

Father of fractals

Benoit Mandelbrot is one of the twentieth century's best known mathematicians. So why, in the twilight of an extraordinary academic career, is he still angry with many of his colleagues? Jim Giles investigates.

“I never learnt the alphabet or times tables,” says Benoit Mandelbrot of his early education in Poland. Instead, it was visual memories that shaped his early years. He still remembers the geometric patterns that covered the rug on which he took his first steps. And when his uncle began teaching him from home, rote learning was never used. “All I did was play chess and read maps,” he says.

His family could not have known it, but they were laying the foundation for a tumultuous career in mathematics — one dominated by remarkable insights and no small amount of feuding. Mandelbrot's skills of visual analysis created new directions in his field: without him, the word ‘fractal’ would not exist and the usefulness of the bizarre shapes the term encompasses may not have been recognized. But that schooling also produced an academic nomad, long unable to find a home amid the territorial disciplines of maths and physics, and often angry because of it.

Mandelbrot, who celebrates his 80th birthday this month, is now widely revered. Yet a fury born of a sense of injustice still seems to simmer below the surface. Throughout his career, some researchers have questioned the importance of his achievements; Mandelbrot has responded with public explosions of anger and written rebuttals. At an age when others might spend their time accepting accolades or reminiscing with colleagues, Mandelbrot is still working — and angrily defending that work.

From the start, his academic career was unorthodox. After a 1952 PhD in mathematical linguistics, he delved into everything from the theory of financial markets to observations about the true lengths of coastlines. His methods were often unusual. Rather than prove theorems, as most mathematicians do, Mandelbrot used patterns and graphs to make conjectures about physical laws.

Take the paper that eventually put him on the map¹: a 1963 work on the fluctuations of cotton prices that he now describes as his “big bang”. At the time, most economists thought that such price changes followed a bell-shaped curve called a gaussian distribution. In effect, they assumed that markets

made frequent small jumps in value and that big fluctuations were rare.

Yet when Mandelbrot looked at charts of cotton prices, he noticed an odd phenomenon. If the label was removed from the time axis, it was impossible to tell whether the charts covered one week or one year; the pattern of peaks and troughs looked the same at each scale. Mandelbrot, then at IBM's Thomas J. Watson Research Center in Yorktown Heights, New York, knew that such ‘self-similar’ systems follow a different distribution, known as a power law. Crucially, big jumps in value are far more common in these distributions. Mandelbrot showed that the same was true for cotton prices — and in doing so he helped to change the way that stock market firms manage risk.

The wanderer

The 1963 paper is now considered a classic, yet its conclusions were disputed at the time. And with few other mathematicians using similar methods, Mandelbrot often had to publish his work in low-impact journals. By his own admission, his limited English and unusual notation also made his work difficult to read. “He almost has his own language,” says Kenneth Falconer, a mathematician at the University of St Andrews in Scotland.

So during the 1960s and 1970s, Mandelbrot remained an academic wanderer. Although IBM gave him a base, his diverse interests — from cosmology to geology — deterred university departments from offering him a permanent position. “I was everyone's favourite visiting professor,” he says. Stints at Yale and Princeton brought prestige, but not the widespread recognition he wanted.

All that changed in the 1980s. Realizing that most scientists were unaware of work he published in obscure journals, Mandelbrot began to bring together his ideas in a book. When the English version² appeared in 1982, the worlds of maths and physics took notice.

At the heart of his book are fractals — beautifully complex shapes that can be produced from simple mathematical equations. Self-similarity is just one of their remarkable



Shaping up: Benoit Mandelbrot (right) changed the face of maths with his work on fractals, such as the set that bears his name (above).

properties. Take the best-known fractal, since dubbed the Mandelbrot set (see above). Zooming into the sea-horse shapes at the edges of the set reveals more miniature Mandelbrot sets. Jump down a magnification level and the same patterns emerge again.

Largely thanks to Mandelbrot, this property is now known to be widespread in the natural world. Coastlines contain similar patterns when viewed on maps of very different scales. A crack snaking along a metal surface can be fractal, as are the branches of human arteries. Even cauliflowers show some degree of self-similarity.

