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EXOTIC MATTER

Another dimension for anyons

Chetan Nayak

Non-Abelian anyons are hypothesized particles that, if found, could form the basis of a fault-tolerant quantum computer. The theoretical finding that they may turn up in three dimensions comes as a surprise.

In an article published in *Physical Review Letters*, Teo and Kane describe[1] a theoretical model for systems in which an exotic state of matter, termed a topological insulator, coexists with superconductivity. They found, surprisingly, that the model predicts hypothesized particles called non-Abelian anyons, and allows these particles to move around in three dimensions, thus breaking free of the bonds that were thought to chain them down to two dimensions.

To appreciate how radical this is, it is useful to recall what non-Abelian anyons are and why they are so interesting. Electrons and photons can be viewed as point-like excitations of the vacuum, or 'empty' space. When space is not empty but instead filled with matter — like that which we encounter in everyday life — the excitations can be quite different. The most surprising and, perhaps, important discovery in physics of the past 30 years is the existence of fractional quantum Hall states: under certain circumstances, the point-like excitations of a material may behave as though they are a fraction of an electron[2]. These excitations, called quasiparticles, are seen in extremely pure gallium–arsenide semiconductor devices in which the electrons are confined to move in a two-dimensional plane at temperatures close to absolute zero (below 1 kelvin) and in high magnetic fields (about 10 tesla, which is 3–10 times stronger than the magnetic field in a clinical magnetic resonance imaging machine). Their electrical charge is one-third that of an electron[3]. Even more shocking, these excitations are anyons[4] — neither bosons (such as photons) nor fermions (such as electrons).

Bosons obey Bose–Einstein statistics: when two identical bosons are exchanged, the quantum-mechanical wavefunction of the system is unchanged. Fermions obey Fermi–Dirac statistics: when two fermions are swapped, the wavefunction changes sign. In both cases, it is unimportant whether the exchange is clockwise or anticlockwise. Abelian anyons are particles that, when swapped, cause the wavefunction to be multiplied by a complex number $e^{i\theta}$, if they are exchanged in an anticlockwise fashion, and by $e^{-i\theta}$ if the exchange is clockwise[4]; θ is a parameter that varies from one type of anyon to another, and is 0 for bosons and π for fermions; i is the imaginary unit of complex numbers. In three dimensions, values of θ other than 0 and π are impossible because an anticlockwise exchange can be continuously deformed into a clockwise exchange. Thus, anyons can occur only in situations in which they are confined to move in a two-dimensional plane. Quasiparticles such as those that emerge in the fractional quantum Hall state in gallium–arsenide semiconductors are anyons with $\theta = \pi/3$. A phase of matter supporting such quasiparticles is called a topological phase of matter or a topologically ordered phase[5].

Non-Abelian anyons[6] are an exotic variant of anyons. In an ensemble of n such particles, there are many degenerate states (that is, states with the same energy), even when the positions of the particles are fixed. The degenerate states form a subspace of the space of all possible states of the system; the degenerate subspace has a dimension that is exponentially large in n. When two particles are swapped, the system can transform from one state into another in the degenerate subspace.

One particular type of non-Abelian anyon, called an Ising anyon, has an n-particle degenerate subspace of dimension $2^{[n/2]-1}$, where $[n/2]$ is the greatest integer less than or equal to $n/2$. Ising anyons have been conjectured[6] to exist in one of the fractional quantum Hall states, the '5/2 state', and experimental evidence has mounted in favour of the conjecture[7–9]. This has caused great excitement in the physics community. Non-Abelian anyons, if found, would not only be a type of particle never before seen in nature, but could also become the basis for a fault-tolerant quantum computer[6,10], a machine that could solve problems beyond the reach of today's computers. The transformations implemented by exchanging quasiparticles would be the logic 'gates' of such a computer, and the irrelevance of small deviations in the exchange route leads to built-in fault tolerance.

Previously, Fu and Kane proposed[11] a device in which a thin superconducting film is grown on top of a special kind of insulating material called a three-dimensional

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50 & 100 YEARS AGO

50 YEARS AGO

When the oldest British general genetical journal left the country with its editor, only one such journal remained ... Not only was it a time when the pressure on space for publication was rapidly increasing in all fields, but it was also a time when the belated recognition by British universities that the Americans were right in recognizing genetics as an essential part of biology was beginning to have results. One journal of general genetics could not be enough for a country such as Britain ... A new journal, *Genetical Research*, has now been launched to meet this need ... It includes a number of important papers among which should be noted especially Pritchard's discussion of recombination, and Waddington's experiments on canalizing selection. The contents of this first number augur well for the future of a journal that was badly needed, and is most welcome.

From *Nature* 2 April 1960.

100 YEARS AGO

The nature and arrangement of the bony armour of the dinosaur Stegosaurus are discussed by Dr. R. S. Lull in the March issue of the *American Journal of Science*. In the specimen restored by Marsh a number of small ossicles were found adhering to the under surface of the lower jaw, and these, in the opinion of Dr. Lull, not only formed a gular shield, but also extended over a considerable part of the body, as it is unreasonable to suppose that any portion of the skin of an armoured reptile would be unprotected. As regards the great vertical dorsal plates and caudal spines, the former of which Marsh regarded as forming a single series, it is practically certain that all were arranged in a double row ... [T]he terminal third of the tail apparently formed a flexible aggressive weapon, in which the laterally divergent spines were inserted in the muscles between the neural spine and the centrum.

From *Nature* 31 March 1910.

