MATHEMATICS

Circularly covering clathrin

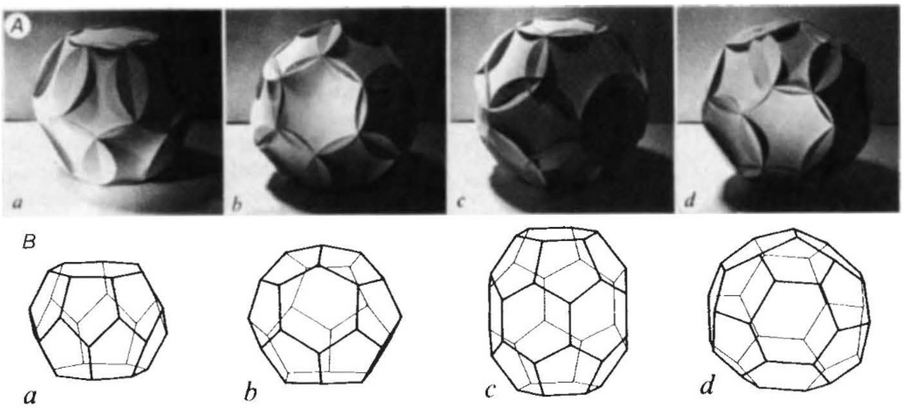
Ian Stewart

MATHEMATICIANS often take inspiration from nature, the latest case in point being the problem of covering a sphere by identical circular disks. In *Journal of Molecular Biology*[1], with details to appear elsewhere[2], Tibor Tarnai announces that several arrangements widely conjectured to be the best possible can in fact be improved. The improvements are derived from the observed form of biological structures known as 'coated vesicles'.

First, the covering problem. An alien race, the Yuppi, are installing a network of cellular telephones on a distant world. The planet has no open bodies of water, and may be considered a perfect sphere. A certain number of radio stations must be placed on the planet so that every point is within receiving distance of at least one station. Because the Yuppi place great store on cost-effectiveness, and stronger radio receivers are more expensive, they wish to minimize that distance. In more orthodox language, if a sphere is to be covered by a number of identical circular disks, how should they be placed to make their common radius as small as possible?

For small numbers of disks the problem is easily solved. For example one disk can be placed anywhere, and its radius should be half the sphere's circumference, so that it wraps itself round the entire sphere. Two disks should be diametrically opposed, like north and south hemispheres. Symmetric arrangements tend to occur; thus four disks should be placed at the vertices of a regular tetrahedron, and twelve at the vertices of a regular dodecahedron.

The best possible solutions are known only when the number of disks is at most 14 (ref. 3). For greater numbers, conjectured optimal coverings have been published only for 16 disks[3] and 20 disks[4] (see figure, *Aa* and *Ac*).



A, Cardboard models of the covering of a sphere by disks of equal size. *a* and *b*, covering by 16 disks; *c* and *d*, covering by 20 disks. *a* and *c* are the conjectured optimal solutions for 16 disks[3] and 20 disks[4]; *b* and *d* are the better solutions offered by Tarnai's study of clathrin cages. *B*, Stick polyhedra of equal edges, each of which corresponds topologically to the similarly labelled photograph in *A*. *a*, 'Heptagonal drum' of 2 individual heptagons and 2 rings of 7 pentagons. *b*, Cage polyhedron of 12 pentagons and 4 hexagons, tetrahedrally arranged. *c*, The 'hexagonal barrel', a cage polyhedron of 2 rings of 6 pentagons, 1 ring of 6 hexagons and 2 individual hexagons. *d*, The 'tennis ball', a cage polyhedron of 2 curved strips of 4 hexagons, and 1 space loop of 12 pentagons analogous to the seam of a tennis ball. (Reproduced from ref. 1.)

Enter the coated vesicles. These have an external lattice structure known as clathrin. R. A. Crowther and colleagues[5] noticed that several common clathrin cages possess 12 pentagonal faces, the rest being hexagonal. The same thing happens in the sphere-covering problem. The arrangement of the disks can be represented by a polyhedron whose vertices are placed at, or radially in line with, their centres; and when there are more than 14 disks, the best known coverings all correspond to polyhedra with 12 pentagonal faces, the remainder being hexagonal. In view of this, Crowther *et al.* suggested that clathrin cages might provide optimal solutions to the sphere-covering problem. For example, the conjectured solution for 20 disks is identical to the clathrin cage known as the 'hexagonal barrel' (see figure, *Bc*).

Tarnai investigates this suggestion in three cases: 16, 20 and 32 disks. For 16 and 20 disks it turns out that the clathrin cages do not correspond to the arrangements conjectured to be optimal by mathematicians. Instead, they provide *better* solutions to the sphere-covering problem (see figure, *A*). Thus for 20 disks the improved solution corresponds not to the hexagonal barrel, but to a clathrin cage known as the 'tennis ball'. The angular radius of the circular disks improves from the conjectured 30.5° to 29.6°. For 16 disks the improvement is from 33.5° to 32.9°. But when the number of disks is 32 the clathrin structure does agree with the conjectured optimal solution — the truncated icosahedron or 'soccer ball'.

Although mathematicians often take inspiration from nature, usually they expect it to provide problems, not answers. Not for the first time, nature has beaten them at their own game. □

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PLANETARY ATMOSPHERES

Jupiter's stratosphere mapped

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OUR knowledge of Jupiter (and the other outer planets) is largely confined to a two-dimensional view at the height of the visible clouds. Motions, patterns, coloration and chemical composition are richly variable at this level, but an understanding of the system will require information from the third dimension. In *Science*, Orton and 14 colleagues[1] present maps of thermal emission from stratospheric levels which will help alleviate the difficulty. The planet was monitored for a decade, a full jovian annual cycle. In spite of the planet's small (3°) obliquity, seasonal effects appear. In addition, there is exciting evidence for variability on a short timescale (only months) which probably indicates upward propagation of disturbances from within or below the visible clouds.

The maps were made at a wavelength of 7.8 μm at the NASA Infrared Telescope Facility on Mauna Kea. At this wavelength, emission is from the middle stratosphere in thermodynamic equilibrium, about 60 km above the visible clouds (Fig. 1). The emitting gas is methane, which is uniformly mixed in the atmosphere of Jupiter. Variations in emission are due to temperature variations in the atmosphere (composition 90% hydrogen, 10% helium and 0.25% methane by number). Horizontal temperature gradients can be produced dynamically by disturbances that penetrate upward from below or by local *in situ* variations in solar heating due to absorption by variable hazes or trace gases. In the latter case the thermal response is governed by a radiative time constant of several years. In the former case fluctuations can be much more rapid, limited only by the wave propagation characteristics of the atmosphere and the rapidity of the excitation processes, which could be as vigorous as terrestrial thunderstorms if they originate in the jovian water clouds, which are very likely to have latent heat effects of the same order as on Earth.

The new data seem to show new phe-