transformations that can perform this function, which provides an important design freedom during the first stage of designing invisibility cloaks. Interested readers are referred to Ref. 17, which provides a systematic survey on different transformations. Here, we proceed with this simple transformation in Figure 1 to explore related electrodynamic problems.

PROPAGATION—WHAT DOES SINGULARITY DO?

Equations (6)–(8) (as a result of the deviations beginning with Equations (1)–(5)), which were originally reported in Ref. 2, are based on an implicit assumption that the transformation is non-singular (i.e., one point is mapped to one point after the transformation). However, the transformation in Figure 1 that can result in an invisibility cloak is a singular mapping (i.e., one point is mapped to a set of points). For example, the origin in Figure 1a is transformed to a surface with radius R_1 in Figure 1b). Although simulations of ray tracing^{2,28} and full-wave finite-element analysis^{29,30} confirmed the effectiveness of the proposed cloaking idea under certain approximations, these simulations are not sufficient to prove that the cloak in Figure 1c can work perfectly, as proposed in Ref. 2. The ray tracing cannot address the 'critical ray' problem, i.e., the ray 'heading directly to the center of the sphere' 'does not know whether to be deviated up or down, left or right'.² The finite-element analysis simulation is inherently associated with scattering²⁹ due to the numerical discretization. Therefore, in theory, the only way to decisively verify the cloaking strategy proposed in Ref. 2 is to solve the propagation problem strictly using Maxwell's equations. References 31–34 used an extended Mie scattering model to calculate the scattering cross-section of an invisibility cloak in both two-dimensional and three-dimensional (3D) cases, and they observed that the cross-section is exactly equal to zero. Ref. 31 explained the 'critical ray' problem that was proposed in Ref. 2 in that this critical ray will lose its energy and completely disappear before it can reach the inner boundary of the cloak. A

cylindrical cloak was treated similarly in Ref. 32, where the sensitivity of cloaking performance on the inner boundary was indicated. Reference 33 extended the cylindrical analysis into 3D by considering oblique incidence and then, together with Ref. 34, attributed the sensitivity to the surface current effect at the inner boundary of a cylindrical cloak because of the infinite values of permeability and permittivity in the azimuth direction.

Although the scattering model can solve the propagation problem for cases where the source is located outside of the cloak, the other problem, i.e., whether the wave can escape from the internal cloaked region, has created different challenges in mathematics and physics. From the viewpoint of transformation theory, the cloaked region, or the 'hole', does not exist before the transformation in the virtual space. Ideally, as explained in Refs. 2 and 35, the cloak is 'a complete separation of electromagnetic domains into a cloaked region and those outside'³⁵ and thus 'no radiation can get into the concealed volume, nor can any radiation get out'² (the recent 'anti-cloak'^{36,37} proposal involves negative refractive index media and will be discussed later). Because the parameters of the cloak in Equations (6)–(8) are reciprocal and the reciprocity theorem^{38,39} should hold, a true cloak should not only cloak passive objects from incoming waves but should also cloak active devices by preventing waves from escaping and being detected. Greenleaf *et al.*⁴⁰ analyzed the case of active sources inside the concealed region of a cloak, and they concluded that this cloak can perfectly cloak active acoustic sources that satisfy the Helmholtz equation; however, for active electromagnetic sources, the 'finite energy solutions' to Maxwell's equations do not exist. In other words, one must seek another solution in the sense of the Schwartz distribution theory. Reference 41 decomposed the electromagnetic fields into transverse-electric and transverse-magnetic waves and observed that at the inner boundary of the cloak, the electrical fields for transverse-magnetic waves and the magnetic fields for transverse-electric waves blow up in the normal direction, which forms a 'surface voltage' effect. This effect is very similar to the model of an extremely thin capacitor composed of two sheets of charges in singular electromagnetics.⁴²

The analysis in Ref. 41 also implied a new set of boundary conditions for which, when the normal components of the D and B fields on a boundary are zero, the electromagnetic fields inside the domain enclosed by the boundary are uniquely determined. This new set of boundary conditions was then systematically studied in the literature.^{43–45} This set of boundary conditions can be categorized into a more general mathematical form of boundary conditions in the sense of the Schwartz distribution theory.^{46,47} Recent studies are interested in the metamaterial models that can potentially realize this particular set of boundary conditions.

Recently, an interesting method based on a negative refractive index 'anti-cloak'^{36,37} has been proposed to defeat invisibility cloaks by simply placing the anti-cloak inside of the cloaked region. Admittedly, an imperfect cloak can be defeated by an anti-cloak because its transformation is continuous without singularity in the Jacobian matrix.³⁶ Regarding the interaction between a perfect cloak and a perfect anti-cloak, however, the electric fields and magnetic fields will simultaneously blow up at the interacting interface because of the singularity,³⁶ which means that an infinite energy oscillation will occur. The singular behavior of interaction between a negative refractive index medium and a positive refractive index medium is a profound research topic of the last decade that is far beyond invisibility cloaking. Some beneficial discussions can be found in Refs. 3 and 48–51.

Note that the cloak in Figure 1 was originally designed only for avoiding detection from outside observers. The primary function of

this cloak is not to confine electromagnetic waves and hide active objects. In contrast, there have been novel transformation proposals for achieving a perfect optical cavity^{52–54} that is specifically designed to confine electromagnetic waves. The cavity can be interpreted as the medium that cloaks the surrounding space.⁵² More discussions can be found in these references.

SCATTERING—IS ENERGY TRANSPORT QUIESCENT OR SUPERLUMINAL?

