Pal *et al.* used a data set of 4,273 deletion effects, each of which is based on two replicate measurements; we used a new data set of 5,937 deletion effects, 4,744 of which are based on more than 10 replicate measurements (data by courtesy of J. Kumm and G. Giaever). Calculating Kendall's correlation coefficient[8] between deletion effect and evolutionary distance, we observed a highly significant negative relationship ($-0.26 < \tau < -0.17$, $P < 0.0001$ in all evolutionary comparisons; manuscript in preparation). To control for different levels of gene expression, we used recent expression data[9] that were not measured in an aneuploid strain of yeast[10]. Calculating Kendall's partial correlation coefficient[8] ($-0.18 < \tau < -0.16$, $P < 0.0001$ in all evolutionary comparisons), we find that the relationship between protein dispensability and evolutionary rate remains highly significant, even when controlling for gene-expression levels.

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COMMUNICATIONS ARISING

Cosmology

Do black holes constrain varying constants?

Tentative observations of shifted spectral lines in distant quasars[1,2] have rekindled interest in Dirac's old idea[3,4] that the fundamental 'constants' of physics may vary over time. Davies *et al.* have argued that black-hole thermodynamics favours theories in which the speed of light, *c*, decreases, and does not favour those in which the fundamental electronic charge, *e*, increases[5]. Here we show, however, that when the entire thermal environment of a black hole is considered, no such conclusion can be drawn. Although black-hole features such as mass quantization may still constrain models with varying 'constants'[6], thermodynamics probably cannot.

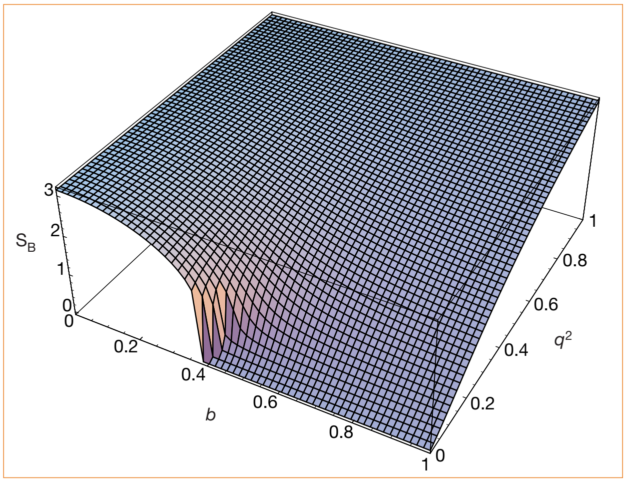

Figure 1 Entropy, S_B, of a black hole and its surroundings as a function of charge, q^2, and inverse temperature, *b*. For any fixed temperature, an increase in the fine-structure constant increases q^2 and therefore S_B.

The observations of refs 1, 2 suggest that the fine-structure constant, α, may have been slightly smaller in the early Universe. As $\alpha = e^2/\hbar c$ depends on *e*, *c* and Planck's constant, $\hbar$, it is natural to ask which of these values varies. Davies *et al.* offer an ingenious argument. A black hole with mass *M* and charge $Q = ne$ has an entropy[7] $S = \pi G/\hbar c[M + (M^2 - n^2e^2/G)^{1/2}]^2$. Evidently, an increase in *e* will reduce the entropy, violating the generalized second law of thermodynamics, whereas a decrease in $\hbar$ or *c* will increase the entropy. This argument involves assumptions that may not be valid for all models[6,8,9], but it offers an interesting starting point.

As Davies *et al.* note, however, such an argument should consider not just the black hole, but also its surroundings. An isolated black hole is never in thermal equilibrium: it decays by Hawking radiation and, if it is charged, by spontaneous emission of charged particles[10]. These processes reduce *S*, but do not violate the second law of thermodynamics because there is a compensating increase in the entropy of the environment.

To investigate the thermodynamics of varying 'constants', one should study a black hole that is in equilibrium with its environment. This can be done by considering a black hole in a 'box' of radius r_B, with fixed boundary temperature *T* and charge *Q* (the canonical ensemble) or electrostatic potential ϕ (the grand canonical ensemble). Note that r_B can be altered only by doing work on the system.

In the canonical ensemble, the entropy is given by $S = \pi r_B^2 x^2$, where *x* is determined by the seventh-order equation[11] $x^5(x - q^2)(x - 1) + b^2(x^2 - q^2)^2 = 0$, where $q = \sqrt{G}Q/r_B c^2$ and $b = \hbar c/4\pi r_B kT$. Figure 1 shows a plot of S/r_B^2 against q^2 and *b*. It is apparent — and may be confirmed numerically — that the entropy increases with increasing α. For the grand canonical ensemble, exact analytical results lead to the same conclusion. Black-hole thermodynamics thus militates against models in which the fundamental charge, *e*, decreases, but places no restriction on increasing *e*.

To compare this result with that of Davies *et al.*, note first that the Hawking temperature of a charged black hole decreases with increasing *e*. A black hole will thus cool below the ambient temperature of the heat bath and will absorb heat, thereby increasing its mass. According to the first law of thermodynamics, the net change in entropy is $dS = 1/T(dE - \phi dQ)$, and it may be verified that the increase in the energy, *E*, dominates.

Of course, such thermodynamic arguments only describe relationships among equilibria, and not the transitions between equilibria. A more detailed analysis, however, requires an explicit, dynamic model. In particular, any theory with a variable fine-structure constant necessarily contains a new scalar field, α itself, the entropy of which cannot be neglected during dynamic processes in which α is changing. Jacobson (personal communication) has suggested that a suitable dynamic version of the second law of thermodynamics[12] will ensure that entropy increases during such a process. Black-hole thermodynamics is therefore insufficient to constrain theories in which α increases.

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