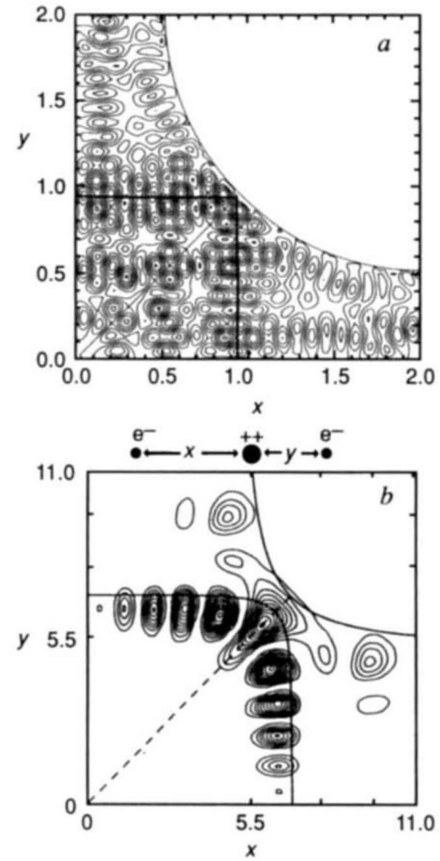

FIG. 3 *a*, A topographic map of a numerically calculated eigenstate in the square Sinai billiard with a scar along the right-angle periodic orbit. Because this trajectory collides twice with the circular scatterer each period, it is more unstable than the diagonal periodic orbits and the associated scars are less distinct than the diagonal scar shown in Fig. 2. (The computer code for this calculation was provided by E. Heller.) *b*, A topographic map of a numerically calculated wavefunction for a doubly excited state with quantum numbers $n = 6$, $L = 0$. The wavefunction is clearly peaked along the right-angle periodic orbit.

cally chaotic helium atom, reported by Ezra *et al.*[3]. The inability of the Bohr–Sommerfeld semiclassical quantization prescription to calculate the energy levels for the helium atom led to the downfall of the old quantum theory in the 1920s. In this, the invariant classical actions were associated with integer quantum numbers multiplied by Planck's constant. But nonintegrable (chaotic) classical systems may have no good action variables to quantize, as recognized in a little known paper[7] by Einstein in 1917.

The classical description of the electrons in a helium atom is a nonintegrable three-body Coulomb problem, so that the old theory failed to determine the energy levels. The new evidence of the role of unstable periodic orbits in such situations suggests a way forward — especially for the doubly excited states that are difficult to describe even with modern quantum mechanics, because of strong electron–electron correlations.

Ezra *et al.* consider a collinear model of the helium atom, with the electrons on either side of the nucleus. The classical configuration space for this simplified model consists of the two-dimensional plane spanned by the distances x and y of the two electrons from the nucleus. The classical motion for this system is fully chaotic and the quantum wavefunctions may be expected to exhibit scars. In fact, a careful implementation of Gutzwiller's semiclassical formula provides excellent predictions for the quantum-mechanical energy levels. The ground-state energy is given to within one per cent and the energy levels for doubly excited states with principal quantum numbers as large eight or nine are accurate to four significant figures.

The connection of these results with scarred wavefunctions is illustrated in Fig. 3*b*, a topographic map of a numerically calculated wavefunction for an $n =$ 6, doubly excited state of helium. This wavefunction is clearly peaked along the path of the classical periodic orbit that is analogous to the right-angle orbit in the Sinai billiard in Fig. 3*a*. In helium this unstable periodic orbit corresponds to an asymmetric stretch of the two electrons. The configuration space in both examples is bounded along the x and y axes (the electrons cannot pass through the nucleus) and to the right by a semicircular potential curve: in the billiard this boundary is a hard wall, in the helium atom it is a smooth barrier determined by the Coulomb attaction of the nucleus.

Because of the similarities, the classical periodic orbits and many of the wavefunctions share common features. But there are also some interesting differences with important physical implications. For example, the doubly excited helium atom is unbounded along the x and y directions, and it is possible for one of the electrons to fall into the nucleus while the other flies off to infinity. In fact, typical chaotic classical trajectories always ionize. Because of this autoionization, the doubly excited states are technically resonances rather than eigenstates, and the quantization of the helium atom is also closely related to the quantum description of chaotic scattering of ballistic electrons in mesoscopic wires with right-angle bends[8]. □

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DAEDALUS

Digital correctness

THE world of extreme hi-fi is riddled with strange postures and pretences. The real aficionado argues the rival merits of amplifiers and loudspeakers in terms of subtleties such as poise, texture and articulation, far beyond mere physical measurement. Fads and fashions abound. For many years valve amplifiers have been held to be purer than solid-state ones; recently even negative feedback has been denounced as too coarse for the truly dedicated; any day now Daedalus expects the original phonograph to be reinstated as the only acoustically correct mode of sound reproduction. And yet, with the coming of digital sound encoding, any sound-reproduction system can be evaluated quite definitively. Daedalus is now doing it.

He simply takes the sound system under test and fits its loudspeaker with a digital position sensor. This generates a stream of binary numbers that encodes its vibration exactly. He then compares this output stream of numbers with the original input stream from the compact disc, digital tape or whatever. This is easily done by feeding both streams simultaneously into a computer. If the sound system is perfect, its output stream of numbers is identical to its input; if not, the deviations show wherein the imperfection lies. Subjective notions of timbre, coherence, coloration and so on, are quite irrelevant.

And this, says Daedalus, is the route to perfect hi-fi. His cunning trick is to feed the input stream to the computer directly, but to delay it slightly on its way to the sound system. The computer is therefore ahead of the sound system, and knows what should be coming next. If the output stream is beginning to deviate from perfection, the computer is in a position to put it right.

In its few milliseconds of running look-ahead time, the computer evaluates the current bit-stream from the speaker. From its mathematical model of the system, it makes an informed guess as to what the next few binary numbers will be. Knowing what they should be, it calculates a corrective 'tweak signal', and feeds it into the amplifier to bring the loudspeaker back on track. The computer's model, derived by continuous evaluation of the sound system's deviations, is steadily updated and optimized. By learning on the job, so to speak, it can work with any sound system at all.

Thus the cheapest amplifier, driving the crudest loudspeaker, can be upgraded to total perfection by one simple accessory. The refined agony of hi-fi comparisons will become a thing of the past. All systems will be perfect, and will sound perfectly alike. DAVID JONES