ASTROPHYSICS

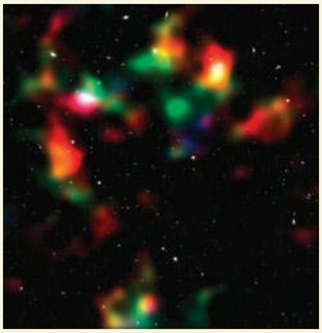
Cosmic acceleration confirmed

If you are still trying to get to grips with the idea that the Universe's expansion is speeding up, check out Schrabback and colleagues' scrutiny of the largest continuous area ever imaged with the Hubble Space Telescope — the COSMOS field (T. Schrabback *et al. Astron. Astrophys.* in the press; preprint available at arXiv:0911.0053). Their study is not only one of the most comprehensive analyses of the 'weak gravitational lensing' effect that is caused by large-scale structures in this distinctive part of the sky, it also uses this effect to provide independent evidence that the Universe is indeed expanding at an increasing rate.

Evidence for cosmic acceleration has been collected, for example, from analyses of stellar explosions known as type Ia supernovae and from studies of the cosmic microwave background radiation (relic radiation from the Big Bang). But when it comes to testing the Universe's dynamics and its fate, the more evidence the better. Different techniques are subject to different uncertainties, so any independent test is most welcome.

So what is the gravitational lensing effect? Picture a distant object in the sky. If there is nothing between the object and an observer on Earth, the observer sees one image of the object. However, if there is a mass concentration in the foreground — the lens — the object's light is deflected by the gravity of that mass. In the strong regime of this effect, the observer sees multiple images of the background object; in the weak mode, the observer sees a subtle distortion of the object's shape.

In their study, Schrabback *et al.* combine space-based measurements of the image distortions of more than 400,000 galaxies in the COSMOS field with ground-based estimates of their redshifts to infer the foreground mass — mostly invisible — in this field. They find that the statistical properties of the deduced mass distribution (pictured; white, cyan and green denote distances typically closer to Earth than do orange and red) can be best explained by a cosmological model in which the Universe's expansion is accelerating.



NASA, ESA, P. SIMON (UNIV. BONN) & T. SCHRABBACK (LEIDEN OBS.)

If that isn't enough, the authors also show that the way in which the weak lensing effect changes with redshift is in accord with what is expected from Einstein's theory of general relativity. What remains to be addressed is the nature of the Universe's most bizarre component, dark energy, which is thought to be driving the cosmic acceleration.

Ana Lopes

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topological insulator[12,13]. Despite sharing the word topological, a topological insulator is quite distinct from a topological phase. A topological insulator is a material whose bulk excitations are ordinary electrons, not anyons, but whose surface excitations are massless electrons. Fu and Kane showed[11] that at the interface between a superconductor and a topological insulator, a topological phase of matter forms that supports Ising anyons. This[11] and other theoretical work[14–16] emphasizes the point that non-Abelian anyons are not intrinsically limited to high magnetic fields, ultra-low temperatures and ultra-pure materials, but could occur, in principle, anywhere in nature. What's more, the proposed interfaces are technologically feasible, and experimental efforts are under way to try to realize them.

One puzzling feature of Fu and Kane's proposal is that it relies on a special property of a three-dimensional topological insulator, the existence of a single branch of massless electron states that occurs at a topological insulator's surface. There is no way for a purely two-dimensional system to have such states. Thus, their model is three-dimensional in an essential way. But one would expect a system supporting anyons to be two-dimensional. There is some tension between these two requirements, and it comes to the fore when we consider situations in which the superconductivity is not confined to a thin layer at the interface, as Teo and Kane consider in their study[1].

Suppose that the superconductivity occurs in a three-dimensional region. For instance, if the topological insulator is doped with copper[17], superconductivity can occur in the bulk of the material. Alternatively, the superconducting film could be thickened into a slab. Further suppose that we can dynamically change the geometry of the superconducting region by making the superconductivity penetrate deeper into the topological insulator or retreat farther away. Fu and Kane's non-Abelian anyons can thereby be moved around in three dimensions. But what happens to their non-Abelian anyonic properties?

To answer this question, Teo and Kane[1] constructed a theoretical model that generalizes Fu and Kane's set-up. In this model, excitations can move around in three dimensions and yet, when there are n such excitations, there are once again many degenerate states that form a $2^{[n/2]-1}$-dimensional subspace of the space of states of the system, and the effect of swapping excitations is the same as in the two-dimensional Ising-anyon case. This is a surprising result, because there can be no point-like excitations that are anyons, let alone non-Abelian anyons, in three dimensions.

The resolution of this apparent paradox is that Teo and Kane's excitations are not point-like. Therefore, the quasiparticles and their exchange routes alone are not a complete description of their model, as they would be for anyons in two dimensions. Instead, one must consider the quasiparticles' surroundings as well, such as the strength of the superconductivity in the region between the quasiparticles. As Teo and Kane's 'quasiparticles' are exchanged, they 'drag' their surroundings around with them. Thus, the set of operations characterizing their model is somewhat richer than simply the set of exchanges. For instance, Teo and Kane found some operations, which they call 'braidless operations', in which the quasiparticles are not moved at all; only their surroundings are changed. However, even accounting for the quasiparticles' surroundings is not enough to explain how non-Abelian anyons can arise.

In fact, the transformations generated by exchanges of Teo and Kane's quasiparticles have the following property (M. H. Freedman *et al.*, personal communication), which mathematicians call a projective representation: even though the braidless operations are physically equivalent regardless of the order in which they are done, their action on the system's wavefunction may differ by a minus sign, depending on the order. It is only through quantum-mechanical 'projection' that three-dimensional excitations — even those that are not point-like — can be an incarnation or avatar of two-dimensional non-Abelian anyons. Once again, quantum mechanics is making a mockery of our classical intuition and showing that any bizarre possibility that is mathematically allowed can and, apparently, will occur. ■

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