Heredity (1982), **48** (1), 145–146 0018-067X/82/01600145$02.00

ASSORTATIVE MATING WITH DOMINANCE

A. E. STARK

School of Community Medicine, The University of New South Wales, P.O. Box 1, Kensington, Australia, 2033

Received 21.v.81

HALDANE (1924) introduced a model of assortative mating with dominance but gave only sufficient of its properties to demonstrate the influence of assortative mating on selection. Here it is compared with the much better understood model of O'Donald (1960).

Let the frequencies at generation t of genotypes gg, gG, and GG be denoted by $f_0(t)$, $f_1(t)$, and $f_2(t)$, respectively. The first is referred to as the *rec* type and the others in combination as the *dom* type.

The mating frequencies in Haldane's system are

$$\begin{aligned} \text{rec female} \times \text{rec male:} &\quad f_0(t) - \theta(t) \\ \text{rec female} \times \text{dom male:} &\quad \theta(t) \\ \text{dom female} \times \text{rec male:} &\quad \theta(t) \\ \text{dom female} \times \text{dom male:} &\quad 1 - f_0(t) - \theta(t), \end{aligned} \tag{1}$$

where $\theta(t)$ is constrained by the constant assortative mating parameter Λ $(-1 \leqq \Lambda < \infty)$ according to

$$(f_0(t) - \theta(t))(1 - f_0(t) - \theta(t)) = (1 + \Lambda)(\theta(t))^2. \tag{2}$$

Rearranging (2) gives

$$\theta(t) = (\{1 + 4\Lambda f_0(t)[1 - f_0(t)]\}^{1/2} - 1)/(2\Lambda).$$

For purposes of analysis the mating frequencies are put in genotypic pairings whose frequencies are denoted by $f_{ij}(t)$ $(i = 0, 1, 2;\ j = 0, 1, 2)$ which are expressed in canonical form

$$f_{ij}(t) = f_i(t) \,.\, f_j(t)(1 + \rho(t) \,.\, x_i(t) \,.\, x_j(t)). \tag{3}$$

Then Haldane's frequencies (1) conform to (3) when:

$$\rho(t) \equiv \rho^*(t) = 1 - \theta(t)/\{f_0(t)(1 - f_0(t))\}; \tag{4}$$

$$x_1(t) = x_2(t) = \{f_0(t)/(1 - f_0(t))\}^{1/2}, \qquad x_0(t) = -(x_1(t))^{-1}. \tag{5}$$

In O'Donald's system a fixed proportion α of phenotypes mate with like phenotype and the remainder mate at random. The mating frequencies under this scheme can be put in form (3) by the substitution $\rho(t) = \rho \equiv \rho' = \alpha$ and $\{x_i(t)\}$ as in (5).

In equilibrium the frequency of genotype gG is given by O'Donald as

$$f_1 = p^2(\alpha - 1) + p\{(1 - \alpha)(p^2(1 - \alpha) + 4q)\}^{1/2},$$

where $q = f_0 + \frac{1}{2}f_1$ and $p = 1 - q$. Using the corresponding f_0, the same mating, and therefore the same offspring frequencies, can be obtained in equilibrium under Haldane's scheme by setting Λ to be

$$\Lambda = \alpha/(f_0(1 - f_0)(1 - \alpha)^2),$$

for then

$$\theta(t) = \theta = f_0(1 - f_0)(1 - \alpha)$$

and $\rho^*(t) = \rho^* = \alpha = \rho'$.

The rate of approach to equilibrium is of the same order in the two systems.

The parameter ρ can be expressed in terms of q and p and the index of homogamy λ, when $f_0 = q^2 + \lambda pq$ etc. From the equilibrium identity $f_{11} = 4f_{02}$, ρ is found to be

$$\rho = \lambda(1 + q(1 - \lambda))/(q + \lambda p). \tag{6}$$

Equation (6), taken in conjunction with (3) and (5), shows that systems in which assortative mating is based on dominance and which give the same genotypic distribution, have identical mating frequencies in equilibrium.

Acknowledgments.—Thanks are due to Professors S. D. Jayakar and C. Matessi, CNR, Pavia, Italy, who read an earlier version of the manuscript and offered constructive comments.

References

HALDANE, J. B. S. 1924. A mathematical theory of natural and artificial selection. Part II. The influence of partial self-fertilisation, inbreeding, assortative mating, and selective fertilisation on the composition of Mendelian populations, and on natural selection. *Proc. Camb. Phil. Soc., Biol. Sc., 1*, 158–163.

O'DONALD, P. 1960. Assortive mating in a population in which two alleles are segregating. *Heredity, 15*, 389–396.