Since the very beginning, when Ref. 2 proposed the cloaking strategy, it was reported that perfect cloaking can only be achieved at a single frequency (the so-called cloaking frequency). An ideal cloak that can work for all wavelengths is practically and theoretically impossible.^{2,55,56} Practically, the required constitutive parameters of a perfect cloak involve singular values, zero and infinity, which cannot be fully fulfilled in practice. Theoretically, because the concealed region in Figure 1 corresponds to a single spot in the virtual space where the wave propagates at the speed of light, the phase of the electromagnetic waves in the physical space must take zero time to pass through the concealed region, which requires an infinite phase velocity. Therefore, the cloak must be strongly dispersive. Consequently, scattering will occur when a non-monochromatic wave with a non-zero bandwidth is incident on the cloak. Moreover, single-frequency studies, such as the cases discussed in the last section, provide no information about the process of energy transport; the field excitation at every point in the medium is already present forever for an infinitely long sine wave.⁵⁷ All energy transport phenomena must be discussed in the context of dispersion with a nonzero bandwidth. In this section, we will focus on the phenomena occurring in the vicinity of the cloaking frequency on the frequency axis and discuss the influence of dispersion on invisibility cloaking.

The most intuitive and thus widely used method to illustrate the interaction of light with an invisibility cloak is through the use of ray theory, which attributes the invisibility to the perfect guiding of light around the concealed region along predetermined trajectories.^{2,28,29} Light energy is supposed to be distributed into a considerable number of rays that precisely follow these ray trajectories during the entire propagation. Extending the use of ray theory to a physical light signal passing through a dispersive cloak, however, has resulted in some concerns about causality^{43,58} and energy gluing at the inner boundary.⁵⁹ Calculations have demonstrated that the energy transport velocity is zero at the inner boundary of the cloak,⁵⁹ whereas the group velocity goes to infinity at the same location.⁵⁸ The former, i.e., zero energy transport velocity, indicates that the energy will be stopped at the inner boundary and thus cannot propagate further. In contrast, the latter, i.e., infinite group velocity, implies that the energy will propagate with an extremely superluminal velocity, which obviously violates Einstein's theory of relativity. Therefore, is the energy transport quiescent or superluminal?

It is very interesting to observe that similar controversies about different types of velocities for the propagation of electromagnetic energy were discussed a long time ago, at least back to 1914, when Sommerfeld and Brillouin conducted a beautiful investigation that may still be read with profit.^{57,60} Before we move to this topic, let us first consider how to define a velocity for the electromagnetic energy. To define the velocity of the energy, one first needs to define its precise location, which is usually characterized as the location of a wave packet. Suppose a well-defined wave packet with a very narrow bandwidth is going to pass through a dispersive medium. In most cases of normal dispersion, this wave packet will slightly deform but still

maintain a well-defined shape, which makes it possible to determine the location of its energy. However, if the wave packet is significantly distorted, the location of its energy becomes elusive. Sommerfeld and Brillouin studied a truncated waveform with a precise starting point or ending point such that the definition of velocity is meaningful. They observed that the only velocity that is always slower than the velocity of light is the velocity of a signal, which is an electromagnetic energy transport sufficient to trigger a physical measurement. All of the other various definitions of energy transport will lose their physical meaning in some unusual cases of materials' dispersion.

In the case of the invisibility cloak in Figure 1, the dispersion of the parameters of the materials can still be normal. However, when an electromagnetic wave approaches the inner boundary, the cloak will have an intrinsic singular phenomenon of infinite fields close to the inner boundary,⁴¹ which will cause the group velocity to lose its physical meaning.⁶¹ Reference 62 chose a Gaussian pulse to measure the optical delay of signals after they passed through the invisibility cloak in the time domain. By monitoring the instantaneous Poynting power passing through a target plane behind the dispersive spherical cloak (as shown in the inset in Figure 2), it can be observed in Figure 2 that the time delay of the transported signals exhibits a volcano-like shape with a pit at the center; the signal transported across the center of the cloak, which should intuitively have the longest path to travel, can be even faster than the others around it. Note that the definition of the velocity of a signal is somewhat ambiguous.⁵⁷ Reference 62 chose the moment when half of the energy has passed as the arrival time. Despite the various definitions for velocity, the velocity of a signal is always slower than the speed of light. When the electromagnetic signals approach the inner boundary of the cloak, they become significantly distorted. Therefore, ray theory cannot correctly predict the energy propagation

process. The only way to calculate a meaningful prediction is to exactly solve Maxwell's equations.

RADIATION—TRANSFORMATION SYMMETRY AND ITS BREAKING

The above sections only discussed passive situations, i.e., the wave is from either the external region or the concealed region, and no radiation is generated from the cloak itself between the external region and the concealed region. Here, we will begin to discuss cases where radiation comes from the cloak itself.

Transformation of active sources inside a transformation medium has been discussed previously.^{63,64} However, pure transformation cannot solve problems such as dispersion and impedance mismatch. A traditional method to approach radiation problems in electromagnetics is to solve the dyadic Green functions,^{65,66} which can provide 3D information about electromagnetic fields. Reference 67 derived the electric dyadic Green function, when an electric dipole is located in the cloak layer, as follows:

$$\bar{\bar{G}}_E^{11} = -\delta(\bar{r}-\bar{r}')\bar{\bar{T}}'^{-1} + \frac{i\pi}{2k_t} \bar{\bar{T}}'^{-1} \sum_{n,m} \frac{(2n+1)(-1)^m}{2\pi^2 n(n+1)} \begin{cases} (\bar{M}_{n,m}^{(1)} \bar{M}'_{n,-m} + \bar{N}_{n,m}^{(1)} \bar{N}'_{n,-m}) \bar{\bar{T}}'^{-1}, & r > r' \\ (\bar{M}_{n,m} \bar{M}'_{n,-m} + \bar{N}_{n,m} \bar{N}'_{n,-m}) \bar{\bar{T}}'^{-1}, & r < r' \end{cases} \quad (9)$$

where $\bar{M}_{n,m}$ and $\bar{N}_{n,m}$ are defined vector spherical waves, the superscript $(\cdot)'$ means replacing (r, θ, ϕ) with (r', θ', ϕ') (the source's location) and $(\cdot)^{(1)}$ means replacing the spherical Bessel function j_n with the spherical Hankel function of the first type $h_n^{(1)}$. By employing the dyadic Green function, it is possible to solve all radiation problems

