

ON MIXTURES OF INBREEDING SYSTEMS

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ONE can consider a large, natural population as one in which some of the matings occur at random while the remainder take place between relatives of various degrees. As is well known (Kempthorne, Ch. 5, 3), the so-called "probabilistic method" is easily extended to work out the *mean* coefficient of inbreeding for such a population. While this can be enough for some limited purpose, one often needs to know the *distribution* of the possible degrees of relationship in the population. It is argued here that such information can also be obtained, at least in principle, through an extension of the probabilistic method.

Consider, for example, an infinitely large population with separate generations, and assume that there is a constant probability of selfing, s , mating at random being the only alternative. Thus all existing inbreeding is due to selfing. For the sake of simplicity, it is also assumed that the parental generation is made up of unrelated individuals. Further, it is slightly more convenient to work in terms of "panmictic index" $P \equiv 1 - F$. The possible values of P at each generation are first derived, referring to the genealogies in the figure. Since the parents are unrelated, in the first



generation there are only two kinds of individuals: unrelated ($P = 1$, like D) and from one generation of selfing ($P = 1/2$, like C). In the F_2 there is a third kind of individual, that coming from two generations of selfing ($P = 1/4$, like G). Thus at the n th generation the possible values of P are given by $(1/2)^i$ with $i = 0, 1, \dots, n$. The frequencies of these values will be denoted by $W_i^{(n)}$.

In our case

$$W_0^{(0)} = 1$$

and, obviously

$$W_0^{(1)} = 1 - s, \quad W_1^{(1)} = s.$$

It is also easy to recognise that

$$W_0^{(2)} = 1 - s, \quad W_1^{(2)} = s(1 - s) \quad \text{and} \quad W_2^{(2)} = s^2$$

and, by induction, that the distribution in a general (n)th generation is

$$\begin{aligned}
 W_0^{(n)} &= 1 - s, \quad W_1^{(n)} = s(1 - s), \quad W_2^{(n)} = s^2(1 - s), \\
 \dots, \quad W_{n-1}^{(n)} &= s^{n-1}(1 - s), \quad W_n^{(n)} = s^n \quad \text{and} \quad W_i^{(n)} = 0; \quad i > n.
 \end{aligned}$$

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Its moments are easily calculated, thus

$$\begin{aligned} \mathcal{E}P_n &= (1-s) \sum_0^{n-1} (s/2)^i + (s/2)^n \\ &= \frac{1-s + (s/2)^{n+1}}{1-s/2}, \end{aligned}$$

in agreement with the value of F_n given in the reference; also

$$\mathcal{E}P_n^2 = \frac{1-s + 3(s/4)^{n+1}}{1-s/4} \quad \text{a.s.o.}$$

As n tends to infinity one gets the stationary distribution of P values

$$W_i = (1-s)^i$$

having mean and variance

$$\begin{aligned} \mathcal{E}P &= 2 \frac{1-s}{2-s} \\ \text{Var } P &= \frac{4s(1-s)}{(4-s)(2-s)^2}. \end{aligned}$$

The same formulae apply to a mixture of random and parent-offspring mating in the case of sex-linked traits or haplo-diploidy, as in bees, provided that "haploid" generations are not counted. More precisely, if n is even, $W_i^{(n)}$ refers to a diploid descendant of a diploid in the parental, and coming, except for $W_0^{(n)}$, from the cross of an $(n-1)$ th generation diploid with his haploid offspring. Odd n refer to lines starting with a haploid and, for them,

$$W_i^{(2k+1)} = W_i^{(2k)}.$$

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EFFECT OF FRACTIONATED DOSES OF RADIATION ON RECOMBINATION

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Low, virtually non-lethal doses of ^{60}Co gamma radiation change gene recombination in *Chlamydomonas reinhardi*, and also chiasma frequency in *Lilium longiflorum* and *Tradescantia paludosa*, only if given during two short meiotic stages (Lawrence, 1965*a*, 1961*a*, *b*). Preleptotene irradiation decreases the frequency of recombination while treatment in late zygotene or pachytene gives an increase. The response at both stages may be due to