A new fish-eye lens based on an idea of James Clark Maxwell.

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To invisibility and beyond

Combining Maxwell's equations with Einstein's general relativity promises perfect images and cloaking devices, explains **Ulf Leonhardt**.

Many everyday products of modern technology — such as mobile phones, television, computers and electric light — would seem almost magical to our ancestors. These all derive from James Clerk Maxwell's unification of the laws of electricity and magnetism 150 years ago. Scientists are still creating new wonders in the laboratory that exploit Maxwell's laws of electromagnetism. Some of these devices are made from extraordinary 'metamaterials' that can perform unusual tricks with light[1] — from optical cloaking to perfect imaging. Just last week, my colleagues and I announced evidence for 'perfect imaging'[2] using a device based on an idea[3] of Maxwell's from 1854: a fitting tribute to a theorist who always thought in practical terms.

The field of metamaterials is barely ten years old. Early metamaterials relied on advances in nanotechnology to build tiny structures, such as metallic rings or wires, that are smaller than the wavelength of light. These nanostructures modify the electromagnetic properties of the metamaterial, sometimes creating optical effects that are not seen in nature. In 2006, for example, US scientists made the first prototype of an electromagnetic 'cloaking device'[4]. This makes a coin-sized object invisible to microwaves of a certain polarization and frequency. Modern metamaterials are also being used to make perfect lenses, that can image details finer than the wavelength of light.

What I find fascinating about metamaterials is how they connect my research area, optics, with Albert Einstein's theory of general relativity. The link is Maxwell's equations. It reminds me how much Einstein owed to Maxwell (a debt he always acknowledged), and also offers a way for the mathematical tools of general relativity to become practically useful in engineering.

In 1861, at the age of 30, Maxwell collected together all the laws of electricity and magnetism, and added one of his own. In doing so, he was the first to unify the concept of light with electricity and magnetism. As he wrote in 1864: "We have strong reason to conclude that light itself — including radiant heat and other radiation, if any — is an electromagnetic disturbance in the form of waves." Without this insight, we would have no understanding of the electromagnetic spectrum, from radio waves and microwaves through the visible to X-rays and gamma rays.

His discovery led to many subsequent advances in our understanding of light and matter. Einstein's theory of special relativity follows directly from Maxwell's equations in empty space — that much is obvious if you read Einstein's original papers. But it surprised me to learn how much Maxwell there is in Einstein's theory of general relativity (his theory of gravity) where, according to American physicist John Wheeler, "mass tells space how to curve and space tells mass how to move".

VIRTUAL SPACE

In fact the connections between Maxwell and Einstein are all around us — optical materials such as glass or water can also be said to curve space[5]. Looking through the front or top of an aquarium, for example, fish inside can appear at different positions and in different sizes, depending on your viewpoint. The glass and the water change your optical perception of space, but the fish still swim happily at their actual locations in physical space.

The aquarium creates a 'virtual' space for what our eyes see, different from ordinary physical space. This virtual space is the space experienced by light — not by the fish — and it has a curved geometry that corresponds exactly to that calculated using Einstein's theory of general relativity. So curved space doesn't just belong to cosmology — it is commonplace.

Rethinking optical materials as virtual spaces is an interesting idea, but is it useful in practice? It turns out to be crucial to the design of

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metamaterials and optical devices. Transformation optics[1], as the combination of general relativity and optics[5] is now known, has inspired engineers, physicists and mathematicians to dream up many wonderful devices. Only a tiny fraction of these ideas will make it out of the laboratory, but some will do the seemingly impossible.

For example, cloaking with metamaterials is easy in theory[6] (see 'Cloaking by transforming space'). In practice, it works only for specific wavelengths[4]. So an object may seem invisible to microwaves, but not to visible light. Scientists working in the visible spectrum have recently succeeded in creating devices that achieve partial cloaking, also known as 'carpet cloaking', whereby a three-dimensional object is made to look flat[7]. This feat of camouflage can be done with almost natural materials (silicon structures that look like tiny woodpiles) rather than complicated ring-and-coil metamaterials.

Carpet cloaking with silicon[7] is related to my work, because I prefer to focus on materials with optical properties that are closer to those found in nature. These are not metamaterials, although they are engineered in ways that obey the rules of transformation optics. For such materials, a mathematical theorem — the Riemann mapping theorem — forbids cloaking transformations. It took me three years to find a way around the theorem[8], but doing so allowed me to extend the framework of transformation optics to a broader range of wavelengths. However, we have not yet demonstrated real cloaking devices constructed from natural materials, and true invisibility for visible light remains impractical with existing materials and fabrication technology.