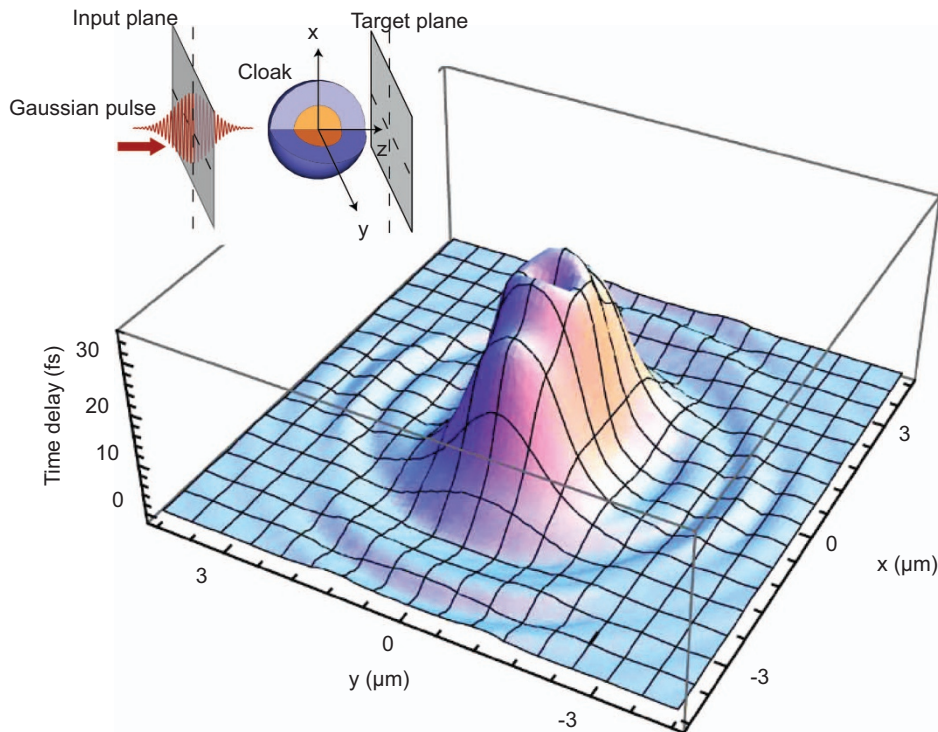


Figure 2 A volcano-shaped time delay distribution of the signal after passing through a dispersive spherical invisibility cloak. Figure reprinted with permission from Ref. 62, © 2009 OSA.

inside an invisibility cloak, including the effects of dispersion and other imperfections.

There are more interesting physics issues underlying this radiation problem. Before we discuss these issues, let us consider a fundamental topic in physics—the symmetry of physical laws. According to Weyl’s definition, ‘a thing is symmetrical if there is something we can do to it so that after we have done it, it looks the same as it did before’.⁶⁸ This definition can perfectly apply in the cloaking phenomena and the principle of transformation optics. In transformation optics, we perform a transformation to an empty space and create a cloak. After we have performed the transformation, the cloak appears the same as the original empty space. This symmetry of coordinate transformation stems from the form invariance of Maxwell’s equations; the coordinate transformation does not change the form of Maxwell’s equations; it only changes the exact permittivity and permeability tensors and the field values. Therefore, a perfect invisibility cloak created from a coordinate transformation is similar to a ‘mirror’ that projects the originally flat electromagnetic space into the physical space by squeezing the electromagnetic space through a transformation. Looking at the ‘mirror’ from the outside, one can only see the illusion of a flat

electromagnetic space, which appears exactly the same as the space before the transformation. As long as we always remain in the electromagnetic space, we will not be able to tell whether the space is curved because there is not a reference for us to make judgment. Therefore, the question is whether there is a ‘ruler’ to measure the curvature of the electromagnetic space and break the symmetry of transformation optics.

The breakthrough was reported in Ref. 67; when the space for Maxwell’s equations (electromagnetic space) is compressed, the space for Newton’s laws of motion (mechanical space) remains flat. Although the transformation of electromagnetic space can completely control the motion of photons, it is not directly applicable to the motion of a charged particle that can perceive both the electromagnetic space (because it has charge) and the mechanical space (because it has mass). This result will give rise to an electromagnetic detection method for ‘seeing’ the invisibility cloak—simply shoot a fast-moving charged particle beam through the cloak, no matter the frequency band the cloak is working in and whether it is a narrow band^{2,69,70} or a broad band.^{56,71–78} Figure 3 demonstrates the different behavior of photons and charged particles in the physical space and virtual

