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Before symbols, complex formulae were expressed in words.

MATHEMATICS

Numbers game

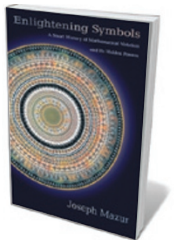
George Szpiro enjoys a history of the slow but irresistible rise of mathematical symbols.

Solving an equation such as $2x - 4 = 10$ is almost ludicrously easy. The symmetry is visible; we just perform identical manipulations on both sides of the equals sign until the symbol for the unknown value, x, remains isolated on the left. But it was not always that simple, as mathematician Joseph Mazur reveals in his history of maths notation, *Enlightening Symbols*.

Until the late Renaissance, there were few symbols to depict clearly and concisely what would later become known as an equation. Numbers and numerical problems were framed in words and sentences. Readers, mostly monks and scholars, had to form mental pictures of the problems at hand to solve, for example, quadratic equations. In the late nineteenth century, even economists still shunned equations and mathematical notation, preferring narrative and anecdotes. In his fascinating narrative, Mazur places the development of mathematical symbolism squarely within cultural evolution.

In the first part of *Enlightening Symbols*, 'Numerals', Mazur traces the depiction of numbers, beginning more than five millennia ago. First, Sumerian farmers calculated the areas of fields using cuneiform numbers; later, Egyptian priests used hieroglyphs for their calendars; and in medieval Europe, astronomers, merchants and officers of courts went about their daily business with abacuses. The chronicle leads from counting on ten fingers, through Egyptian, Hebrew, Greek, Chinese and Roman numerals, to the positional system that we use today, with units in the right-hand column, tens in the next column to the left, hundreds in the next, and so on.

A revolution was fomented in the seventh century AD by Hindu scholars, who invented zero both as a number to denote nothingness and as a placeholder for the positional system. Dissemination of the system in Europe began with translations into Arabic in the late eighth century, and into Latin in the early twelfth century. Mazur discusses many notable early mathematicians, including the Indian Brahmagupta in the seventh century, the Persian Muhammad ibn Musa Al-Khwarizmi in the ninth and the Italian Leonardo Pisano Bigollo — better known as Fibonacci — in the thirteenth.



Enlightening Symbols: A Short History of Mathematical Notation and Its Hidden Powers
JOSEPH MAZUR
Princeton University Press: 2014.

But numerals are only part of the story. There must also be signs to indicate negative and imaginary numbers, symbols to denote special constants, characters to represent unknowns and parameters, and pictograms for operators. Mazur's second section, 'Algebra', covers these. In AD 830, Al-Khwarizmi asked: "What must be the amount of two squares which, when summed up and added to ten times the root of one of them, make up a sum of forty eight dirhems?" Compare such verbosity to our concise $2x^2 + 10x = 48$, and then imagine the even more frightening impression left on readers by the Italian mathematician Scipio del Ferro's wordy prescription of how to solve cubic equations in 1505.

About 50 years later, the Welsh physician Robert Recorde introduced the equals sign, =, "to auoide the tediouse repetition of these woordes: is equalle to … because noe. 2. thynges, can be moare equalle" than a pair of parallels. Recorde was following a trend to symbolize algebra, first with word abbreviations and later with specialized symbols, that had been started by mathematicians in the Maghreb about two centuries earlier.

'The Power of Symbols', the book's third part, is the most challenging. In it, Mazur turns to the theory of mind. Does one need language to have mathematical thought? Drawing on material from luminaries including writer Fyodor Dostoyevsky, philosopher Ludwig Wittgenstein, cognitive neuroscientist Stanislas Dehaene and psychologist Daniel Kahneman, he wrestles with the question.

In literature, for example, meaning comes from associative experiences, and so it is at times with reading mathematics. Mazur argues that symbols, like poems, make connections between experience and the unknown to convey meaning. But when symbols acquire meaning, he warns, one may lose sight of the object represented, and continue with mechanical manipulations without grasping the essence of what is being done — as when one juggles equations that contain π without comprehending the connection to a circle's circumference.

This is a nuanced, intelligently framed chronicle packed with nuggets — such as the fact that Hindus, not Arabs, introduced Arabic numerals. In a word: enlightening. ■

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