Secrets of successful stone-skipping

Hitting the water at a magic angle gives top performance in a time-honoured pastime.

Skipping stones across water has been a popular pastime for thousands of years — the rules of the game have remained unchanged since the time of the ancient Greeks[1] — and the world record, set by J. Coleman-McGhee in 1992, is believed to be 38 rebounds[2]. Following earlier attempts[3–6] to analyse the physics of this ancestral human activity, we focus here on the crucial moment in stone skipping: when the stone bounces on the water's surface. By monitoring the collision of a spinning disc with water, we have discovered that an angle of about 20° between the stone and the water's surface is optimal with respect to the throwing conditions and yields the maximum possible number of bounces.

A stone-skipping throw involves four parameters (Fig. 1): U and Ω are the translational and spin velocities, respectively, α is the 'attack' angle of the stone in relation to the water's surface, and β is the impact angle of the translational velocity. Our experimental set-up is designed to control for each of these parameters independently. Collision sequences were recorded using a high-speed video camera (Fig. 1a), allowing factors such as collision time, change in orientation of the stone, and the shape of the liquid cavity to be determined.

Regarding the role of spin velocity, rotation is found to stabilize the stone (as expected[4]) owing to a gyroscopic effect. We focus our analysis on the high-spin velocity limit, at which α remains constant along the impact. A dynamic phase diagram can then be constructed using the three remaining control parameters (U, α and β), highlighting the necessary conditions for a successful bounce (the 'skipping-stone' domain). Cross-sections in the $\{U, \alpha\}$ and $\{\alpha, \beta\}$ variables are shown in Fig. 1b, c.

The value $\alpha \approx 20°$ is unexpectedly found to play a specific role in this phase diagram: the lowest velocity for a rebound, U_{min}, reaches a minimum for $\alpha \approx 20°$, whereas the maximal successful domain in the impact angle, β, is also achieved for this specific value of α. It can be seen that no rebound is possible for impact angles that are larger than 45°. In a quantitative analysis of the collision, experimental measurements for the collision time, τ, are shown in Fig. 1d: the main feature on this plot is the existence of a minimal value of the collision time, τ_{min}, which is obtained for $\alpha \approx \alpha_{min} \approx 20°$ for all velocities.

This minimal collision time is found to obey a simple scaling when the velocity, U, radius, R, and thickness, e, of the stone are varied: namely, $\tau_{min} \propto \sqrt{(eR)}/U$ (for fixed α and β). This scaling is inferred from a simple dimensional analysis. As the lift force, F_{lift}, is the key point in the rebounding process, a collision time can be constructed from the dynamical law as $\tau \propto \sqrt{(mR/F_{lift})}$, where m is the mass of the stone.

For the velocities under consideration, the lift force is expected to scale as $F_{lift} \propto \rho_w S_{wetted} U^2$, where ρ_w is the mass density of water and the wetted area scales as $S_{wetted} \propto \pi R^2$ (refs 7–9). Using $m = \rho_s e \pi R^2$, where ρ_s is the mass density of the stone, then $\tau \propto \sqrt{(\rho_s/\rho_w)}\sqrt{(eR)}/U$, as measured experimentally. This simple argument, although informative, does not help in understanding the existence of a minimum collision time, a behaviour that requires a more detailed description of the time-dependent hydrodynamic flow around the stone.

The 'magic' angle $\alpha \approx 20°$ is accordingly expected to maximize the number of bounces because the amount of energy dissipated during a collision is directly proportional to the collision time[4]. The ancient art of stone-skipping may therefore benefit from modern scientific insight.

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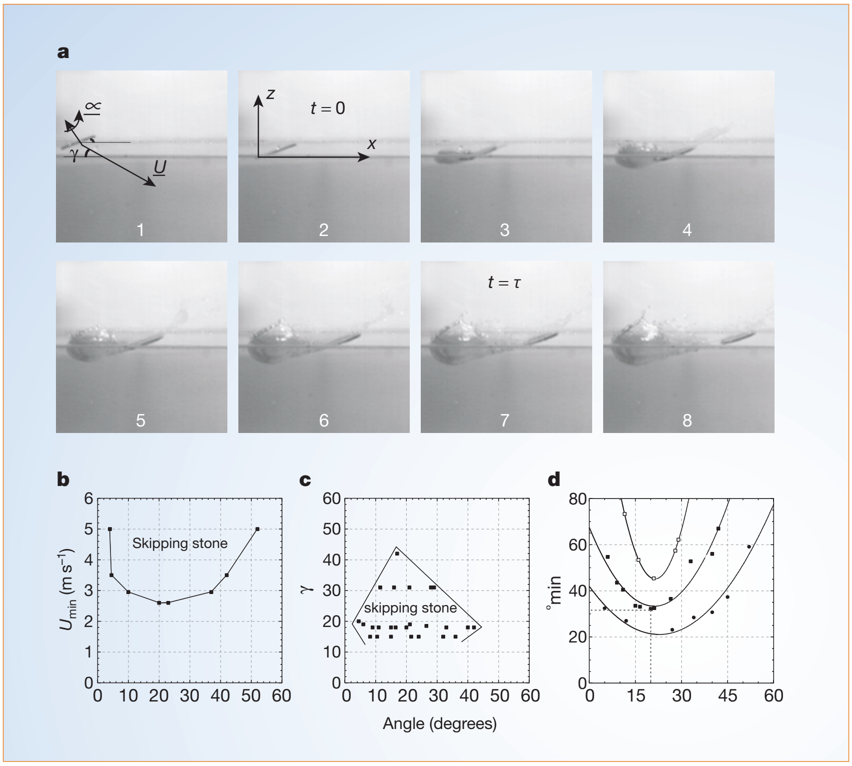

Figure 1 Analysis of stone-skipping. **a,** Chronological photography of a skipping stone, using an aluminium disc as a model stone (radius, $R = 2.5$ cm; thickness, $e = 2.75$ mm; translation velocity, $U = 3.5$ m s^{-1}; angular velocity, $\Omega = 65$ rotations s^{-1}; attack angle, $\alpha = 20°$; and trajectory angle, $\beta = 20°$). Time increases from left to right and from top to bottom, with time step $\Delta t = 6.5$ ms. **b–d,** Definition of the skipping-stone domain (for the model stone in **a**): **b,** domain of the skipping stone in the $\{\alpha, U_{min}\}$ plane for a fixed $\beta = 20°$; **c,** domain of the skipping stone in the $\{\alpha, \beta\}$ plane for a fixed $U = 3.5$ m s^{-1}. The domain $\beta < 15°$ was not attainable in our experimental set-up. **d,** Evolution of the collision time, τ, as a function of the attack angle, α, under various conditions: filled squares, $U = 3.5$ m s^{-1}, $\beta = 20°$; open squares, $U = 3.5$ m s^{-1}, $\beta = 30°$; circles, $U = 5.0$ m s^{-1}, $\beta = 20°$. The density ratio of the aluminium stone (s) to water (w) was $\rho_s/\rho_w \approx 2.7$ in all experiments.

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