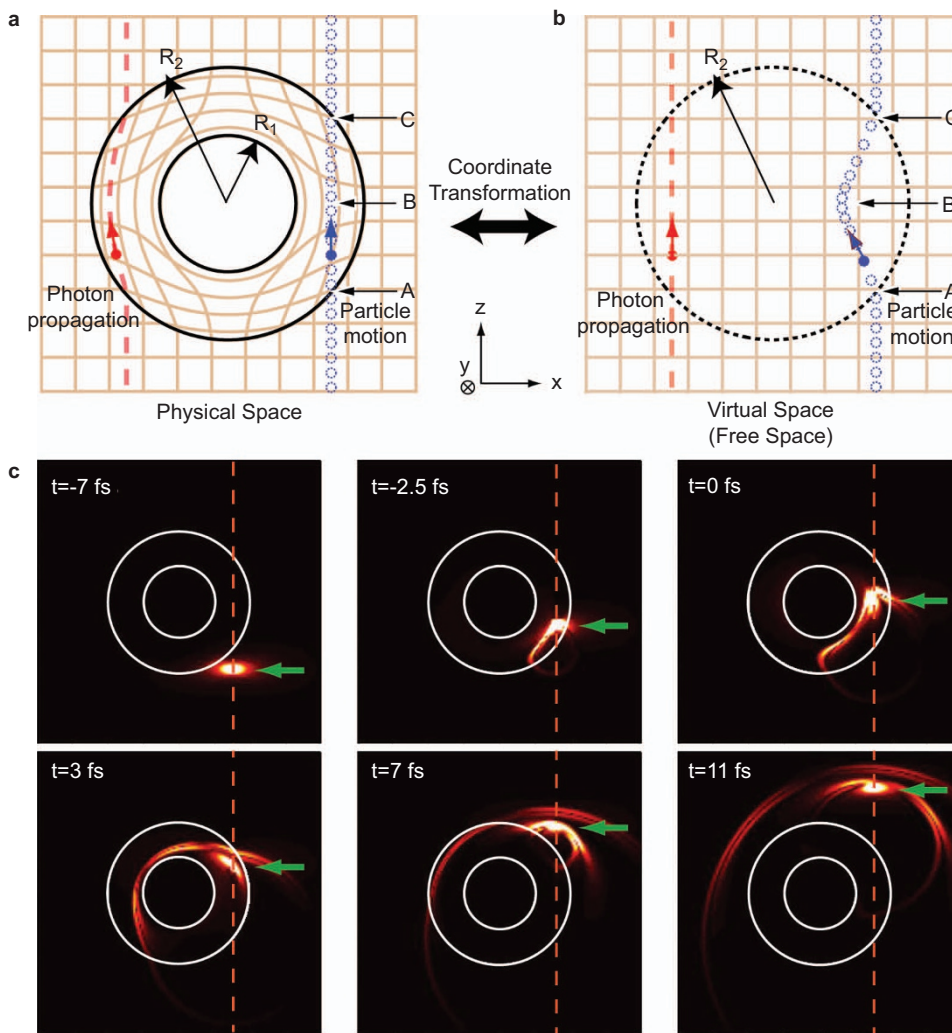


Figure 3 Trajectory of a fast-moving, charged particle inside the cloak compared with the trajectory of a photon in the physical space in **a** and in the virtual space in **b**. Background is the mesh of electromagnetic space. **(c)** The radiation from a charged particle passing through a spherical invisibility cloak. The dotted line and the small arrow indicate the trajectory and the exact location of the particle, respectively. Figure reprinted with permission from Ref. 67, © 2009 APS.

space, respectively, and the radiation generated from the charged particle passing through an invisibility cloak.⁶⁷

FABRICATION—METAMATERIALS OR NATURAL MATERIALS?

It is very interesting to look back at the development of methods for fabricating invisibility cloaks over the past few years. Figure 4 provides a brief chronological roadmap of some milestones towards achieving invisibility cloaking. Each milestone was established by providing a solution to previous limitations.

The first invisibility cloak (Figure 4a) was constructed at microwave frequencies using simplified parameters based on traditional splitting-resonator (SRR) metamaterial technologies⁶⁹ for a particular polarization. Similar SRR experiments have also been reported for the other polarization.⁷⁰ Due to the resonant nature of the SRR structure, this type of cloak must operate in a narrow band, as predicted from the original theory.² Essentially, this limitation arises from the singularity in Figure 1, as we have discussed in the previous sections.

A significant breakthrough subsequently followed, which was able to bypass the bandwidth limitation and push the working frequencies into the optical spectrum. Reference 71 proposed that an object sitting on a flat ground plane can be made invisible under a fully dielectric gradient-refractive-index ‘carpet’ cloak generated by quasi-conformal mapping. This idea is illustrated in Figure 5a. Apparently, after abandoning the free-standing requirement and imposing the restriction that the object must sit on a ground plane, the ‘hole’ for hiding the object can be effectively flattened to a flat surface rather than a single point, and thus, the singularity in Figure 1 can be avoided.

Quasi-conformal mapping can efficiently generate orthogonal meshes that have almost square shapes, which can be possibly implemented using isotropic materials.⁷¹

This exciting carpet cloak model with quasi-conformal mapping has resulted in a considerable number of subsequent experimental implementations (Figure 4b–4f). The first implementation (Figure 4b) based on this model was also at microwave frequencies, but the traditional SRR structure was changed to a non-resonant but still metallic structure.⁷² It is widely known that metal is significantly lossy at optical frequencies. To push the working frequency into the optical regime, Refs. 76–78 utilized dielectric silicon to implement a prototype carpet cloak on a thin-film waveguide at infrared frequencies (Figure 4c and 4d). Silicon is widely used in thin-film optical lithography, but it is very difficult to fabricate in 3D. Therefore, Refs. 74 and 75 utilized a polymer and fabricated a 3D cloak at microwave (Figure 4e) and infrared (Figure 4f) frequencies, respectively. Another strategy using a tapered waveguide with some similarity to transformation optics also had successful results in both microwave⁷⁹ and optical frequencies,⁸⁰ but it is out of the scope of our discussion.

However, these successfully demonstrated carpet cloaks still have limitations. First, as indicated in Ref. 81, the quasi-conformal mapping strategy will generally result in a lateral shift of the scattered wave, whose value is comparable to the height of the hidden object, which makes the object detectable. An illustration with geometrical parameters adopted from Ref. 71 is provided in Figure 5b, where meshes that are approximately square are generated from quasi-conformal mapping. We can see that although the bump (in reference to the

