$$P(b|a)\,\mathrm{d}b = \left(\int P(a|b')P(b')\mathrm{d}b'\right)^{-1} P(a|b)\,P(b)\,\mathrm{d}b$$

for the second factor in the integral, we find

$$P(R=r|T_{\mathrm{past}}=t_{\mathrm{p}})\mathrm{d}r=\left(\int_{t_{\mathrm{p}}}^{\infty} P(T=t)t^{-1}\mathrm{d}t\right)^{-1} P(T=(r+1)t_{\mathrm{p}})\,(r+1)^{-1}\,\mathrm{d}r \qquad (6)$$

Contrary to equation (2), this depends on the *a priori* distribution $P(T)$ (this is the deeper meaning of Bayes' theorem: you cannot invert a condition probability distribution unless you have an *a priori* probability). It can easily be proved that there is no normalized probability distribution that makes equations (2) and (6) match.

Now consider, as a special case, a constant extinction rate λ_0. The distribution of intervals of potential observation is exponential, and for an actually observed interval

$$P(T=t)\,\mathrm{d}t=\lambda_0^{\,2}\,t\exp(-\lambda_0 t)\,\mathrm{d}t \qquad (7)$$

where an 'anthropic' factor of t accounts for the higher probability of observing a longer interval. Inserting equation (7) into equation (6) yields

$$P(R=r|T_{\mathrm{past}}=t_{\mathrm{p}})\,\mathrm{d}r =\lambda_0 t_{\mathrm{p}}\exp(-\lambda_0 t)\,\mathrm{d}r \qquad (8)$$

For the variable T_{future},

$$P(T_{\mathrm{future}}=t_{\mathrm{f}}|T_{\mathrm{past}}=t_{\mathrm{p}})\,\mathrm{d}t_{\mathrm{f}} =\lambda_0\exp(-\lambda_0 t_{\mathrm{f}})\,\mathrm{d}t_{\mathrm{f}} \qquad (9)$$

Hence the future behaviour of a system with exponential decay does not depend on its past, but only on its intrinsic parameter λ_0. This result is well known from particle physics.

To summarize, Gott first treated the past and future of a system in a symmetric manner. Later, he switched over to a derivation of the future from the past. Thus he changed the conditions under which his equations were derived. By keeping his equations unchanged he obtained some surprising results. Here I have tried to show that these results are wrong. Gott's prediction of the future from nothing but the past does not reflect the reality we are all living in, including intelligent observers.

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GOTT REPLIES — Goodman fails to note that while Keynes did show some examples where he thought it was inappropriate to apply the principle of indifference, he also gave examples where even he thought the principle of indifference could be used safely. He said[1]: "Suppose, for instance, that a point lies on a line of length *m.l.*, we may write the alternative 'the interval of length l on which the point lies is the xth interval of that length as we move along the line from left to right' = $\Phi(x)$; and the Principle of Indifference can then be applied safely to the m alternatives $\Phi(1)$, $\Phi(2)$. . . $\Phi(m)$, the number m increasing as the length of l of the intervals is diminished. There is no reason why l should not be of any definite length however small." My application[2], which involves dividing the observed interval into 40 equivalent subintervals of equal length, is similarly conservative. Statisticians have long argued about which axioms to adopt in this context. According to the Copernican principle, you expect your observation of whatever you are measuring to be located randomly on the chronological list of such observations.

Buch would like to know the expected (posterior) probability distribution for the total longevity L of our species after we observe that our past age is $t_{\mathrm{p}} = 200{,}000$ yr, using Bayesian statistics[3].

$$P_{\mathrm{posterior}}(L)\,\mathrm{d}L \propto \text{Likelihood}\,(t_{\mathrm{p}} = 200{,}000\,\mathrm{yr}) \times P_{\mathrm{prior}}(L)\,\mathrm{d}L \qquad (1)$$

Often P_{prior} is based on some real data, but as we have no data on the longevities of other intelligent species we are not free to adopt just any prior but are commanded to adopt an appropriate vague Bayesian prior. We know that L is positive but we do not know anything about its magnitude. In such a situation we should adopt as the appropriate vague Bayesian prior[3]

$$P_{\mathrm{prior}}(L)\mathrm{d}L \propto \mathrm{d}\ln L \propto L^{-1}\,\mathrm{d}L \qquad (2)$$

This means that there is a *priori* an equal chance of L being in any order of magnitude interval. The likelihood of observing $t_{\mathrm{p}} = 200{,}000$ yr is 0 if $L < 200{,}000$ years, so

$$P_{\mathrm{posterior}}(L)\mathrm{d}L = 0 \text{ if } L < 200{,}000\,\mathrm{yr} \qquad (3)$$