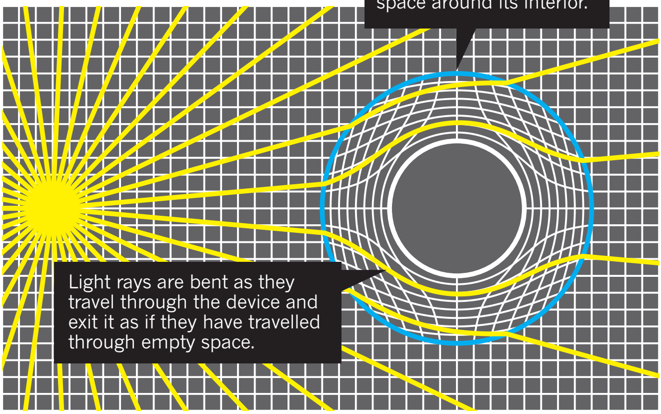
THE PERFECT IMAGE

In my opinion, the most promising potential of transformation optics lies in imaging. The idea of perfect imaging is what launched the field of metamaterials — and it takes us neatly back to Maxwell. Ordinary imaging devices such as microscopes suffer from a fundamental problem: they cannot image structures much smaller than half the wavelength of light, the diffraction limit. One cannot take a snapshot of atoms and molecules, because they are too small. In 2000, John Pendry introduced the concept of metamaterials[9], showing that materials with negative refraction can, in theory, make a perfect lens that beats the diffraction limit. Materials with negative refraction can bend light in a direction that would not occur normally (all natural materials have positive refraction).

By 2006, Pendry's perfect lens could be understood by using the tools of transformation optics[10]. The lens appears to have folded space: a plane of physical space appears like a folded sheet of paper in virtual space. The electromagnetic waves in the folded regions are absolutely identical, which explains why the image is a perfect copy of the original and no information is lost. But if light waves in two regions are identical, then light must instantly hop from one region of space to another, which, according to Einstein, cannot happen. Therefore, in reality, perfect imaging with negative refraction must be impossible for any useful device over practical distances. To get a negatively refracting perfect lens to work, imaging is limited to distances far smaller than the wavelength of light used.

CLOAKING BY TRANSFORMING SPACE

Objects can be rendered invisible to certain wavelengths of light.



A cloaking device deforms the grid of space around its interior.

Light rays are bent as they travel through the device and exit it as if they have travelled through empty space.

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Anything inside the device is hidden from an external viewer, and also doesn't affect the light rays carrying the image of the surrounding scenery, which creates the ultimate optical illusion: invisibility.

An alternative approach can produce perfect imaging without using negatively refracting materials[2]; it is inspired by another idea of (who else?) Maxwell[3]. As a mathematics student at Trinity College at the University of Cambridge, UK, Maxwell dreamed up an optical device that reminded him of the eye of a fish. In Maxwell's 'fish-eye' lens, light travels in physical space as if it were confined to the surface of a virtual sphere. On this virtual sphere, light rays would go round in circles so that all light waves emitted from one point would meet again, perfectly, on the opposite side, just because of the symmetry of the sphere. Maxwell showed in 1854 that this fish-eye lens would give perfect resolution — that is, a point source appears as a point image. When Maxwell proposed his fish-eye lens he knew nothing about the wave nature of light — his electromagnetic discoveries were still five years away. And so for the past 150 years it was assumed that the wave nature of light would, in practice, restrict the resolution of a fish-eye lens to the diffraction limit.

In 2009 I argued, using transformation optics, that a fish-eye lens should in fact image waves with perfect resolution. As with Pendry's prediction[9] of perfect imaging, this proposal created much controversy, in part because it sounds too good to be true and in part because it contradicts accepted wisdom. However, we have recently demonstrated perfect imaging for microwaves[2] using a two-dimensional version of the fish-eye lens (see image). We built our perfect lens using a metamaterial constructed from concentric bands of copper circuit board surrounded by a metallic mirror. Without a detector, the microwaves are reflected back and forth between source and image. But with a detector array in place (similar in principle to a digital camera) the fish eye can resolve two point sources that in ordinary imaging would appear blurred together. In principle, this route to perfect imaging should be achievable without using structured metamaterials. The next step is to demonstrate the same device for light, rather than microwaves.

What is truly remarkable about transformation optics is that by connecting Maxwell to Einstein, a theory as abstract as general relativity has actually become useful in engineering. Both men are known for their beautiful theories, but they were practical theoretical physicists: Maxwell performed experiments of his own, and Einstein always enjoyed making inventions and filing patents. Of course, in an ideal world run by wise politicians, we would not need to worry about justifying science as practical or fundamental. My mentor Stig Stenholm said that "the discovery of Maxwell's equations has already paid for all fundamental research for the following 500 years", because it laid the foundations of most of modern technology. We ought to have 350 years to go, no questions asked, thanks to Maxwell. ■

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