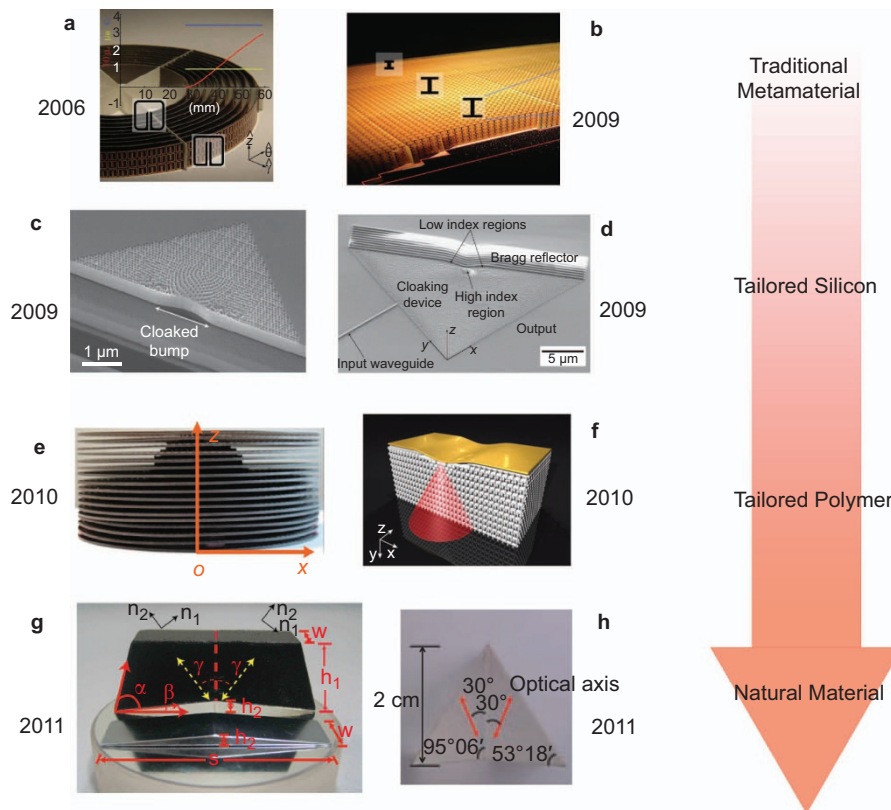


Figure 4 Milestones established on the path to implementing invisibility cloaks. **a** and **b** used traditional metamaterial technology. **c** and **d** were achieved by tailoring the local density of a silicon substrate. **e** and **f** utilized a polymer as a bulk material to implement three-dimensional cloaks. **g** and **h** adopted calcite as a natural material directly. Figure reprinted with permission: **a**, Ref. 69, © 2006 AAAS; **b**, Ref. 72, © 2009 AAAS; **c**, Ref. 76, © 2009 NPG; **d**, Ref. 77, © 2009 NPG; **e**, Ref. 74, © 2010 NPG; **f**, Ref. 75, © 2010 AAAS; **g**, Ref. 84, © 2011 APS; **h**, Ref. 85, © 2011 NPG.

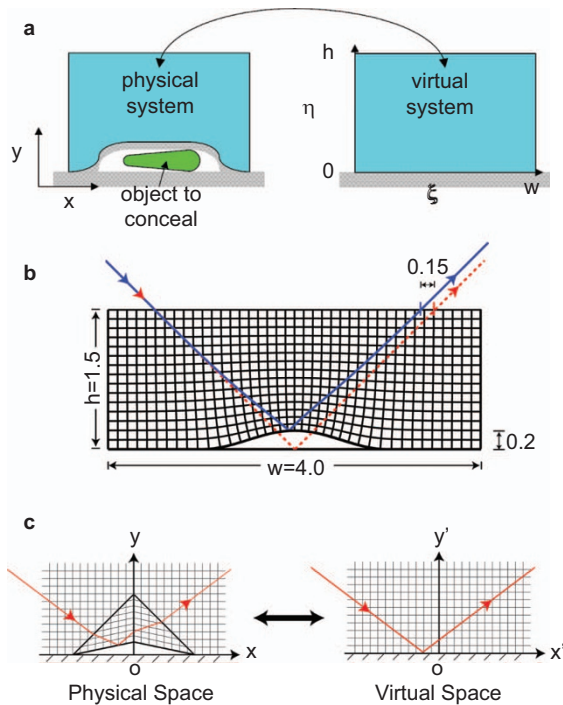


Figure 5 (a) An object sitting on top of the ground plane can be effectively flattened and thus be hidden under a ‘carpet’ cloak.⁷¹ The transformation does not have the singularity in Figure 1. (b) The lateral shift⁸¹ of the reflected ray (in blue) from a ‘carpet’ cloak that is generated from quasi-conformal mapping.⁷¹ The red dotted line indicates specular reflection when nothing is on top of the ground plane. (c) The ‘carpet’ cloak generated from an affine transformation adopted in Refs. 84 and 85 to overcome the lateral shift limitation. The reflected ray is exactly restored as if nothing was on top of the ground plane. Figure reprinted with permission: a, Ref. 71, © 2008 APS.

object to be cloaked) is successfully flattened such that the reflected wave can restore its angle of reflection, it appears that the flattened bottom surface (cf., the bottom of the ‘virtual system’ in Figure 5a) has been slightly lifted; therefore, the reflected wave is shifted towards the incidence direction. The lifted position of the flattened bottom surface is between the top and the bottom of the bump. Mathematically, this result corresponds to the bounds of the conformal module in conformal mapping.⁸² The reason for this lateral shift in Figure 5b is because the meshes generated from quasi-conformal mapping are not exactly square, and, consequently, the required transformation medium is not exactly isotropic. However, the resulting weak anisotropy is dropped as an approximation in an isotropic carpet cloak. Although the anisotropy is a result of the unavoidable mismatch of the conformal module in carpet cloaks, note that in some other applications, it is possible to choose a proper conformal module⁸³ to reduce the extra distortion. Another limitation that should be emphasized is the small size. Previous optical carpet cloaks can only work at the nanoscale to hide a tiny object that can only be observed under a microscope. To observe the cloaking effect with the naked eye, one must cloak a macroscopic object in visible light.

To overcome these limitations of the lateral shift and the small size, Refs. 84 and 85 adopted an affine transformation by utilizing the anisotropy rather than abandoning it, as in previous strategies. An illustration of this transformation is shown in Figure 5c. Then, calcite, which is a natural anisotropic optical material, was utilized to construct a macroscopic cloak that can work in broadband visible light (Figure 4g and 4h). Recently, Ref. 86 used a similar approach and

constructed a sapphire cloak at terahertz frequencies. Reference 87 used natural calcite material to cloak an isolated macroscopic cylindrical object from multiple angles. Note that macroscopic cloaking still complies with the delay-bandwidth and delay-loss limitations in Ref. 88, which can serve as guidance for the design and implementation of practical cloaking.