And the likelihood of observing $t_{\mathrm{p}} = 200{,}000$ yr if $L > 200{,}000$ yr is proportional to L^{-1}, so

$$P_{\mathrm{posterior}}(L)\mathrm{d}L \propto L^{-1} \times L^{-1}\,\mathrm{d}L \propto L^{-2}\,\mathrm{d}L \text{ if } L > 200{,}000\,\mathrm{yr} \qquad (4)$$

Pick a longevity $L_{\mathrm{o}} > 200{,}000$ yr. Intregrating $P_{\mathrm{posterior}}(L)\mathrm{d}L$ from L_{o} to infinity and normalizing, we find that

$$P_{\mathrm{posterior}}(L > L_{\mathrm{o}}) = (200{,}000\,\mathrm{yr}/L_{\mathrm{o}}) \text{ for } L_{\mathrm{o}} > 200{,}000\,\mathrm{yr} \qquad (5)$$

Thus, $P_{\mathrm{posterior}}(L > 200{,}000\,\mathrm{yr}) = 1$ and $P_{\mathrm{posterior}}(L > 400{,}000\,\mathrm{yr}) = 0.5$, and so forth. Define Y by the equation $L_{\mathrm{o}} = (Y + 1)\,t_{\mathrm{p}} = (Y + 1) \times 200{,}000$ yr. Because $L = t_{\mathrm{f}} + t_{\mathrm{p}} = [(t_{\mathrm{f}}/t_{\mathrm{p}}) + 1]\,t_{\mathrm{p}}$, $L > L_{\mathrm{o}}$ is equivalent to $(t_{\mathrm{f}}/t_{\mathrm{p}}) > Y$ so

$$P_{\mathrm{posterior}}([t_{\mathrm{f}}/t_{\mathrm{p}}] > Y) = 1/(Y + 1) \qquad (6)$$

which is identical to the distribution found in the delta t argument[2]. It is not necessary that the vague Bayesian prior be normalizable, just that the posterior probability is[3]. If one still demanded a normalized prior one could establish lower and upper cutoffs of 10 yr and $10^{5{,}000{,}000}$ yr, respectively, corresponding to the longevity of an individual and of our inflationary domain in the Linde cosmology, without significantly altering the results.

Buch's second question concerns the problem of a constant extinction rate λ_0. He is right that if one knew λ_0, one would have the perfect prior and would be able to determine the probability distribution for t_{future} $P(t_{\mathrm{f}})\mathrm{d}t_{\mathrm{f}} = \lambda_0 \exp(-\lambda_0 t_{\mathrm{f}})\,\mathrm{d}t_{\mathrm{f}}$, independently of the particular observed value of t_{past} in this case. But we do not know λ_0. We may, however, learn something about it by inspecting t_{past}. Because the dispersions in the relevant galactic and stellar evolution timescales are measured in billions of years and we originated approximately midway in the main sequence lifetime of the Sun, the rate at which intelligent species have been forming in the Universe over the past millions of years has been approximately constant[4]. Thus, $P(t_{\mathrm{p}})\,\mathrm{d}t_{\mathrm{p}} = \lambda_0 \exp(-\lambda_0 t_{\mathrm{p}})\,\mathrm{d}t_{\mathrm{p}}$, because, of all those intelligent species formed at an epoch t_{p} ago, only a fraction $f = \exp(-\lambda_0 t_{\mathrm{p}})$ are still around today[2]. Thus $\langle t_{\mathrm{p}} \rangle = \lambda_0^{-1}$ and there is a 95% chance that $\lambda_0^{-1} < 19.5 t_{\mathrm{p}}$. Also $P(t_{\mathrm{p}}, t_{\mathrm{f}})\,\mathrm{d}t_{\mathrm{p}}\mathrm{d}t_{\mathrm{f}} = \lambda_0^{2} \exp(-\lambda_0[t_{\mathrm{p}} + t_{\mathrm{f}}])\,\mathrm{d}t_{\mathrm{p}}\mathrm{d}t_{\mathrm{f}}$, giving $P(L)\mathrm{d}L = \lambda_0^{2} L \exp(-\lambda_0 L)\,\mathrm{d}L$ (where $L = t_{\mathrm{p}} + t_{\mathrm{f}}$) and the distribution in eq. (6).

Mackay misquotes the delta t argument. The correct statement is that there is a 50% chance that the future longevity lies between ⅓ and 3 times the past longevity and a 95% chance that the future longevity lies between 1/39 and 39 times the past longevity as Buch correctly noted in his equation. Because Mackay started the program and picked the time interval after which to look, it could be argued that it might be special because he may have become impatient only when it had run longer than others, or he may have consciously picked a wait time shorter than most programs. Here's an example that should work, however. If Mackay happened to have a program running when he first heard of my paper, then the delta t argument should tell him how much longer after that the program would run: if it had been running for 1 hour, he could say that there was a 50% chance that it would continue to run for more than 20 minutes but less than 3 hours.

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