Repeat performance

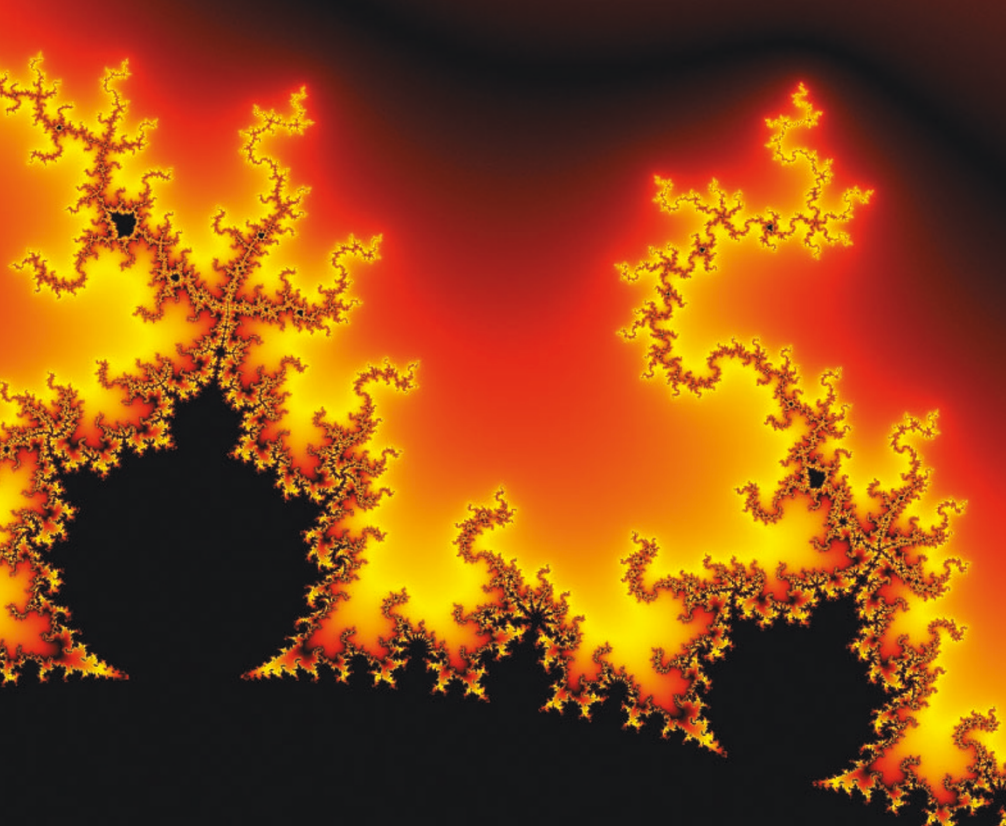
Mandelbrot had wanted recognition, and here it was. Physicists working on topics from cloud formation to metallurgy began using fractals in their work. His book quickly generated hundreds, and eventually thousands, of citations. And stunning plots of fractals, made possible by new computer technology, featured in everything from greetings cards to *Star Trek* movies.

But it is around this time that Mandelbrot developed a reputation for being confrontational. As so often happens in academia, questions of precedence were central. No one denies that Mandelbrot single-handedly put fractals on the scientific map. But in many of the examples he cited in his book, other researchers covered at least part of the ground before him.

Similar studies to Mandelbrot's work on power laws in economics, for example, were written up by Italian economist Vilfredo Pareto at the beginning of the last century³.

“Mandelbrot has changed the questions mathematicians ask.”

— Ian Stewart



Lungs are just one example of natural structures that show the self-repeating nature of fractals.

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And the famous Mandelbrot set may not have been first plotted by Mandelbrot — Robert Brooks and Peter Matelski, mathematicians then at the University of Maryland and the State University of New York at Stony Brook, respectively, did so at about the same time⁴.

When this latter case was pointed out in a book review in 1989, Mandelbrot penned a rebuttal claiming that Brooks and Matelski had plotted a “crude version” of the set and had given “no thought” to its special nature⁵. And that is far from the only time he has made such comments about other researchers’ contributions. “He can get up in the middle of other peoples’ lectures and claim to have done the same work years ago,” says one mathematician who has worked on fractals. “He can be really quite aggressive.”

Gene Stanley, a theoretical physicist at Boston University, Massachusetts, has exper-

rienced some of that aggression. An argument between Stanley and Mandelbrot brought a 1996 conference on fractals, held at New England College in Henniker, New Hampshire, to a temporary standstill. Researchers who were present say that the row, which centred on a dispute over who would chair the next conference, degenerated into an all-out shouting match. One delegate was prompted to cry out, tongue-in-cheek: “Why can’t we all just get along?” Most academics can recall similar feuds in their own fields, albeit probably less fiercely fought ones. But Mandelbrot, who has been at Yale since 1999, has been accused of more unusual behaviour in relation to his *Selecta* books — a collection of reprints of Mandelbrot’s original papers. When the books are compared to those originals, differences emerge.

In his “big bang” paper on stock-market statistics, for example, many references to Pareto have been removed from the reprint⁶. The “Paretian hypothesis”, which refers to Mandelbrot’s new way of thinking about the

statistics of markets, becomes the “L-stable hypothesis”, after the French mathematician Paul Lévy, who worked on the same problem. Another theorem, the Pareto–Doebelin–Gnedenko conditions, is in some places renamed the Doebelin–Gnedenko conditions. And one reprint, contributed by Eugene Fama, an economist at the University of Chicago in Illinois, has had its 1963 title changed from “Mandelbrot and the stable paretian hypothesis” to “Mandelbrot on price variation”⁶.

Time for heroes

So has Mandelbrot attempted to write Pareto out of his papers? One researcher, who asked not to be named, has made just this accusation to Springer, the Berlin-based publishers of the *Selecta* books. But Mandelbrot denies that this is the case, saying that Pareto, whose achievements he acknowledges at length in his latest book⁷, is one of his heroes. He adds that the changes were made so that the terminology was consistent. Fama was not aware of the changes to his paper, but backed Mandelbrot’s explanation when *Nature* made him aware of the editing.

Despite these controversies, Mandelbrot’s colleagues are generally more interested in what he has achieved than how he may have behaved. Some feel fractals are over-hyped, but the majority say that their introduction has changed the way physicists think about natural phenomena — for which they owe Mandelbrot a considerable debt.

Even researchers who have been the subject of Mandelbrot’s attacks praise his contributions to maths. And several physicists told *Nature* that they altered the focus of their research after hearing him speak. Ian Stewart, a mathematician at the University of Warwick, UK, sums it up: “He has changed the questions we ask.”

Hearing such praise earlier in his career might have mellowed Mandelbrot. But ask him today whether he has received the recognition he deserves, and he delivers a typically combative reply. He says that many new books on finance omit to mention his work, and colleagues still pursue grudges against him. Is this normal behaviour for a distinguished mathematician in the twilight of his career? Perhaps not. But, as Falconer points out, Mandelbrot is not a normal mathematician. ■

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