If we examine the above development procedure, as shown in Figure 4, we can observe that there is a clear trend for the transition of traditional metamaterials to natural materials. The inspiration we can obtain from this trend is twofold. First, this trend demonstrates that transformation optics is a powerful and general design strategy that can be used in a considerably broader manner by combining metamaterials and natural materials, although it was originally inspired from the development of metamaterials. Second, as we mentioned in the beginning, the development towards a specific function in transformation optics is a type of ‘loop’; the function determines what materials we need, whereas the properties of the materials control the performance of the function we desire. The evolution of this ‘loop’ development will eventually guide the technology into practical solutions that can be used to solve realistic problems.

THE ROAD AHEAD—CHALLENGES AND OPPORTUNITIES

As we have observed thus far, invisibility cloaking and transformation optics are a very exciting and rapidly developing field of research. However, there are still many challenges in the road ahead: how to implement a free-standing 3D cloak in practice is still unknown; how to use the cloaking strategy in free space is very challenging; how to extend the bandwidth is still under active exploration; and even the problem of how to properly characterize errors from all types of approximations for creating a real cloak has not been satisfactorily solved. After all, there are still many unsolved problems in the broad field of metamaterial research, not excepting invisibility cloaking.

Two of the key challenges for real applications of invisibility cloaking, from the author’s viewpoint, are the limitations of bandwidth and 3D implementation. Fortunately, there have been recent breakthroughs on non-Euclidian transformation optics^{73,89} that can potentially overcome at least the first difficulty. The application of non-Euclidian transformations may eventually produce a broadband free-standing invisibility cloak, which is worth anticipating. Another direction is static magnetic cloaking, which has essentially no bandwidth demand.^{90–92} Breaking the 3D fabrication limitation will take a long time, but it is still doable, especially at microwave frequencies.

However, transformation optics, as a new perspective and way of thinking when we perceive and design various types of optical materials, is not necessarily limited to the framework of metamaterials, although it was originally inspired from metamaterial research. In contrast, transformation optics should be able to integrate into a considerably broader regime of optics, especially traditional optics, including lens design, diffractive optics, optical fibers and holography. At microwave frequencies, this methodology should find appropriate applications in conformal antenna design and the miniaturization of current radiofrequency devices. Each significant step forward in these directions will bring forth a new market-ready technology, where original studies of invisibility cloaking will play the role of an impellent cause and a catalyst in technology creation. Therefore, it is time for more engineers together with scientists from wide branches of optics/electromagnetics to participate in this development in close cooperation. The ingenious idea of the inverse design from ‘function’ to ‘material’ has inspired many of our imaginations in the past few years, whereas creation in the direction from ‘material’ to ‘function’ is of

equal importance at this stage. In this manner, practically useful solutions will finally be generated during the ‘loop’ development. As commented in Ref. 19, ‘the idea of transformation optics is so beautiful that it would seem a profligate waste of inspiration if it didn’t lead to something useful’.

ACKNOWLEDGMENTS

The author is grateful for many discussions with Professor G Barbastathis and for the financial support from Nanyang Technological University.

- 1 Leonhardt U. Optical conformal mapping. *Science* 2006; **312**: 1777–1780.
- 2 Pendry JB, Schurig D, Smith DR. Controlling electromagnetic fields. *Science* 2006; **312**: 1780–1782.
- 3 Pendry JB. Negative refraction makes a perfect lens. *Phys Rev Lett* 2000; **85**: 3966–3969.
- 4 Leonhardt U, Philbin TG. Transformation optics and the geometry of light. *Prog Opt* 2009; **53**: 69–153.
- 5 Dantzig DV. The fundamental equations of electromagnetism, independent of metrical geometry. *Proc Cambridge Philos Soc* 1934; **30**: 421–427.
- 6 Dolin SL. On a possibility of comparing three-dimensional electromagnetic systems with inhomogeneous filling. *Izv Vyssh Uchebn Zaved Radiofiz* 1961; **4**: 964–967.
- 7 Lax M, Nelson DF. Maxwell equations in material form. *Phys Rev B* 1976; **13**: 1777–1784.
- 8 Post EJ. *Formal structure of electromagnetics: general covariance and electromagnetics*. New York: Interscience Publishers, 1962.
- 9 Ward AJ, Pendry JB. Refraction and geometry in Maxwell’s equations. *J Mod Opt* 1996; **43**: 773–793.
- 10 Teixeira FL, Chew WC. Lattice electromagnetic theory from a topological viewpoint. *J Math Phys* 1999; **40**: 169–187.
- 11 Teixeira FL, Chew WC. Differential forms, metrics, and the reflectionless absorption of electromagnetic waves. *J Electromagn Waves Appl* 1999; **13**: 665–686.
- 12 Schurig D, Pendry JB, Smith DR. Transformation designed optical elements. *Opt Express* 2007; **15**: 14772–14782.
- 13 Liu Y, Zentgraf T, Bartal G, Zhang X. Transformational plasmon optics. *Nano Lett* 2010; **10**: 1991–1997.
- 14 Aubry A, Lei DY, Fernández-Domínguez AI, Sonnefraud Y, Maier SA *et al.* Plasmonic light-harvesting devices over the whole visible spectrum. *Nano Lett* 2010; **10**: 2574–2579.
- 15 Chen H, Chan CT, Sheng P. Transformation optics and metamaterials. *Nat Mater* 2010; **9**: 387–396.
- 16 Chen H, Chan CT. Acoustic cloaking and transformation acoustics. *J Phys D: Appl Phys* 2010; **43**: 113001.
- 17 Dubinov AE, Mytareva LA. Invisible cloaking of material bodies using the wave flow method. *Phys Usp* 2010; **53**: 455–479.
- 18 Shalaev VM. Transforming light. *Science* 2008; **322**: 384–386.
- 19 Service RF, Cho A. Strange new tricks with light. *Science* 2010; **330**: 1622.
- 20 Leonhardt U, Philbin TG. *Geometry and light: the science of invisibility*. Mineola, NY: Dover, 2010.
- 21 Leonhardt U. To invisibility and beyond. *Nature* 2011; **471**: 292–293.
- 22 Alu A, Engheta N. Achieving transparency with plasmonic and metamaterial coatings. *Phys Rev E* 2005; **72**: 016623.
- 23 Alu A, Engheta N. Cloaking a sensor. *Phys Rev Lett* 2009; **102**: 233901.
- 24 Fan P, Chettiar U, Cao L, Afshinmanesh F, Engheta N *et al.* An invisible metal-semiconductor photodetector. *Nat Photon* 2012; **6**: 380–385.
- 25 Lai Y, Chen H, Zhang ZQ, Chan CT. Complementary media invisibility cloak that cloaks objects at a distance outside the cloaking shell. *Phys Rev Lett* 2009; **102**: 093901.
- 26 Greenleaf A, Lassa M, Uhlmann G. On nonuniqueness for Calderon’s inverse problem. *Math Res Lett* 2003; **10**: 685–693.
- 27 Greenleaf A, Lassa M, Uhlmann G. Anisotropic conductivities that cannot be detected by EIT. *Physiol Meas* 2003; **24**: 413–419.
- 28 Schurig D, Pendry JB, Smith DR. Calculation of material properties and ray tracing in transformation media. *Opt Express* 2006; **99**: 9794–9894.
- 29 Cummer SA, Popa BI, Schurig D, Smith DR, Pendry JB. Full-wave simulations of electromagnetic cloaking structure. *Phys Rev E* 2006; **74**: 036621.
- 30 Zolla F, Guenneau S, Nicolet A, Pendry JB. Electromagnetic analysis of cylindrical invisibility cloaks and the mirage effect. *Opt Lett* 2007; **32**: 1069–1071.
- 31 Chen H, Wu BI, Zhang B, Kong JA. Electromagnetic wave interactions with a metamaterial cloak. *Phys Rev Lett* 2007; **99**: 063903.
- 32 Ruan X, Yan M, Neff CW, Qiu M. Ideal cylindrical cloak: perfect but sensitive to tiny perturbations. *Phys Rev Lett* 2007; **99**: 113903.
- 33 Zhang B, Chen H, Wu BI, Luo Y, Ran L *et al.* Response of a cylindrical invisibility cloak to electromagnetic waves. *Phys Rev B* 2007; **76**: 121101(R).
- 34 Greenleaf A, Kurylev Y, Lassa M, Uhlmann G. Improvement of cylindrical cloaking with the SHS lining. *Opt Express* 2007; **15**: 12717–12734.
- 35 Sheng P. Waves on the horizon. *Science* 2006; **313**: 1399–1400.
- 36 Chen H, Luo X, Ma H. The anti-cloak. *Opt Express* 2008; **16**: 14603–14608.
- 37 Castaldi G, Gollina I, Galdi V, Alu A, Engheta N. Cloak/anti-cloak interactions. *Opt Express* 2009; **16**: 106343.
- 38 Kong JA. *Electromagnetic wave theory*. Cambridge, MA: EMW Publishing, 2008.
- 39 Landau LD, Lifshitz EM. *Electrodynamics of continuous media*. Oxford: Pergamon Press, 1984.
- 40 Greenleaf A, Kurylev Y, Lassa M, Uhlmann G. Full-wave invisibility of active devices at all frequencies. *Commun Math Phys* 2007; **275**: 749–789.
- 41 Zhang B, Chen H, Wu BI, Kong JA. Extraordinary surface voltage effect in the invisibility cloak with an active device inside. *Phys Rev Lett* 2008; **100**: 063904.
- 42 Bladel JV. *Singular electromagnetic fields and sources*. New York: IEEE Press, 1991.
- 43 Yaghjian AD, Maci S. Alternative derivation of electromagnetic cloaks and concentrators. *New J Phys* 2008; **10**: 115022.
- 44 Weder R. The boundary conditions for point transformed electromagnetic invisibility cloaks. *J Phys A Math Theor* 2008; **41**: 415401.
- 45 Lindell IV, Sihvola AH. Electromagnetic boundary conditions defined in terms of normal field components. *IEEE Trans Antennas Propag* 2010; **58**: 1128–1135.
- 46 Michalski KA. Missing boundary conditions of electromagnetics. *Electron Lett* 1986; **22**: 921–922.
- 47 Idemen M, Serbest AH. Boundary conditions of the electromagnetic field. *Electron Lett* 1987; **23**: 704–708.
- 48 Garcia N, Nieto-Vesperinas M. Left-handed materials do not make a perfect lens. *Phys Rev Lett* 2002; **88**: 207403.
- 49 Pendry JB. Comment on left-handed materials do not make a perfect lens. *Phys Rev Lett* 2003; **91**: 099701.
- 50 Chew WC. Sommerfeld integrals for lefthanded materials. *Microwave Opt Technol Lett* 2004; **42**: 369–373.
- 51 Wee WH, Pendry JB. Universal evolution of perfect lenses. *Phys Rev Lett* 2011; **106**: 165503.
- 52 Ginis V, Tassin P, Soukoulis CM, Veretennico I. Confining light in deep subwavelength electromagnetic cavities. *Phys Rev B* 2010; **82**: 113102.
- 53 Zhai T, Zhou Y, Shi J, Wang Z, Liu D *et al.* Electromagnetic localization based on transformation optics. *Opt Express* 2010; **18**: 11891.
- 54 Ginis V, Tassin P, Danckaert J, Soukoulis CM, Veretennico I. Creating electromagnetic cavities using transformation optics. *New J Phys* 2012; **14**: 033007.
- 55 Miller DA. On perfect cloaking. *Opt Express* 2006; **14**: 12457–12466.
- 56 Chen H, Liang ZX, Yao PJ, Jiang XY, Ma HR *et al.* Extending the bandwidth of electromagnetic cloaks. *Phys Rev B* 2007; **76**: 241104(R).
- 57 Brillouin L. *Wave propagation and group velocity*. New York/London: Academic Press, 1960.
- 58 Yao P, Liang Z, Jiang X. Limitation of the electromagnetic cloak with dispersive material. *Appl Phys Lett* 2008; **92**: 031111.
- 59 Chen H, Chan CT. Time delays and energy transport velocities in three dimensional ideal cloaking devices. *J Appl Phys* 2008; **104**: 033113.
- 60 Stratton JA. *Electromagnetic theory*. New York/London: McGraw-Hill, 1941.
- 61 Zhang B, Wu BI, Chen H, Kong JA. Rainbow and blueshift effect of a dispersive spherical invisibility cloak impinged on by a nonmonochromatic plane wave. *Phys Rev Lett* 2008; **101**: 063902.
- 62 Zhang B, Wu BI, Chen H. Optical delay of a signal through a dispersive invisibility cloak. *Opt Express* 2009; **17**: 6721–6726.
- 63 Kundtz N, Roberts DA, Allen J, Cummer S, Smith DR. Optical source transformations. *Opt Express* 2008; **16**: 21215–21222.
- 64 Luo Y, Zhang J, Chen H, Huangfu J, Ran L. High-directivity antenna with small antenna aperture. *Appl Phys Lett* 2009; **95**: 193506.
- 65 Tai CT. *Dyadic green functions in electromagnetic theory*. New York: IEEE Press, 1994.
- 66 Yaghjian AD. Electric dyadic Green’s functions in the source region. *Proc IEEE* 1980; **68**: 248–263.
- 67 Zhang B, Wu BI. Electromagnetic detection of a perfect invisibility cloak. *Phys Rev Lett* 2009; **103**: 243901.
- 68 Feynman RP, Leighton RB, Sands M. *The Feynman lectures on physics. Vol. I*. Redwood City, CA: Addison-Wesley, 1989.
- 69 Schurig D, Mock J, Justice B, Cummer S, Pendry J *et al.* Metamaterial electromagnetic cloak at microwave frequencies. *Science* 2006; **314**: 977–980.
- 70 Kant B, Germain D, de Lustrac A. Experimental demonstration of a nonmagnetic metamaterial cloak at microwave frequencies. *Phys Rev B* 2009; **80**: 201104(R).
- 71 Li J, Pendry JB. Hiding under the carpet: a new strategy for cloaking. *Phys Rev Lett* 2008; **101**: 203901.
- 72 Liu R, Ji C, Mock JJ, Chin JY, Cui TJ *et al.* Broadband ground-plane cloak. *Science* 2009; **323**: 366–369.
- 73 Leonhardt U, Tyc T. Broadband invisibility by non-Euclidean cloaking. *Science* 2009; **323**: 110–112.
- 74 Ma HF, Cui TJ. Three-dimensional broadband ground-plane cloak made of metamaterials. *Nat Commun* 2010; **1**: 21.
- 75 Ergin T, Stenger N, Brenner P, Pendry JB, Wegener M. Three-dimensional invisibility cloak at optical wavelengths. *Science* 2010; **328**: 337–339.
- 76 Valentine J, Li J, Zentgraf T, Bartal G, Zhang X. An optical cloak made of dielectrics. *Nat Mater* 2009; **8**: 568–571.
- 77 Gabrielli LH, Cardenas J, Poitras CB, Lipson M. Silicon nanostructure cloak operating at optical frequencies. *Nat Photon* 2009; **3**: 461–463.
- 78 Lee JH, Blair J, Tamma VA, Wu Q, Rhee SJ *et al.* Direct visualization of optical frequency invisibility cloak based on silicon nanorod array. *Opt Express* 2009; **17**: 12922–12928.

- 79 Tretyakov S, Alitalo P, Luukkonen O, Simonvski C. Broadband electromagnetic cloaking of long cylindrical objects. *Phys Rev Lett* 2009; **103**: 103905.
- 80 Smolyaninov II, Smolyaninova VN, Kildishev AV, Shalaev VM. Anisotropic metamaterial emulated by tapered waveguides: application to optical cloaking. *Phys Rev Lett* 2009; **102**: 213901.
- 81 Zhang B, Chan T, Wu BI. Lateral shift makes a ground-plane cloak detectable. *Phys Rev Lett* 2010; **104**: 233903.
- 82 Challis NV, Burley DM. A numerical method for conformal mapping. *IMA J Numer Anal* 1982; **2**: 161–181.
- 83 Landy NI, Kundtz N, Smith DR. Designing three-dimensional transformation optical media using quasi-conformal coordinate transformations. *Phys Rev Lett* 2010; **105**: 193902.
- 84 Zhang B, Luo Y, Liu X, Barbastathis G. Macroscopic invisibility cloak for visible light. *Phys Rev Lett* 2011; **106**: 033901.
- 85 Chen X, Luo Y, Zhang J, Jiang K, Pendry JB *et al*. Macroscopic invisibility cloaking of visible light. *Nat Commun* 2011; **2**: 176.
- 86 Liang D, Gu JQ, Han JG, Yang YM, Zhang S *et al*. Robust large dimension terahertz cloaking. *Adv Mater* 2012; **24**: 916–921.
- 87 Chen H, Zheng B. Broadband polygonal invisibility cloak for visible light. *Sci Rep* 2012; **2**: 255.
- 88 Hashemi H, Zhang B, Joannopoulos JD, Johnson SG. Delay-bandwidth and delay-loss limitations for cloaking of large objects. *Phys Rev Lett* 2010; **104**: 253903.
- 89 Perczel J, Tyc T, Leonhardt U. Invisibility cloaking without superluminal propagation. *New J Phys* 2011; **13**: 083007.
- 90 Wood B, Pendry JB. Metamaterials at zero frequency. *J Phys Condens Matter* 2007; **19**: 076208.
- 91 Narayana S, Sato Y. DC magnetic cloak. *Adv Mater* 2012; **24**: 71–74.
- 92 Gomory F, Solovyov M, Souc J, Navau C, Prat-Camps J *et al*. Experimental realization of a magnetic cloak. *Science* 2012; **335**: 1466–1468.



This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivative Works 3.0 Unported License. To view a copy of this license, visit <http://creativecommons.org/licenses/by-nc-nd/3.0>